Galileo, the Telescope, and the Science of Optics in the Sixteenth Century

A Case Study of Instrumental Practice in Art and Science

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I. Galileo's Telescope in the History of Science

1. The Telescope Was Never Invented

The telescope was never invented. Only more than a year after Galileo published his first telescopic discoveries in his 'Sidereus Nuncius', on a banquet on 14 April 1611, thrown by Federico Cesi, founder of the Accademia dei Lincei, in honor of Galileo, one of the participants, the Greek immigrant Demisiani, coined the term 'telescopium' to refer to the instrument that Galileo himself called his 'occhiale' or 'perspicillum'.[1] However, that the telescope was never invented is a more than a matter of semantics. The meaning of the paradox is that the instrument given the name of 'telescopium' on 14 April 1611 was not the consequence of any single deliberate decision to make an optical instrument that had the design properties of the 'telescopium' and of which the purpose was astronomical observation. Only with the hindsight of 14 April 1611, it could be said that a telescope was invented, and that, consequently, a name could be given to the instrument with its specific characteristics anno 1611. At that time, the telescope had already a history in which the design properties of the instrument itself were subject to change. The 'inventor' at the beginning of line of the development of the 'telescopium' could not only not foresee the purpose of the instrument in 1611, but also not the properties of the 'telescopium' of 14 April 1611. Consequently, who the inventor of the telescope was, can only be answered by making rather arbitrary decisions about what is considered to be a telescope.[2] Therefore, this dissertation is not about who invented the telescope. It will try to recover the path that led to the instrument to which on 14 April 1611 the name 'telescopium' was given.
What are the characteristics of the 'telescopium'? Two of Galileo's telescopes and one broken objective lens have been preserved in the Museum of the History of Science of Florence.[3] The first telescope consists of a tube, made of wood, and covered with leather.[4] (Figure 1.1 – Figure 1.2)

[1] Rosen, Edward. *The Naming of the Telescope*. New York: Henry Schuman, 1947, in particular, pp. 60-1.

[2] For the historical problem of assigning an inventor to the telescope, see Van Helden, Albert. 'The Historical Problem of the Invention of the Telescope.' *History of Science* 13 (1975): 251-63. See also Pantin, Isabelle. 'La Lunette Astronomique: Une Invention en Quête d' Auteurs.' In *Inventions et Découvertes au Temps de la Renaissance*, edited by M.T. Jones-Davies, 159-74. Paris: Klincksieck, 1994; Rosen, Edward. 'Did Galileo Claim He Invented the Telescope?' *Proceedings of the American Philosophical Society* 98 (1954): 304-12.

[3] Miniati, Mara. *Museo di Storia della Scienza: Catalogo*. Firenze: Giunti, 1991, p. 60. All data on Galileo's lenses and telescopes are taken from Greco, Vincenzo, Giuseppe Molesini, and Franco Quercioli. 'Optical Tests of Galileo's Lenses.' *Nature* 358 (1992): 101; Van Helden, Albert. *Catalogue of Early Telescopes*. Firenze: Giunti Istituto e Museo do Storia della Scienza, 1999, pp. 30-3. See also Greco, Vincenzo, Giuseppe Molesini, and Franco Quercioli. 'Esame Ottico dei Cannocchiali di Galileo.' *Nuncius* 8 (1993): 305-11; Greco, Vincenzo, Giuseppe Molesini, and Franco Quercioli. 'I Cannocchiali di Galileo.' *Nuovo Saggiatore* 8 (1992): 48-54; Greco, Vincenzo, Giuseppe Molesini, and Franco Quercioli. 'Modern Optical Testing on the Lenses of Galileo.' In *Homage to Galileo*, edited by P. Mazzoldi, 113-24. Padova: Cleup Editrice, 1996; Molesini, Giuseppe, and Vincenzo Greco. 'Galileo Galilei: Research and Development of the Telescope.' In *Trends in Optics: Research, Developments and Applications*, edited by Anna Consortini, 423-38. San Diego: Academic Press, 1996; Miniati, Mara, Vincenzo Greco, Giuseppe Molesini, and Franco Quercioli. 'Examination of an Antique Telescope.' *Nuncius* 9 (1994): 677-82.

[4] For another replica of Galileo's telescope, see Gainer, Michael K. 'Construction of a 17th Century Telescope: An Experiment in the History of Astronomy.' *The Physics Teacher* 19 (1981): 22-25.

Figure 1.1

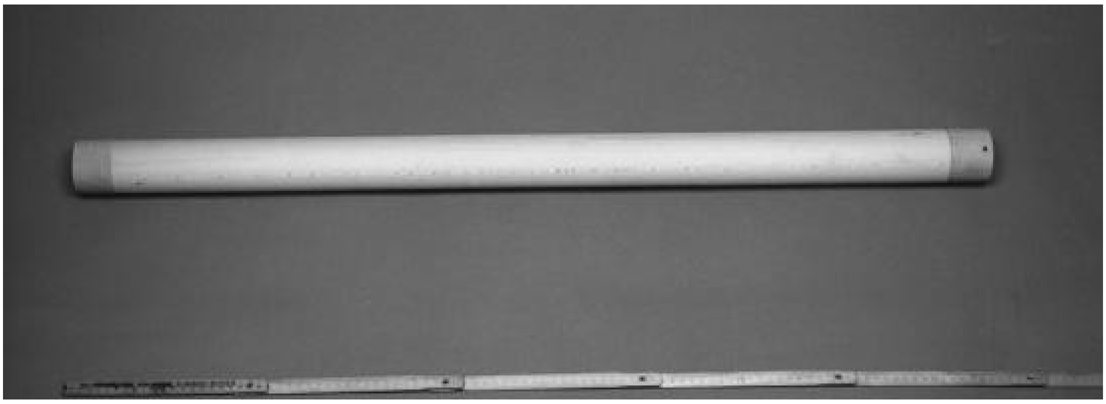

Figure 1.2

At both ends of the tube, there are housings, also made of wood, and covered with leather, that hold the objective lens and the ocular. The overall length of the instrument is 98 cm. The objective lens is plano-convex. Its focal length is 980 mm. The ocular is a biconcave lens, with a focal length of 47.5 mm. It can be pulled out slightly for focusing. This lens is not original, but a later replacement. The instrument has a magnification of 21 and a field of view of 15 arc minutes. The second instrument consists likewise of a wooden tube, made of two hollowed-out channels held together with copper wires, covered with paper. (Figure 1.3)

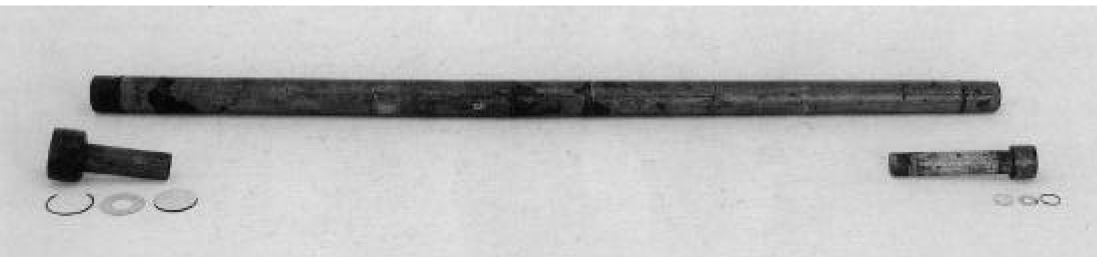

Figure 1.3

The ocular is a plano-concave lens. Its focal length is 94 mm. The overall length of the instrument is 136 cm. This instrument has a magnification of 14 and a field of view of 15 arc minutes. Finally, the single broken objective lens is also nearly plano-convex and it has a focal length of 1710 mm. (Figure 1.4)

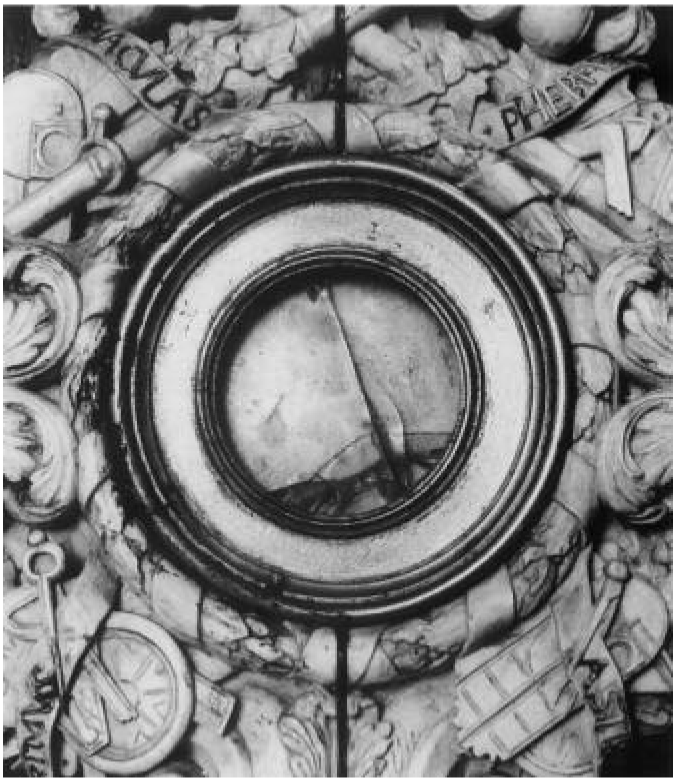

Figure 1.4

Consequently, a 'Galilean' telescope consists of an objective made of a convex lens and an ocular made of a concave lens. The distance between these two lenses is determined by the focal lengths of the lenses. (Figure 1.5)

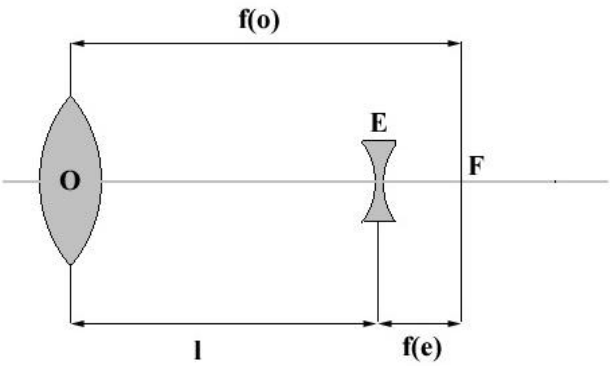

Figure 1.5

The lenses are so placed that the focal point of the convex lens and the focal point of the concave lens coincide in F. The distance between the lenses l is then equal to the focal length f(o) of the convex lens minus the focal length f(e) of the concave lens. Such a combination does not allow the rays refracted by the convex lens to come together inside the telescope and, consequently, it will yield a virtual image. Typically, it also has a small field of view. Looking through a ‘Galilean’ telescope gives the impression of looking through a tube at a small light at the end of the tube. The magnification of the instrument is determined by the proportion of the focal lengths of the lenses: m = f(o)/f(e). The magnifications of the preserved telescopes are representative for the magnifications of Galileo’s instruments, which, as is evident from his correspondence and publications, did certainly not exceed 30.[5] A magnification of 20 would have been more convenient, because of the wider field of view at lower magnifications.
The convex lenses have larger diameters than the concave lenses. In the first telescope, the convex lens has a diameter of 37 mm and the concave lens a diameter of 22 mm. The convex and concave lens of the second telescope have diameters of respectively 51 mm and 26 mm. However, the optical performance of Galileo’s lenses was troubled by aberrations, most importantly spherical and chromatic aberrations. Therefore, the diameter of the objective lenses is stopped down with a cardboard ring. (Figure 1.6 – Figure 1.7)

Figure 1.6

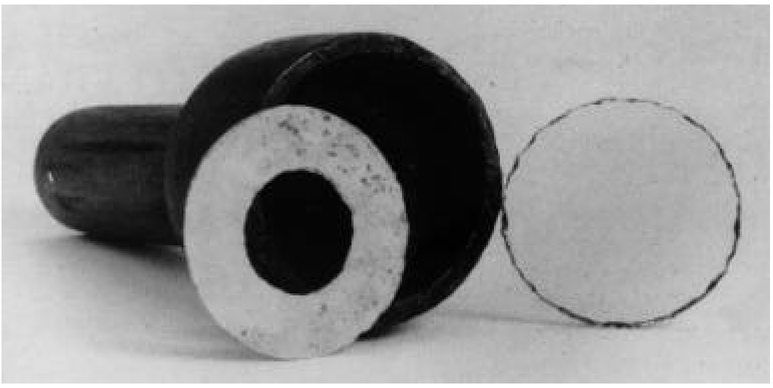

Figure 1.7

[5] Drake, Stillman. ‘Galileo's First Telescopic Observations.’ *Journal for the History of Astronomy* 7 (1976): 153-68.

While the full diameter of the objective lens of the first telescope is 37 mm, it has only an aperture of a diameter of 16 mm. The aperture of the objective lens of the second lens has a diameter of 26 mm (against a full diameter of 51 mm). Also the apertures of the concave lenses have been stopped down with cardboard rings to respectively 16 mm (against 22 mm full diameter) and 11 mm (against 26 mm full diameter). From observations made with the original telescopes, Abetti has estimated the resolution of both instruments.[6] The first telescope has a resolution of 10 arc seconds, while the second instrument performed less well than the first with a resolution of 20 arc seconds.

As is well known, Galileo did not have to start from scratch when making these telescopes. In September 1608, a Dutch lens-maker from Middelburg, Hans Lipperhey, had applied for a patent to the States-General in The Hague for 'a certain device by means of which all things at a very great distance can be seen as if they were nearby, by looking through glasses'.[7] In December 1608, the patent application was rejected, because at that moment it had become clear that the instrument could be too easily imitated to be applicable for a patent. Two other Dutch lens-makers, Sacharias Janssen and Jacob Metius, had come forward with the claim that they were the actual inventors of this telescopic device. In the nine months following the patent application of Lipperhey, the instrument spread like fire across Europe. Already at the autumn book fair in Frankfurt in 1608, a Dutchman is said to have shown such a telescopic device to Johann Philipp Fuchs. Moreover, that at the moment that Lipperhey applied for a patent a most important international peace conference gathered in The Hague was most instrumental in the spread of the news of the instrument and the instrument itself, because its military advantages were widely recognized.

Two of the participants of the conference were Ambrogio Spinola, chief commander of the Spanish army in the Low Countries, and Pierre Jeannin, head of the French delegation.[8] After the meeting Spinola returned to Brussels, where he informed Archduke Albrecht of the device. From a letter from the papal nuncio in Brussels, Guido Bentivoglio, of 2 April 1609, it is evident that Archduke Albrecht was in the possession of a Dutch telescope by March 1609.[9] Bentivoglio also sent an instrument to the Pope in Rome, where it found its way to the Jesuit mathematicians of the Collegio Romano. In December 1608, Jeannin informed the king of France of the Dutch telescope, but already in November 1608 the news of the telescope from The Hague had reached Pierre de l' Estoile in Paris. The latter also informed us that telescopes were for sale in Paris in

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[6] Abetti, Giorgio. 'I Cannocchiali di Galileo e dei Suoi Discepoli.' *L' Universo* 4 (1923): 685-92. See also Ronchi, Vasco. 'Sopra i Cannocchiali di Galileo.' *L' Universo* 4 (1923): 791-806; Baxandall, David. 'Replicas of Two Galileo Telescopes.' *Transactions of the Optical Society* 25 (1923-24): 141-44.

[7] 'seeckere conste … daer mede men seer verre alle dingen can sien al oft die naer by waeren by middel van gesichten van glasen'. Letter from the Committee of Councillors of the States of Zeeland in Middelburg to the Zeeland delegation at the States-General in The Hague, 25 September 1608. Reproduced and translated in Van Helden, Albert. *The Invention of the Telescope*. Transactions of the American Philosophical Society 67. Philadelphia: American Philosophical Society, 1977, p. 35. For the invention of the Dutch telescope, see Ibid., pp. 20-5; De Waard, C. *De Uitvinding der Verrekijkers: Een Bijdrage tot de Beschavingsgeschiedenis*. 's-Gravenhage: De Nederl. Boek- en Steendrukkerij, 1906; De Waard, C. 'L' Invention du Télescope.' *Ciel et Terre* 28 (1907): 3-18.

[8] For the rapid diffusion of the telescope from Holland, see Sluiter, Engel. 'The Telescope before Galileo.' *Journal for the History of Astronomy* 28 (1997): 223-34. See also Sluiter, Engel. 'The First Known Telescopes Carried to America, Asia and the Arctic, 1614-39.' *Journal for the History of Astronomy* 28 (1997): 141-45.

[9] For a reproduction of this letter, see Hensen, A. H. L. 'De Verrekijkers van Prins Maurits en van Aartshertog Albertus.' *Mededeelingen van het Nederlandsch Historisch Instituut te Rome* 3 (1923): 199-204, pp. 203-4.

April 1609. From France, the instrument travelled to Milan, where in May 1609 a French soldier presented it to the local Spanish commander. At that time, the instrument had also crossed the Channel, reaching Thomas Harriot in England, who started his observations of the moon.
Sometime in November 1608, a newsletter announcing the Dutch telescope had already reached Paolo Sarpi, a close associate of Galileo, in Venice.[10] In March 1609, Sarpi inquired a friend, Jacques Badovere about the instrument. Badovere was in Paris, where telescopes were already for sale in spectacle-makers' shops. Badovere's reply to Sarpi's inquiry is lost, but from Galileo's own words in the 'Sidereus Nuncius' he did not know of the instrument from Badovere before June 1609.[11] Since Sarpi was already abreast of the Dutch telescope in November 1608, it is very unlikely that Galileo did not know about the instrument before June 1609. Badovere's letter must have provided more details about the design of the instruments that were for sale in Paris. These instruments might be called telescopes, but they were not identical to Galileo's 'telescopium'. First, the Dutch telescopes, made from lenses available in ordinary spectacle-makers' shops had low magnifications, presumably not exceeding a magnification of 3.[12] In August 1609, Galileo succeeded in making a telescope, presented to the Venetian authorities, with a magnification of 9.[13] By November 1609, when observing the moon with the instrument, he made telescopes that magnified more than twenty times.[14] Second, as already shown, Galileo stopped down the objective lens of his telescope. Thus, to recover the path of Galileo that led to the 'telescopium', I will show three things, (1) how Galileo understood the telescope when he first encountered it in June 1609, (2) how he succeeded in improving the magnification of the instrument, and (3) why he stopped down the objective lens of his telescopes. None of these questions have been satisfactorily answered. To make that clear, I will confront my approach with other approaches that have so far been taken to the development of Galileo's telescope.

2. Context, Cognition, and Skill

Four different approaches have been applied to Galileo's telescope. They are not always strictly exclusive, as different elements of each approach might be mixed in a historical account, but I deal with them separately for reasons of clarity. First, Ronchi has argued that Galileo's telescope was the consequence of an 'act of faith'.[15] I will call Ronchi's thesis the immigration approach. It is based on two assumptions, (1) lenses were considered unreliable by contemporaneous optics

[10] For a reproduction of this newletter, see Drake, Stillman. *The Unsung Journalist and the Origin of the Telescope.* Los Angeles: Zeitlin & Ver Brugge, 1976, pp. 20-24.

[11] Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 60. For discussion, see Rosen, Edward. 'When Did Galileo Make His First Telescope?' *Centaurus* 2 (1951): 44-51; Drake, Stillman. 'Galileo Gleanings VI: Galileo's First Telescopes at Padua and Venice.' *Isis* 50 (1959): 245-54.

[12] Van Helden, *The Invention of the Telescope*, p. 11.

[13] Galileo to Benedetto Landucci, 29 August 1609, in Galileo, *Opere*, Vol. 10, pp. 253-4.

[14] For the dating of Galileo's lunar observations and the magnification of the instrument used, see Drake, 'Galileo's First Telescopes at Padua and Venice', pp. 153-7; Whitaker, Ewen A. 'Galileo's Lunar Observations and the Dating of the Composition of 'Sidereus Nuncius'.' *Journal for the History of Astronomy* 9 (1978): 155-69.

[15] Ronchi, Vasco. 'The Influence of the Early Development of Optics on Science and Philosophy.' In *Galileo, Man of Science*, edited by Ernan McMullin, 195-206. New York London: Basic Books, Inc., 1967, pp. 199-202. See also Ronchi, Vasco. *Il Cannocchiale di Galileo e la Scienza del Seicento*. Torino: Edizioni Scientifiche Einaudi, 1958.

up to the time of Galileo, and (2) Galileo was not acquainted with the science of optics. Therefore, since Galileo was an immigrant in the field of optics, he trusted his telescopic observations. As to why Galileo trusted the telescope, beside ignorance, Ronchi did not give the answer. The Galileo that Feyerabend portrayed can be considered an extreme version of Ronchi's Galileo.[16] Feyerabend has argued that Galileo was not acquainted with the optics of his time, in particular, Kepler's 'Paralipomena'. Therefore, Galileo did not understand the optics of his telescope, because, as Feyerabend claimed, Kepler's 'Paralipomena' was the only way to theoretically support the optics of Galileo's telescope, and his only way out was to take recourse to a series of propaganda tactics. There is no reason to dwell further on these propaganda tactics, since Feyerabend's thesis is well known. Let it suffice to point out that, as will be shown, Ronchi and Feyerabend have the facts wrong. It is not only that lenses were not generally considered unreliable in contemporaneous optics, even more important, it will be shown that Galileo was acquainted with contemporaneous optics. Another extreme version of Ronchi's Galileo is Drake's who simply considered Galileo's construction of the telescope and his telescopic observations to be unproblematic.[17] Drake portrayed Galileo as the experimental scientist who was confronted with a bunch of stubborn philosophers, who simply refused to take the unproblematic empirical evidence offered by the telescope for granted. However, the telescope and the telescopic observations are considered problematic in the second and third approach.

A second attempt at understanding Galileo's telescope is what I will call a literary approach. It is framed within a larger account of experience and experiment in the scientific revolution. Dear has argued that what was new around 1650 was not experience and experiment as such, but a new way of talking about experience and experiment.[18] Experience was a well-established category within the Aristotelian mixed sciences of the sixteenth century. However, in the Aristotelian mixed sciences, experience was expressed as ordinary or universal experience. Only in the scientific revolution, a language was found to express experience under the form of an eyewitness report of a particular experience. Hamou has applied this framework to Galileo's telescope.[19] He has argued that Galileo used the methodology found in the Aristotelian tradition. Wallace has shown that Galileo was familiar with this Aristotelian framework from his studies of courses of the Jesuits at the Collegio Romano, who were, according to Dear, primary advocates of the

[16] Feyerabend, P. K. 'Problems of Empiricism II.' In *The Nature and Function of Scientific Theories: Essays in Contemporary Science and Philosophy*, edited by Robert G. Colodny, 275-353. Pittsburgh: University of Pittsburgh Press, 1970. For a critical review, see Machamer, Peter K. 'Feyerabend and Galileo: The Interaction of Theories, and the Reinterpretation of Experience.' *Studies in the History and Philosophy of Science* 4 (1973): 1-46.

[17] Drake focussed on Galileo's process of discovery, in particular in Drake, 'Galileo's First Telescopic Observations', pp. 153-68, but he considered Galileo's telescopic discoveries self-evident to replicate shows that he considered them unproblematical, once a good telescope is available. It is of course the 'invention' of a good telescope that is not self-evident. For Drake deproblematization of the telescope, see for example Drake, Stillman. *Galileo at Work: His Scientific Biography*. Chicago London: The University of Chicago Press, 1978, pp. 159-68.

[18] Dear, Peter. *Discipline & Experience: The Mathematical Way in the Scientific Revolution*. Edited by David L. Hull, *Science and Its Conceptual Foundations*. Chicago London: The University of Chicago Press, 1995. See also Dear, Peter. 'Miracles, Experiments, and the Ordinary Course of Nature.' *Isis* 81 (1990): 663-83.

[19] Hamou, Philippe. *Le Mutation du Visible: Essai sur la Portée Épistémologique des Instruments d' Optique au XVIIe Siécle*. 2 vols. Vol. 1: Du Sidereus Nuncius de Galilée à la Dioptrique cartésienne. Villeneuve d' Ascq: Presses Universitaires du Septentrion, 1999, in particular, pp. 29-144.

Aristotelian universal experience.[20] Notwithstanding that the telescope was a new kind of research instrument, and, consequently, problematic, Hamou has argued that Galileo cast his experience with the instrument on the Aristotelian category of universal experience, dealing with the observations through the telescope as comparable to naked-eye observations. Thus, Galileo considered the telescope as a kind of black box yielding universal experiences. Moreover, he discussed his telescope in terms of perspective, a well-established Aristotelian mixed science.
A third approach is sociological. It has in common with the literary approach that the telescope is presented as a problem. Shapin and Schaffer have claimed that science developed a social technology aimed at witnessing that had to guarantee the trustworthiness of experimental reports.[21] Applied to Galileo's telescope, Winkler and Van Helden traced the emergence of a new visual language in seventeenth century astronomy, starting with Galileo's engravings of almost day-to-day observations of the sunspots, which aimed at the possibility of 'virtual witnessing'.[22] By rendering the process of the making of telescopes, observing with the instruments, and representing what was seen with them invisible or authorative, the visual representations gave the opportunity to the reader of a 'direct look through the telescope'. Consequently, there are parallels between the craft of the painter and the art of the experimental scientist. Shapin and Schaffer have claimed that both aimed at making representations that imitated the act of unmediated seeing, and they observed a similarity with Alpers' approach to seventeenth century Dutch art as an 'art of describing', that is representing only 'what the eye can see'.[23] In both cases, the direct mirroring of nature is a consequence of a sociological process that makes the craft of the painter and the experimenter involved in making the representation invisible.[24]
Biagioli has portrayed Galileo as caught between two economies, on the one hand, the economy of the mathematician who could obtain a financial reward for his invention of the instrument by applying for a patent, and, on the other hand, the economy of the natural philosopher who wants credit for his telescopic discoveries.[25] He has argued that Galileo first tried the tactic of the mathematician by successfully trying to obtain a higher salary at the University of Padua by

[20] See for example Wallace, William. *Galileo and His Sources: The Heritage of the Collegio Romano in Galileo's Science*. Princeton: Princeton University Press, 1984; Wallace, William A. *Galileo's Logic of Discovery and Proof: The Background, Content, and Use of His Appropriated Treatises on Aristotle's Posterior Analytics*. Edited by Robert S. Cohen. Vol. 137, *Boston Studies in the Philosophy of Science*. Dordrecht Boston London: Kluwer Academic Publishers, 1992; Wallace, William A. *Galileo's Logical Treatises*. Edited by Robert S. Cohen. Vol. 138, *Boston Studies in the Philosophy of Science*. Dordrecht Boston London: Kluwer Academic Publishers, 1992.

[21] Shapin, Stevin, and Simon Schaffer. *Leviathan and the Air-Pump: Hobbes, Boyle, and the Experimental Life*. Princeton: Princeton University Press, 1985, in particular, pp. 22-79.

[22] Winkler, Mary G., and Albert Van Helden. 'Johannes Hevelius and the Visual Language of Astronomy.' In *Renaissance and Revolution: Humanists, Scholars, Craftsmen and Natural Philosophers in Early Modern Europe*, edited by J.V. Field and Frank A. J. L. James, 97- 116. Cambridge: Cambridge University Press, 1993.

[23] Alpers, Svetlana. *De Kunst van het Kijken: Nederlandse Schilderkunst in de Zeventiende Eeuw*. Translated by Christien Jonkheer. Amsterdam: Uitgeverij Bert Bakker, 1989. See Shapin and Schaffer, *Leviathan and the Air-Pump: Hobbes, Boyle, and the Experimental Life*, in particular, pp. 17-8.

[24] See Shapin, Stevin. *A Social History of Truth: Civility and Science in Seventeenth-Century England*. Edited by David L. Hull, *Science and Its Conceptual Foundations*. Chicago London: The University of Chicago Press, 1994; Shapin, Stevin. 'The Invisible Technician.' *American Scientist* 77 (1989): 554-63; Van Helden, Albert. 'Telescopes and Authority from Galileo to Cassini.' *Osiris* 9 (1994): 9-29.

[25] Biagioli, Mario. 'Replication or Monopoly? The Economics of Invention and Discovery in Galileo's Observations of 1610.' *Science in Context* 13 (2000): 547-92.

applying to the Venetian state for a patent according to the conventions of early patent application. This tactic explains, according to Biagioli, why Galileo was secretive about the skills involved in making telescopes and why he never published any details on the optical processes of image formation in the telescope.[26] Galileo was very reluctant to give information on the construction of the telescope in private correspondence. He did not regularly provide other astronomers with telescopes of his hand. Biagioli has argued that in this way Galileo tried to monopolize the discoveries that could be made with the instrument, and that this attempt at a monopoly had priority on other astronomers replicating his discoveries. Galileo sent his telescopes not to fellow mathematicians, but to prospective patrons, to obtain the title of philosopher and the social promotion that went along with such a title. Biagioli has argued that Galileo circumvented the problem of replication, by dedicating his discoveries, for example of the satellites of Jupiter or the 'Medicean Planets', to future patrons who were able to socially sanction his discoveries.[27]

Neither the literary nor the sociological approach is however capable of providing a model of how Galileo made the telescope. As to how the telescope is actually made, the sociological approach refers to the skill involved in making the instrument, which is subsequently made invisible by sociological processes. The primary advocate of the skill-approach is Van Helden, who appears to understand skill as, on the one hand, some kind of tacit gift at observing, and, on the other hand, the kind of manual labor that is involved in making lenses, in particular, of long focal lengths, a technological problem with which Galileo was confronted in order to make telescopes with higher magnifications.[28] However, since Van Helden considered the invention of the Dutch telescope by Lipperhey as the product of trial and error (with which I do not necessarily disagree), he is unable to explain how Galileo understood the optics of his telescope and how he found out how it magnified.[29] Consequently, Van Helden did not explain how Galileo discovered that lenses of longer focal lengths result in telescopes with higher magnifications, a problem that is more basic than the technological problem of making lenses of longer focal lengths. Therefore, while I agree with the skill-approach as an entrance into the understanding of the development of Galileo's 'telescopium', I will use a model of skill that allows explaining how Galileo understood the optics of his telescope and how he was able to improve his instrument.

The skill-approach, which I will use to account for the development of Galileo's 'telescopium', draws inspiration from Piaget, whose importance has been stressed by De Mey and Gooding.[30] Piaget has argued that the roots of knowledge lie in action, the bodily functions and their

[26] Ibid., pp. 577-80.

[27] Biagioli, Mario. 'Galileo the Emblem Maker.' *Isis* 81 (1990): 230-58; Biagioli, Mario. 'Galileo's System of Patronage.' *History of Science* 28 (1990): 1-62; Biagioli, Mario. *Galileo, Courtier: The Practice of Science in the Culture of Absolutism*. Edited by David Hull, *Science and Its Conceptual Foundations*. Chicago London: The University of Chicago Press, 1993.

[28] Van Helden, Albert. 'Galileo and the Telescope.' In *Optics in Sixteenth-Century Italy*, edited by Paolo Galluzzi, 149-58. Firenze: Giunti Barberà, 1984.

[29] Van Helden, *The Invention of the Telescope*, pp. 16-20.

[30] De Mey, Marc. *The Cognitive Paradigm: An Integrated Understanding of Scientific Development*. Chicago: The University of Chicago Press, 1992, pp. 230-6; Gooding, David. *Experiment and the Making of Meaning: Human Agency in Scientific Observation and Experiment*. Dordrecht: Kluwer Academic Publishers, 1990, pp. 159-61. See also Machamer, Peter. 'Galileo's Machines, His Mathematics, and His Experiments.' In *The Cambridge Companion to Galileo*, edited by Peter Machamer, 53-79. Cambridge: Cambridge University Press, 1998.

interaction with the environment.[31] Knowledge starts from trial and error actions and observing the resulting effects of these actions on the objects. Skills are developed by the gradually growing awareness of the interaction of the bodily movements with the objects. Consequently, in Piaget's account, there is a second stage in which reflexive abstraction, a reflexivity on the actions and their effects on the object, becomes a substitute for sheer trial and error. I will argue that the imaging effects of moving a candlelight back and forward with respect to a concave mirror lead to the representation of the imaging effects resulting from this action in a notion of the point of inversion of a concave mirror in the mid-sixteenth century. Moreover, it will be shown that Galileo's understanding of the telescope was based on this representation of skill in a notion of point of inversion. It is important to point out that this point of inversion is not a high-level theoretical concept, but an immediate abstraction of practice, that set limits to the practice of reproducing the imaging effects of a mirror. I will use the formulation 'point of inversion' as a shortcut to this package of skills. As a second caveat to show how close the point of inversion is to actual practice, it needs to be pointed out that the formulation 'point of inversion' was not used until about fifty years after the skill involved was first represented.

How can such a concept of skill be historicized? The sociological approach has called attention to the social context of scientific practice. However, I will focus not so much on how sociological processes made the skill invisible, but on the skill itself embodied in this context of scientific practice. As concerns the telescope, this context is the optical instrumental practice of mathematical practitioners, instrument-designers and spectacle-makers of the sixteenth century. I refer to this context as the instrumental practice of art and science. It needs to be pointed out that art in a sixteenth century context does not exclusively refer to the fine arts, foremost the visual arts or even more limited painting, strictly disjunctive of science, with which it is associated today.[32] Instead, art and science refer to a whole range of sixteenth century mathematical practices, of which painting is nevertheless an important one, involving the design and use of instruments. Consequently, I will look at these practices not to make a sociological argument that the origin of science is in the arts, providing science with its material technology, but to highlight the knowledge embodied in these practices, and how this knowledge became to be used in a research context.[33] It will be argued that Galileo understood the optics of his telescope, in particular, that he gained an understanding of how it magnified, on the basis of the appropriation of skills, represented in the point of inversion, embodied in contemporaneous instrumental practices of instrument designers, and in the workshop practices of spectacle-makers.

3. Galileo's Experiments in the History of Science

The telescope has the characteristic that it is as much a technological invention as the basis of the development of modern geometrical optics in the seventeenth century. The telescope can be considered an experiment, however with hindsight, since Galileo did not intend it to be an

[31] Piaget, Jean. *La Prise de Conscience*. Paris: Presses Universitaires de France, 1974, pp. 261-82.

[32] For the conjunct of art and science in the sixteenth century, see for example Bennett, Jim. 'Projection and the Ubiquitous Virtue of Geometry in the Renaissance.' In *Making Space for Science: Territorial Themes in the Shaping of Knowledge*, edited by Crosbie Smith and Jon Agar, 27-38. London: Macmillian Press Ltd, 1998.

[33] For the origin thesis, see for example Rossi, Paolo. *Les Philosophes et les Machines 1400-1700*. Translated by Patrick Vighetti. Paris: Presses Universitaires de France, 1996.

experiment in the sense of a deliberate experimental set-up either to test some theoretical optical assumptions to discover laws unknown to the contemporaneous science of optics. Galileo's only aim was to develop an instrument and as such he was not different from most of the instrument-designers of the sixteenth century, who, as will be shown, also were engaged in an experimental practice that was however not intended to be an experiment, because it had only the practical aim of the development of an instrument.[34] If however with this caveat Galileo's telescope can be considered an experiment, then what is its significance for the debate between theory-driven and experiment-driven models of Galileo's research that, mostly in the context of Galileo's study of mechanics and the laws of motion, has dominated the history of science since the 1960s? I will give a short overview of the debate in order to place Galileo's telescope against this background.

Starting point of the debate was Koyré's argument that the hypothetico-deductive method is the only valid way to do physics at any time. Therefore, Galileo was a hypothetico-deductivist. In his 'Metaphysics and Measurement', Koyré argued that good physics is made a priori. Theory precedes fact. Experience is useless because before any experience we are already in possession of the knowledge we are seeking for. Fundamental laws of motion (and of rest), laws that determine the spatio-temporal behaviour of material bodies, are laws of a mathematical nature. Of the same nature as those which govern relations and laws of figures and of numbers. We find and discover them not in Nature, but in ourselves, in our mind, in our memory, as Plato long ago has taught us'.[35] Koyré did not even grant to Galileo the experimental confirmation of his theoretical results. According to Koyré, the imperfection of Galileo's instruments and experimental procedures did not enable any real experiments. After quoting Galileo's description of the inclined plane experiment, where Galileo claimed to have used the weight of the water running out of a water-vessel as a time-keeper, Koyré remarked: 'A bronze ball rolling in a 'smooth and polished' wooden groove! A vessel of water with a small hole through which it runs out and which one collects in a small glass in order to weigh it afterwards and thus measure the times of descent (the Roman water-clock, that of Ctesebius, had been already a much better instrument): what an accumulation of sources of error and inexactitude!'[36] Thus, according to Koyré, Galileo's experiments were imaginary experiments, with, at most, a pedagogical value to teach Galileo's readers what they already knew anyway.

In 1961, Settle replicated the inclined plane experiment described by Galileo in the 'Discorsi' with an instrumentation as close as possible to the one that Galileo claimed to have used. He obtained results with the accuracy that Galileo had claimed for them. It granted to Galileo the experimental confirmation of his theoretical results, but the successful replication did not prove Koyré's hypothetico-deductivism wrong. Settle concluded: 'Thus far I can only reproduce the end

[34] For the parallel between technological invention and experiment, see Gooding, David. 'Mapping Experiment as a Learning Process: How the First Electromagnetic Motor Was Invented.' *Science, Technology & Human Values* 15 (1990): 165-201, pp. 192-6. For the concept of instrument and the telescope as an instrument, see Van Helden, Albert. 'The Birth of the Modern Scientific Instrument, 1550-1700.' In *The Uses of Science in the Age of Newton*, edited by John G. Burke, 49-84. Berkeley: University of California Press, 1983; Field, J.V. 'What Is Scientific About a Scientific Instrument?' *Nuncius* 3 (1988): 3-26; Warner, Deborah Jean. 'What Is a Scientific Instrument, When Did It Become One, and Why?' *British Journal for the History of Science* 23(1990): 83-93; Van Helden, Albert, and Thomas L. Hankins. 'Introduction: Instruments in the History of Science.' *Osiris* 9 (1994): 1-6.

[35] Koyré, Alexandre. *Metaphysics and Measurement: Essays in the Scientific Revolution*. Cambridge, Massachusetts: Harvard University Press, 1968, p. 13.

[36] Ibid., p. 94.

product of a process of evolution (in Galileo’s own mind) which may have covered 20 years’.[37] At the time of his replication of the inclined plane experiment, Settle was still sceptical about Galileo actually dropping balls from a height, because it seemed that one of Galileo’s observations could not be reasonably accounted for.[38] Galileo recorded that lighter balls would fall faster than heavier ones at the start, before arriving almost simultaneously at the ground. It threw serious doubt on Galileo having actually performed the experiment. However, Settle replicated the experiment and concluded that this was what Galileo actually would have perceived if he had dropped balls from a height, and that the effect was due to physiology. We tend to let go lighter objects a fraction of time earlier than heavier objects.[39] Also, it became clear that this experiment urged Galileo to perform inclined plane experiments. At the time of the original experiment, presumably somewhere around 1590, Galileo thought that a body in free fall moves with a characteristic uniform speed, which is directly proportional to its specific gravity. In other words, a body with twice the density should arrive at the ground in half the time, but this was not what Galileo actually saw. As mentioned, both balls tend to reach the ground simultaneously. Galileo tried to account for these observations by introducing an impressed force that steadily decayed during the fall. In the short time that it takes a body to fall from a height accessible to an experimenter, for instance a tower, the entire impressed force did not have time to decay, and both objects hit the ground simultaneously. Consequently, Galileo started looking for a system, the inclined plane, to systematically slow down motion. He seemed to hope that the smaller quantity of impetus would dissolve rapidly enough to allow observing the characteristic uniform motion before the ball attained the end of the inclined plane.[40]

Due to Settle’s replication of Galileo’s experiments, it became clear that Galileo’s own hypothetico-deductive account of his discoveries in the ‘Discorsi’ was a rhetorical reconstruction ‘after the facts’ aimed at persuading the audience.[41] If no longer taken for granted as the way Galileo discovered the law of free fall, Koyré’s hypothetico-deductivism was under severe doubt. In 1967, Settle made painstakingly clear what was at stake here, arguing that ‘most accounts of the discovery of the properties of natural acceleration have relied fairly exclusively on Galileo’s own exposition of the subject in the Third Day of the Discorsi. With minor variations, the common interpretation has Galileo following the ‘hypothetico-deductive’ model, performing the inclined plane experiment, if at all, only to test deductions from a theoretical presupposition, postulated in advance as the true definition of natural acceleration. Very little attempt is ever made to find the order in which Galileo discovered the various propositions or how he may have

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[37] Settle, Thomas B. ‘An Experiment in the History of Science.’ *Science* 133 (1961): 19-23, p. 20. For Galileo’s experiments in mechanics and dynamics, the locus classicus still is Settle, Thomas B. ‘Galilean Science: Essays in the Mechanics and Dynamics of the Discorsi.’ Ph.D, Cornell University, 1966.

[38] Settle, Thomas B. ‘Galileo's Use of Experiment as a Tool of Investigation.’ In *Galileo, Man of Science,* edited by Ernan McMullin, 315-37. New York London: Basic Books, Inc., 1967, p. 325.

[39] Settle, Thomas B. ‘Galileo and Early Experimentation.’ In *Springs of Scientific Creativity: Essays on Founders of Modern Science*, edited by Rutherford Aris, H. Ted Davis and Roger H. Stuewer, 3-20. Minneapolis: University of Minnesota Press, 1983, pp. 12-4.

[40] Settle, ‘Galileo's Use of Experiment as a Tool of Investigation’, pp. 333-5.

[41] For Galileo’s use of rhetoric in the context of experimentation, see Naylor, R.H. ‘Galileo's Experimental Discourse.’ In *The Uses of Experiment*, edited by David Gooding, Trevor Pinch and Simon Schaffer, 117-34. Cambridge: Cambridge University Press, 1989, but see Cantor’s criticism in the same volume: Cantor, Geoffrey. ‘The Rhetoric of Experiment.’ In *The Uses of Experiment*, edited by David Gooding, Trevor Pinch and Simon Schaffer, 159-80. Cambridge: Cambridge University Press, 1989.

come upon them. We are often left with the conclusion that by and large the sequence of exposition of the Third Day follows the same order as that in which Galileo originally gained his knowledge, almost that the text is a transcription of a daily research journal'.[42] Other scholars, foremost Drake and Naylor, joined Settle in replicating Galilean experiments on motion.[43] However, Drake's Galileo became an unproblematical experimental scientist, simply gathering his accurate data from an experimental set-up, unproblematical in itself, to generate his theory of motion.

Settle emphasized that Galileo's experiments were problematical, involving experimental reasoning and skill. In more recent publications, Settle has emphasized the 'experimental trajectory' of Galileo's research, arguing that the real question is 'the ways in which his many experimental initiatives both were interconnected or cross-linked among themselves and evolved over a period of time. … it has become quite clear that with regard to Galileo there was no such thing as single, stand-alone experiment. Individual experiments, even if we can speak of them as such, were always linked to other individual experiments, both synchronically and diachronically, as in a web or network that has depth both in space and time'.[44] For example, Settle has shown that Galileo was preoccupied with oscillators during most of his life, but in different ways, depending on the research context.[45] Presumably, Galileo considered the pendulum a way to slow down motion in order to study free fall before he discovered isochronism, in the same way he wanted to slow down motion along inclined planes to study free fall. Once he discovered the isochronic movement of the pendulum, it became a device to refine the water-vessel for the measurement of time during inclined plane experiments. Moreover, quite soon in Galileo's scientific career the pendulum experiments became connected with experiments in a semi-circular track, considered a collection of very small inclined planes, in order to find a relation between circular-curvilinear acceleration and linear acceleration. Consequently, for Settle, the inclined plane experiment that Galileo described in the 'Discorsi' was a reconstructed result, ready to replicate and unproblematical aimed at confirmation, from a long experimental line of research, that started more than forty years earlier.

Thus, Galileo's experiments were not always unproblematical measurements with an unproblematic instrument to confirm a theory. The way in which Settle reconstructed Galileo's trajectory of experiments has found confirmation in more recent studies of experimentalism in other domains of the history of science. Experiments are often generative, sometimes even exploratory and open-ended, involving experimental reasoning and skill in manipulating the experimental apparatus.[46]

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[42] Settle, 'Galileo's Use of Experiment as a Tool of Investigation', p. 316.

[43] See for example Drake, Stillman. 'Galileo's Experimental Confirmation of Horizontal Inertia: Unpublished Manuscripts (Galileo Gleanings XXII).' *Isis* 64 (1973): 291- 305; Drake, Stillman. 'The Role of Music in Galileo's Experiments.' *Scientific American* 232 (1975): 98-104, and several articles reprinted in Drake, Stillman. *Essays on Galileo and the History and Philosophy of Science*. Selected and introduced by N. M. Swerdlow and T. H. Levere ed. Toronto Buffalo London: University of Toronto Press, 1999, part VI; Naylor, R.H. 'The Role of Experiment in Galileo's Early Work on the Law of Fall.' *Annals of Science* 37 (1980): 363-78.

[44] Settle, Thomas B. 'Experimental Research and Galilean Mechanics.' In *Galileo Scientist: His Years at Padua and Venice*, edited by Milla Baldo Ceolin, 39-60. Venezia: Istituto Nazionale di Fisica Nucleare Istituto Veneto di Scienze, Lettere ed Arti Dipartimento di Fisica "Galileo Galilei" dell' Università degli Studi di Padova, 1992, p. 39.

[45] Settle, Thomas B. 'La Rete degli Esperimenti Galileiani.' In *Galileo e la Scienza Sperimentale*, edited by Milla Baldo Ceolin, 11-62. Padova: Dipartimento di Fisica "Galileo Galilei", 1995.

[46] Gooding, David. *Experiment and the Making of Meaning: Human Agency in Scientific Observation and Experiment*. Dordrecht Boston London: Kluwer Academic Publishers, 1990. For exploratory experimentation, see Steinle, Friedrich. 'Entering New Fields: Exploratory Uses of Experimentation.' *Philosophy of Science* 64 (1997): S65-S74; Steinle, Friedrich. 'Exploratives vs. Theoriebestimmtes Experimentieren: Ampères Erste Arbeiten zum

Consequently, the instruments used in these experimental trajectories matter. They are not simply antiquarian artefacts in a museum. They are to be placed in their research context. It will be argued that Galileo’s telescope also needs to be described as an instrument part of a research context in order to understand the design of the ready-made ‘telescopium’. On the one hand, the actual instruments in the museum need to be taken into account. Galileo’s use of aperture stops on his telescopes is not mentioned in his publications, even rarely in his correspondence. The only way to know about it is from the actual instruments. On the other hand, Galileo’s telescope is to be placed within the historical research practice. It will be argued that Galileo’s telescope was used in an experimental trajectory already established before Galileo built his first telescopes, and that the results of that research involving another instrument influenced the introduction of the diaphragm, which in turn allowed to further the initial research questions already present before the telescope.

4. Sixteenth Century Optics, Galileo’s Optics and the Telescope

Two narratives will be intertwined. One the one hand, there is a broader narrative on sixteenth century optics and its relation to the telescope. On the other hand, there is a more limited narrative on Galileo’s appropriation of sixteenth century optics, his background in optics, and its influence on Galileo’s development of the telescope in to the ‘telescopium’ of 1611. Since our primary focus is Galileo’s development of the telescope, there are two limitations to the time frame. On the hand, since our interest is in the immediate context of Galileo’s optics, attention is primarily given to sixteenth century optics, although the characteristics of sixteenth century optics will be positioned against the optics of antiquity and the Middle Ages. Second, our narrative on Galileo will be limited to the period between 1585 and 1611, from his mathematical education with Ostilio Ricci to the development of the telescope in its final design in 1611. Therefore, Galileo’s optics after 1611, interesting as, for example, his work on comets between 1618 and 1623 may be, strictly falls outside our scope, although sometimes some cross-references to his later work will be discussed. However, in most cases, it will concern optics, published at a later date, but of which the development can be dated to the period before 1611.

We will start with the broader narrative on sixteenth century optics. Sixteenth century optics has received relatively little attention with respect to, on the one hand, the optics of antiquity and the Middle Ages, and, on the other hand, seventeenth century optics. In chapter 2, the appropriation of the optics of antiquity and the Middle Ages by sixteenth century mathematical practitioners will be discussed. The chapter is not meant to be an exhaustive study of sixteenth century optics, since it focuses on one particular tradition within sixteenth century optics, that of the mathematical practitioners, with relatively little attention given to the sixteenth century Aristotelian commentary tradition on vision. This is a choice, but for good reasons. In the early period of Galileo, with which we deal, Galileo himself was such a mathematical practitioner, and, if he had a background in optics, it would have been foremost determined by the optics of his

Elektromagnetismus.’ In *Experimental Essays - Versuche Zum Experiment*, edited by Michael Heidelberger and Friedrich Steinle, 272-97. Baden-Baden: Nomos Verlagsgesellschaft, 1998. For the more general context of experimentalism, see in particular Hentschel, Klaus. ‘Historiographische Anmerkungen zum Verhältnis von Experiment, Instrumentation und Theorie.’ In *Instrument - Experiment: Historische Studien*, edited by Christoph Meinel, 13-51. Berlin Diepholz: Verlag für Geschichte der Naturwissenschaften und der Technik, 2000; the overview given in Ackermann, Robert. ‘The New Experimentalism.’ *British Journal for the Philosophy of Science* 40 (1989): 185-90; Hacking, Ian. *Representing and Intervening*. Cambridge: Cambridge University Press, 1983.

fellow mathematical practitioners. We will concentrate not so much on the technical content of sixteenth century optics – little truly innovative in optics was offered by sixteenth century mathematical practitioners – but on the shifts of the focus and the scope of sixteenth century optics with respect to medieval optics as a consequence of its appropriation by mathematical practitioners. Four conclusions will be drawn, (1) vision was considered within a Euclidean framework, with hardly any attention given to the problems of physiology, psychology and epistemology that had dominated medieval optics, (2) optics made claims to be considered important to decide on natural philosophical and cosmological issues, (3) there was a shift of focus from the study of vision to the study of propagation of light, and (4) optics was closely connected with the interest in instrument design and astrology of mathematical practitioners.

The third and the fourth chapter will refine this study of the appropriation of optics by sixteenth century mathematical practitioners by concentrating on one such mathematical practitioner, the hardly studied Ettore Ausonio. Ausonio is important to the study of Galileo's optics, because, as will be shown in chapter 4, Galileo copied from Ausonio's manuscripts, and, as such, Ausonio is one of the more important sources of information on Galileo's early optics. The focus will be on Ausonio's design of optical instruments, and on how his activity as an instrument designer influenced his appropriation of optics. In chapter 3, Ausonio's refractive sundials will be discussed. Contrary to the common opinion on the knowledge of sixteenth century instrument designers, it will be argued that Ausonio's refractive dials were made by implementation of his knowledge of refraction. Moreover, it will be shown how the occupation with instrument design determined a highly instrumental (though not necessarily experimental) view of medieval optics. In chapter 4, another optical instrument, Ausonio's burning mirror, will be discussed. It will be shown that Ausonio's catoptrics is a visual representation of the point of inversion, based on actual practice with a concave spherical mirror, and that this notion was not present in the optics of antiquity and the Middle Ages. Moreover, it will be shown that Galileo copied Ausonio's 'Theorica speculi concave sphaerici', in which the point of inversion is represented.

Chapter 5 is the beginning of our second narrative specifically on Galileo. While Galileo is most often positioned against medieval optics, the study of sixteenth century optics in the previous chapters will allow placing Galileo against his immediate background. It will be shown that his interest in optics fits the pattern and scope established by sixteenth century mathematical practitioners. Thus, in this chapter, Galileo's background in optics as a mathematical practitioner prior to his development of the telescope will be discussed. First, it will be argued that Galileo was familiar with perspective, not only by his acquaintance with contemporary painting, but also by his interest in mathematical instrument design. Mathematical instruments embodied perspective. Second, contrary to common opinion, it will be shown that Galileo was also familiar with catoptrics (and dioptrics). His sources will be identified, and it will be shown that Galileo's copy of Ausonio's 'Theorica' fits an already established pattern of optical interests. However, it will be argued that Galileo's copy of Ausonio's 'Theorica' was not directly connected with the development of the telescope. Most likely, it concerned Galileo's involvement in a program of designing burning mirrors, quite typical for contemporaneous optical interests and practice.

In chapter 6, we will return to our broader narrative on sixteenth century optics and the telescope. First, the influence of Ausonio's point of inversion on the optics of the second half of the sixteenth century will be discussed. While Ausonio used it to represent image formation in a concave spherical mirror, it will be shown that the point of inversion was subsequently transferred from concave mirrors to convex lenses. Second, it will be argued that the point of inversion, making for the first time a connection between the focal properties of a mirror or lens, and vision or image formation in a mirror or lens, was at the basis of the attempts in the 1570s and 1580s to make a reflecting telescope, in particular, by Digges and Bourne in Elizabethan England. It will be concluded that the absence of the point of inversion from optics before the second half of the sixteenth century explains why the invention of the telescope was delayed although the optical components necessary to make a telescope, and the idea of telescopic magnification, were already present much earlier. The second line of argument in chapter 6 is again specifically about Galileo. It will be argued that Galileo understood the optics of the refracting telescope on the basis of the point of inversion, when he made his first telescopes. Moreover, it will be shown that due to his understanding of the optics of the telescope on the basis of the point of inversion, in combination with the knowledge embodied in a workshop practice of spectacle-makers, Galileo considered the magnification of a telescope depended upon the focal length of the convex lens, which allowed improving the magnification of his instrument. It will be concluded that an insufficient understanding of magnification explains why the earlier reflecting telescopes of Digges and Bourne did not cause the take-off of the telescope.

Finally, in chapter 7, it will be shown how Galileo arrived at the application of the diaphragm by locating the telescope within Galileo's broader network of research on celestial light. It is not only that the use of the diaphragm could only occur when the telescope is applied to astronomical observation, but it will be argued that the telescope replaced another optical instrument, the mirror, with which Galileo until 1610 had researched the question of celestial light, and that the diaphragm, for Galileo not connected with optical aberrations of lenses but with irradiation, was transferred from his research on celestial light prior to 1610 to the telescope. Moreover, the influence of Leonardo's notebooks will be assessed, showing that Galileo's understanding of irrradiation and the cause of irradiation were similar to Leonardo's. While on the macro-level it appears as if Galileo simply changed his initial theory on celestial light on the basis of his telescopic observations, it will be argued that a study of Galileo's experimental trajectory from 1604 to 1611 on the micro-level shows a much more complex interaction between theory, observation, and instrument to which neither a theory-first account nor an account that considers experiments as simply data-gathering, even if generative, can do completely justice. Thus, with an understanding of magnification and the application of the diaphragm, discussed in chapter 6 and chapter 7, Galileo arrived at the 'telescopium' of 1611.

II. The Appropriation of Optics in the Sixteenth Century: Sixteenth Century Mathematical Practitioners and Medieval Optics

1. The mathematicians' appropriation of medieval optics

'Thus the sixteenth century saw neither a revolution nor the beginnings of a revolution in optics; and when Galileo encountered the science of optics, as we know he did, it was the optics of antiquity and the Middle Ages. The optical revolution was yet to come', as David Lindberg has concluded his review of sixteenth century optics.[1] Sixteenth century optics have been dealt with in terms of continuity and revolution within a larger historical framework which has Kepler's theory of vision in his 'Paralipomena ad Vitellionem' (1604) as its main focus. While, on the one hand, Ronchi, Crombie and Straker have argued that Kepler's optics presented a radically new departure away from medieval perspectivistic optics, Lindberg, on the other hand, has discussed Kepler's optics of the 'Paralipomena' as the summit of medieval optics.[2] Their views of sixteenth century optics are dependent upon this continuity-revolution rupture. The 'revolutionaries' have argued that the discussion of the eye as a camera obscura in sixteenth century optics was highly instrumental to Kepler's optical account of vision, considered to be informed by a mechanistic worldview. The adepts of continuity have rather stressed that Kepler drew upon the perspectivistic means of medieval optics to which sixteenth century optics had little to contribute.

Arguments on both sides of the continuity-revolution rupture have poorly enlightened the scope of sixteenth century optics. Ronchi has argued that 'the first half of the sixteenth century was uneventful in the history of optics', while Lindberg has commented that 'scholarship on sixteenth-century optics has not been one of the bigger 'growth industries' in the economy of the history of science'.[3] Their accounts of sixteenth century optics are largely based upon an analysis of the new optical works produced in the second half of the sixteenth century in Italy by Francesco Maurolico and Giambattista Della Porta. In this chapter, I will attempt to clearify the scope of sixteenth century optics, not by taking side in the continuity-revolution debate, but by considering how the optics of antiquity and the Middle Ages was appropriated in the sixteenth century. The sixteenth century saw the print and reprint of many of the major optical texts of antiquity and the Middle Ages.[4] Why? As Grafton has emphasized, the transmission of texts is a matter of active choice.[5] If this obvious, but also too frequently neglected, fact is the case, then

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[1] Lindberg, David C. 'Optics in Sixteenth-Century Italy.' In *Novità e Crisi del Sapere*, edited by Paolo Galluzzi, 131-48. Firenze: Giunti Barberà, 1984, p. 148.

[2] Ronchi, Vasco. *The Nature of Light: An Historical Survey.* Translated by V. Barocas. Cambridge, Masachusetts: Harvard University Press, 1970, pp. 78-109; Crombie, A.C. 'The Mechanistic Hypothesis and the Scientific Study of Vision: Some Optical Ideas as a Background to the Invention of the Microscope.' In *Historical Aspects of Microscopy*, edited by S. Bradbury and G. L'E. Turner, 3-112. Cambridge: W. Heffer & Sons Ltd., 1967; Straker, Stephen Mory. 'Kepler's Optics: A Study in the Development of Seventeenth Century Natural Philosophy.' Ph. D., Indiana University, 1971, pp. 480-523; Lindberg, David C. *Theories of Vision: From Al-Kindi to Kepler*. Chicago London: The University of Chicago Press, 1976, pp. 178-208.

[3] Ronchi, *The nature of light*, p. 78; Lindberg, 'Optics in Sixteenth-Century Italy', p. 131.

[4] See the information on sixteenth century publications in Lindberg, David C. *A Catalogue of Mediaeval and Renaissance Optical Manuscripts*. Toronto: Pontifical Institute of mediaeval studies, 1975.

[5] Grafton, Anthony. 'Notes from Underground on Cultural Transmission.' In *The Transmission of Culture in Early Modern Europe*, edited by Antony Grafton and Ann Blair, 1-7. Philadelphia: University of Pennsylvania Press, 1990.

the cultural context of the transmission of texts should clearify why the sixteenth century bothered with Euclid, Bacon, Pecham and Witelo. Scholarship caught in the continuity-revolution dichotomy has assumed that the optics of antiquity and the Middle Ages was not only adopted in the sixteenth century, but also that it was adopted without any change. Appropriation is a more dynamic concept than transmission. As Barker has used this concept,

> Appropriation involves more than the adoption of an old idea, theory, technique, or practice, in a new place and time. Whatever is appropriated is also changed in a way characteristic of its new historical location and perhaps alien to the preferences of its previous owners. All appropriation, then, changes what is appropriated.[6]

There is appropriation instead of transmission, because the cultural context of the appropriation changes over time. The sixteenth century still read the optics of antiquity and the Middle Ages, but it read the same optical texts differently and put them to other cognitive ends.

In this process of appropriation, optics itself might change meaning, as it becomes a different socio-cognitive discipline with a different scope and different goals. The scope of the medieval perspectivistic tradition, developed in the thirteenth century, was not limited to 'geometrical optics', although the study of light propagation and its application to mathematical demonstrations concerning image formation in reflection and refraction, was part of what the perspectivists considered optics. The fundamental aim of perspectivistic optics was the study of vision, not only of visual perception, but also of visual cognition. Thus, the scope of perspectivistic optics was much broader than the scope of 'geometrical optics', as it crossed the disciplines of physics, physiology, psychology and epistemology. The final aim of medieval perspectiva was to describe the successive stages of the mind's formation of a picture of reality.[7]

Who studied the perspectivistic optics of antiquity and the Middle Ages in the sixteenth century? For our purposes, the focus will be limited to mathematicians. A first approach to the study of how optics was appropriated in the sixteenth century, needs to understand the disciplinary and professional traditions to which sixteenth century mathematicians belonged. In general, there are two professional traditions to which sixteenth century mathematicians can be related.[8] First, there is the professional tradition of medicine and astrology. In sixteenth century medical practice,

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[6] Barker, Peter. 'Understanding Change and Continuity.' In *Tradition, Transmission, Transformation: Proceedings of Two Conferences on Pre-Modern Science Held at the University of Oklahoma*, edited by Jamil F. Ragep, Sally P. Ragep and P. Livesey, 527-52. Leiden New York Köln: E. J. Brill, 1996, p. 528. For an attempt to understand Kepler's optics from this point of view, see also Chen Morris, Raz D. 'Optics, Imagination and the Construction of Scientific Observation in Kepler's New Science.' *The Monist* 84 (2001): forthcoming; Chen Morris, Raz D. 'The Typology and Transformation of the Renaissance Discourse of Vision from Alberti to Kepler.' In *Systèmes de Pensée Précartésiens: Etudes d' après le Colloque International organisé à Haïfa en 1994*, edited by Ilana Zinguer and Heinz Schott, 19-33. Paris: Honoré Champion, 1998.

[7] Smith, A. Mark. 'Getting the Big Picture in Perspectivist Optics.' *Isis* 72 (1981): 568-89; for the fourteenth century, see also Tachau, Katherine H. *Vision and Certitude in the Age of Ockham: Optics, Epistemology and the Foundation of Semantics 1250-1345*. Edited by Albert Zimmermann, *Studien Und Texte Zur Geistesgeschichte Des Mittelalters*. Leiden New York Kobenhavn Köln: E. J. Brill, 1988.

[8] Westman, Robert S. 'The Astronomer's Role in the Sixteenth Century: A Preliminary Study.' *History of Science* 18 (1980): 105-47, pp. 116-27; Biagioli, Mario. 'The Social Status of Italian Mathematicians, 1450-1600.' *History of Science* 27 (1989): 41-95; Smith, Eugene. 'Medicine and Mathematics in the Sixteenth Century.' *Annals of Medical History* 1 (1917): 125-40; Vérin, Hélène. *La Gloire Des Ingénieurs: L' Intelligence Technique Du XVIe Au XVIIIe Sciècle*. Paris: Albin Michel, 1993, in particular, pp. 40-2, 62-4.

there was a quite general belief that the visible heavens control the human body. As such, this provided a cognitive link between medicine and the astrologer-mathematician, who needed to calculate horoscopes. Moreover, the cognitive link was built into the institutionalized career pattern of a mathematician located at the typical sixteenth century university. At the university, mathematics was a propaedeutic discipline with respect to medicine. Thus, the university professor of mathematics, astronomy or astrology would teach mathematics to medical students, while he himself was often tenured in his position while still studying for a higher degree in medicine. Second, there is the professional tradition of the artist-engineer or the 'mathematical practitioner', as I will henceforward refer to practitioner of this tradition. Mathematical practitioners were active in the fields of surveying, dialling, painting, architecture, military fortification, cartography and a range of other practical activities which involved the designing and making of mathematical instruments.[9] Although White's suggestion that all instrument designers had astrological concerns and that astrologers were the primary instrument designers of the sixteenth century cannot be confirmed, both professional traditions were not mutually exclusive, in particular, during the second half of the sixteenth century.[10]

These professional roles of sixteenth century mathematicians determined how they appropriated the optics of antiquity and the Middle Ages. It determined not only their particular points of focus on the optical heritage they received, but also what they considered to be optics, the scope and aims of sixteenth century optics, different from the eventually epistemological concerns of medieval optics. In this chapter, as a preliminary account of this appropriation, the primary focus will be on two sixteenth century prefaces, Jean Pena's 'De usu optices' (1557), the preface to his edition of Euclid's 'Optics' and 'Catoptrics', and John Dee's 'Mathematicall Praeface' (1570) to Billingsley's first English translation of Euclid's 'Elements'. In the two next chapters, this preliminary account of sixteenth century mathematicians' appropriation of optics will be further substantiated by focussing on the optical work of the little known sixteenth century mathematician, Ettore Ausonio. It is of the utmost importance to analyze the appropriation of optics in the sixteenth century in order to understand Galileo's background in optics. While it might be true that, as Lindberg has claimed in the quote with which this chapter started, that, when Galileo encountered the science of optics, he encountered the transmitted texts of medieval optics, he encountered medieval optics as seen through sixteenth century glasses.

[9] See in particular the work of Jim Bennett, for example, Bennett, J.A. 'The Challenge of Practical Mathematics.' In *Science, Culture and Popular Belief in Renaissance Europe,* edited by Stephen Pumfrey, Paolo L. Rossi and Maurice Slawinski, 176-90. Manchester New York: Manchester University Press, 1991; Bennett, Jim. 'Practical Geometry and Operative Knowledge.' *Configurations* 6 (1998): 195-222.

[10] White, Lynn, Jr. 'Medical astrologers and Late Medieval Technology.' In *Medieval Religion and Technology: Collected Essays,* 297-316. Berkeley Los Angeles London: University of California Press, 1978.

2. Petrus Ramus and Optics: 'Optica est Ars Bene Videndi'

2.1. Ramus, his Students, and the Edition of Optical Texts

The humanist, Petrus Ramus (Pierre de la Ramée, 1515-1572), born in the small Picard village of Cuts, took his MA degree at the age of twenty-one in Paris.[11] Immediately afterwards, he started teaching philosophy at some of the smaller colleges in Paris. However, after he was forbidden to to teach publicly or write on philosophy, because of the condemnation of his 'Aristotelicae Animadversiones' (1544), he turned his teaching and intellectual interests towards mathematics. In 1545, he was invited to teach at the more prestigious Collège de Presles. Due to his friendship with the cardinal Charles de Lorraine, by 1551, Ramus was appointed Regius Professor of Eloquency and Philosophy at the Collège Royal. The following years were of primary importance for Ramus' influence on the history of optics, because, in those years, Ramus' plan of a 'Corpus Matheseos' was first formulated, and, partly realized, due to the contribution of, among others, one of his brightest students, Jean Pena, who came to Paris to study with Ramus ca. 1550.[12]

It is not precisely known when Jean Pena (1528-1558), born in Moustiers in the Provence, came to Paris to study at the Collège de Presles.[13] According to Ramus' biographer, Nancel, it was Nancel, who brought Pena, 'a lonely and solitary person, never imposing himself on anyone, always hiding himself away in the library, content with poor clothes and food, a small, thin, graceful man, and somewhat consumptive as the manner of his death was to prove, but remarkably studious and diligent', to Ramus' attention as a secretary, because of his fine handwriting.[14] However, Ramus must have soon discovered Pena's mathematical and linguistic talents, because in 1556, with Ramus' support, he was appointed Royal Professor of Mathematics. Already in August 1558, Pena died, although not before having contributed to Ramus' 'Corpus Matheseos'. The 'Corpus Matheseos' was Ramus' version of an ambitious publication program of editions and translations of mathematical works, mostly of antiquity, quite typical of a Renaissance humanism interested in mathematics. Although Pena's sudden death made a further contribution impossible, he managed to publish Euclid's 'Rudimenta musices' and, for our purposes, highly important, his 'Optica et Catoptrica', both in 1557.[15]

[11] Ong, Walter J., S. J. *Ramus, Method and the Decay of Dialogue: From the Art of Discourse to the Art of Reason*. Cambridge London: Harvard University Press, 1983, pp. 17-35; Hooykaas, R. *Humanisme, Science et Réforme: Pierre De La Ramée (1515-1572)*. Leyde: E. J. Brill, 1958, pp. 4-6; Verdonk, Johannes Jacobus. *Petrus Ramus en de Wiskunde*. Assen: Van Gorcum & Comp. N. V. - Dr. H. J. Prakke & H. M. G. Prakke, 1966, pp. 5-35. For a bibliography on Ramus; see Sharratt, Peter. 'Recent Work on Peter Ramus (1970-1986).' *Rhetorica* 5 (1987): 7-58; Sharratt, Peter. 'The Present State of Studies on Ramus.' *Studi francesi* 16 (1972): 201-13.

[12] On Ramus' 'Corpus Mathesos', see Verdonk, *Petrus Ramus en de Wiskunde*, pp. 85-106.

[13] Ibid., pp. 59-65.

[14] '… [solus ac] solitarius ac nemini sese obtrudens, in bibliotheca semper abditus, satis tenui cultu victuque contentus, homo parvus [et gracilis] et valde macilentus, [ut cumque ... ut exitus docuit], mirum vero in modum studiosus ac perdiligens'. Sharratt, Peter. 'Nicolaus Nancelius, Petri Rami Vita, Edited with an English Translation.' *Humanistica Lovaniensia* 24 (1975): 161-277, pp. 198-9.

[15] A first complete Latin version of Euclid's 'Optica et Catoptrica' by Bartolomeo Zamberti appeared in Venice in 1505. Pena was the first to publish the complete Greek text together with a Latin translation. For these and subsequent editions of Euclid, see Euclide. *L' Optique et la Catoptrique*. Translated by Paul Ver Eecke. Paris: Librairie Scientifique et Technique Albert Blanchard, 1959, pp. xxxvi-xlvii.

Herewith, Ramus' influence on the history of optics has not been exhausted. After Ramus had to leave Paris for a short period in July 1561, because of his sympathizing with the Hugenots, he returned to Paris in March 1563. He stayed until September 1567, when he had to leave again, because of a new outbreak of civil war. During those years, Ramus studied optics with Frederic Reisner (Fredericus Risner, ?-1580/1), who must have met Ramus around 1565.[16] When Ramus left Paris on a trip abroad in 1568, he took Reisner with him to continue work on an edition of Alhazen and Witelo. In 1570, when this edition was still unfinished, Ramus left Reisner, who had to stay in Nürnberg as part of a deal with the goldsmiths Wenzel Jamnitzer (1508-1585) and Hans Lencker (Johannes or Johann, ?-1585), who both left publications on perspective.[17]

The story of their meeting has been reported by Freigius, one of Ramus' early biographers, according to whom, Lencker would only allow Ramus to see his perspective instrument, if he would leave Reisner in Nürnberg to teach him optics.[18] Shortly afterwards, Lencker fulfilled his part of the deal by publishing his 'Perspectiva' (1571) to explain how the drawings in his 'Perspectiva litteraria' (1567) were made, according to its preface, on the specific request of Ramus.[19] Only in 1572, Reisner returned to Basel to finish the edition of Alhazen and Witelo, which was finally published in this year. As shown by Reisner's own preface to this edition and Ramus' correspondence concerning its publication history, as studied by Verdonck, the edition, which would turn out to be very influential in the history of optics, was the result of the collaborative work of both Ramus and Reisner, although Ramus did most work on Alhazen, while Reisner concentrated on Witelo.[20] Herewith, Reisner's work on optics was not finished.

In his testament, Ramus sponsored a position as Professor of Mathematics at the Collège Royal. Moreover, he stipulated that Reisner should be appointed to this position. Reisner's fellowship would be renewed if he met the requirements as laid out by Ramus.

> I bequaeth five hundred as a salary for a Professor of Mathematics who will teach from a chair at the Collège Royal, Arithmetic, Music, Geometry, Optics, Mechanics, Astronomy and Geography, for three years, not according the opinion of men but according to logical truth. First I choose, nominate and create Frederic

[16] Verdonk, *Petrus Ramus en de Wiskunde*, pp. 66-72.

[17] On Wenzel Jamnitzer, Hans Lencker and the mathematical and artistic context of Nuremberg, see Doppelmayr, Johann Gabriel. *Historische Nachricht von den Nürnbergischen Mathematicis und Künstlern*. Hildesheim New York: Georg Olms Verlag, 1972, pp. 159-161, 205-6; *Wenzel Jamnitzer und die Nürnberger Goldschmiedekunst 1500-1700: Eine Ausstellung im Germanischen Nationalmuseum Nürnberg vom 28. Juni - 15. September 1985.* München: Klinkhardt & Biermann Verlagsbuchhandlung, 1985; Smith, Jeffrey Chipps. *Nuremberg: A Renaissance City, 1500-1618*. Austin: University of Texas Press, 1983, in particular, pp. 79-84, 304; Strauss, Gerald. *Nuremberg in the Sixteenth Century: City Politics and Life between Middle Ages and Modern Times*. Bloomington London: Indiana University Press, 1976, in particular, pp. 133-40; see also, Franke, Ilse. 'Wenzel Jamnitzers Zeichnungen Zur Perspectiva.' *Münchner Jahrbuch der bildenden Kunst* 23 (1972): 165-86.

[18] Waddington, Charles. *Ramus (Pierre De La Ramée): Sa Vie, ses Écrits et ses Opinions*. Paris: Librairie de Ch. Meyrueis et Ce, 1855, pp. 209-10.

[19] 'So bin ich entlich auff anregen viler fürnemer und kunstliebenden personen / und besonder des hochberhümbten unnd hochgelerten Herrn P. Ramus Königlicher Maiestet zu Franckreich Ordinarii Professoris in der weitbeümbten Universitet zu Pariss / der mich selbst eigner person (und neben ime Er Friederich Reisener Mathematischer kunst liebhaber und förderer) zu hauss ersucht / und darumb gebeten hat / dahin bewegt worden / zubewilligen / solch mein (von Gott verliehen) geringes pfündlein zu plubicirn'. Lencker, Hans. *Perspectiva*. Nürmberg: Dietrich Gerlak, 1571, quoted from the 'Vorrede zum Leser'.

[20] Verdonk, *Petrus Ramus en de Wiskunde*, p. 71.

> Reisner as professor for the first three years, so that he will be able to finish the work on optics which we began by our common labours, and especially the work on astronomy, and if during this time he takes care to finish and perfect it according to the method laid down in my *Introduction to Mathematics*, I offer him a further period of three years. Otherwise, when the three years are up, or when the six years are up if he has acted according to my desires and wishes, I want a new election to be instituted by the Royal Professors.[21]

Ramus' last will shows that he was collaborating with Reisner on another book on optics. This is the 'Opticae libri quatuor ex voto Petri Rami', posthumously published in 1606, under the auspices of Nicolas Crugius. Ramus' scepticism about Reisner's perseverance, as shown in his testament, was proven legitimate, because the 'Opticae' needed eventually to be published unfinished. Only the first book was completely finished. Verdonck has argued that, with the possible exception of the references to Alhazen and Witelo and some of the mathematical demonstrations, attributed to Reisner, the 'Opticae' was actually the work of Ramus.[22] Anyway, although Reisner accepted the position as Professor of Mathematics at the Collège Royal, he hold the chair only for a couple of months, before he had to resign because of illness.

Ramus' and Reisner's meeting with Jamnitzer and Lencker in Nürnberg was typical for Ramus' interest in practical mathematics. It has been pointed out that Ramus was a regular visitor of artisans' workshops, in Paris and abroad. Moreover, according to one of his biographies, his library not only contained books, but also a fair amount of mathematical instruments.[23] Hooykaas and Verdonck have shown that this attitude was reflected in Ramus' concept of mathematics in general, and geometry in particular.[24] Ramus' concept of mathematics was directed by its practical utility. When he edited Euclid's 'Elements', he left out anything that did not stand the test of being useful, that is useful for artisans and mathematical practitioners. A similar attitude is found in his optics, as is evident from Pena's 'De usu optices', which, in order to emphasize its importance for Ramus' optics, was also reprinted in the 'Opticae' of Ramus and Reisner.

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[21] 'lego quigentas in stipendium Mathematici professoris, qui triennio Arithmeticam, Musicam, Geometriam, Opticam, Mechanicam, Astrologiam, Geographiam, non ad hominum opinionem, sed ad Logicam veritatem in Regia cathedra doceat. Primum Fridericum Reisnerum in tres primos annos professorem eligo, nomino, creoque, ut inchoata communibus vigiliis opera Optica, praesertim et Astrologica perficiat. Quo tempore si ad methodum Mathematico prooemio propositum, perfecta aut effecta studiose seduloque curaverit, triennium alterum prorogo. Exacto triennio siquid secus, aut sexennio, si ex optato votoque faxit, novam electionem a professoribus regiis sic institui volo.' Sharratt, 'Nicolaus Nancelius', pp. 274-5.

[22] Verdonk, *Petrus Ramus en de Wiskunde*, pp. 72-3.

[23] 'Witness all the demonstrations, not only manuscript and reduced to written form, but also skillfully made models, fashioned with cut-out charts, and by iron rods or plates, and bronze obelisks or daggers, in a large variety of forms. Witness his large Pythagorean solids, and machines which he kept carefully at home in display-cabinets'. From Nancel's 'Petri Rami Vita', translation in Sharratt, 'Nicolaus Nancelius', p. 203.

[24] Hooykaas, *Humanisme, Science et Réforme*, pp. 91-6; Verdonk, *Petrus Ramus en de Wiskunde*, pp. 341-69; see also, Cifoletti, Giovanna. 'L' Utilité des Mathématiques selon la Ramée: Brèves Notes.' *Revue des Sciences Philosophiques et Théologiques* 70 (1986): 99-100; Oldrini, Guido. 'Sul Rapporto al Quotidiano in Ramo e nel Ramismo.' In *Scienze, Credenze Occulte, Livelli di Cultura*, 65-85. Firenze: Leo S. Olschki Editore, 1982; Grafton, Anthony, and Lisa Jardine. *From Humanism to the Humanities: Education and the Liberal Arts in Fifteenth- and Sixteenth-Century Europe*. Cambridge: Harvard University Press, 1986, pp. 163-4. For the general background of mathematical teaching by Ramus and in France, see Sharratt, P. 'La Ramée's Early Mathematical Teaching.' *Bibliothèque d' Humanisme et Renaissance* 28 (1966): 605-14; Margolin, Jean-Claude. 'L' Enseignement des Mathématiques en France (1540-70): Charles De Bovelles, Fine, Peletier, Ramus.' In *French Renaissance Studies, 1540-70: Humanism and the Encyclopedia*, edited by Peter Sharratt, 109-55. Edinburgh: University Press, 1976.

In his preface, Pena lamented the poor state of contemporaneous interest in optics, and consequently its low status, while he considered optics the science 'which, like the sun, throws its light on all the other sciences'[25]. Pena hoped to remedy this lack of interest by showing the utility of optics to other mathematical disciplines. In this respect, Pena is far from an isolated case in the sixteenth century. Pena's defense of the utility of optics was a strong echo of the Nürnberg tradition of perspective, initiated by Dürer's 'Underweysung der Messung' (1525), for which Ramus, in his geometrical work, did not hide his appreciation.[26] In the preface to this work, Dürer claimed that his 'Underweysung' was 'profitable for all those who desire to know better their art, not only painters, but also goldsmiths, sculptors and stone cutters, carpenters and all those who need to measure'.[27] A similar defense of the utility of optics on the basis of its wide applicability to a whole range of mathematical disciplines, in combination with a stress on the unity of theory and practice, is also found in the sixteenth century Vitruvian commentary tradition from Ryff or Rivius (1548), in Nürnberg, to Daniele Barbaro (1556), in Venice.[28]

Finally, Danti's preface to his Italian translation of Euclid's 'Optica et Catoptrica' shows remarkable resemblances to Pena's in its defense of the utility of optics. Egnatio Danti (1536-1586), born in a family of goldsmiths in Perugia, entered the Dominican Order when he was nineteen.[29] In 1563, he came to Florence, where he was appointed the court cosmographer of the Grand Duke of Tuscany. His professional duties included the making of the maps for the Sala di Geografia in the Palazzo Vecchio, the building of instruments, for example the meridiana in the Santa Maria Novella to aid the reform of the calendar, and the teaching of mathematics.[30] The

[25] 'quae Solis instar, reliquis lucem impertiat'. Pena, Jean. 'De Usu Optices Praefatio.' In *Petrus Ramus - Audomarus Talaeus: Collectanae Praefationes, Epistolae, Orationes*, edited by Walter J. Ong, 140-58. Hildesheim: Georg Olms Verlagsbuchhandlung, 1969, p. 140. Future references are to page numbers in the edition of Ong. There is a French translation in De Rochas, A. 'Une Leçon d' Ouverture au Collège de France en l' an 1555.' *Cosmos* 41 (1899): 311-14, 40-3, 402-4. Unless otherwise indicated, translations are mine.

[26] On Ramus and Dürer, see in particular Verdonk, J. J. 'Über die Geometrie des Petrus Ramus.' *Sudhoffs Archiv* 52 (1968): 371-80, p. 375.

[27] 'allen kunstbegirigen zu gut geschicht/ und auch nicht allein den maleren/ sonder Goldschmiden Bildhameren Steinmessen Schreineren und allen den so sich das mass gebrauche dienstlich sein mag'. Dürer, Albrecht. *Underweysung der Messung mit dem Zirckel und Richtscheit*. Nürnberg, 1525, f. Ajv.

[28] Ryff wrote on the unity of theory and practice that 'die Architectur iren ursprung hab aus zweyen ding/ Als ertslichen aus der Fabrica/ das ist der handt arbeit/ dadurch man ein ding in das werck bringen mag/ Zum andern aus gewisser ursach/ warumb uns gefalle/ und fur gut ansehe/ eind ding jekundt geradt solcher gestalt in das werck zu bringen.' Rivius. *Vitruvius Teutsch*. Nuremberg: Johan Petreius, 1548, f. 6v. In his commentary on Vitruvius, Barbaro emphasized the importance of practice for a proper understanding of sundials. 'Io voglio avvertiti quelli, à iquali pareranno queste cose difficili, che se penseranno intenderle bene senza farne la prova, si potranno facilmete igannare, ne bisogna dire, che siano scritte difficilmente, perche in ogni esperienza e difficulta, dove non e stato essercitio, e veramente io posso affermare d' haverne inteso, e questo molto piu facendo, e isperimentando, che leggendo, pure i principii sono di grande importanza'. Barbaro, Daniele. *I dieci Libri dell' Architettura di M. Vitruvio.* Venetia: Francesco Marcolini, 1556, p. 243. See Long, Pamela Olivia. *The Vitruvian Commentary Tradition and Rational Architecture in the Sixteenth Century: A Study in the History of Ideas*. Ph.D., University of Maryland, 1979, pp. 83-98; Long, Pamela O. 'The Contribution of Architectural Writers to a 'Scientific' Outlook in the Fifteenth and Sixteenth Centuries.' *Journal of Medieval and Renaissance Studies* 15 (1985): 265-98.

[29] Settle, Thomas B. 'Egnazio Danti and Mathematical Education in Late Sixteenth-Century Florence.' In *New Perspectives on Renaissance Thought: Essays in the History of Science, Education and Philosophy*, edited by John Henry and Sarah Hutton, 24-37. London: Duckworth, 1990.

[30] Heilbron, J.L. *The Sun in the Church: Cathedrals as Solar Observatories*. Cambridge London: Harvard University Press, 1999, pp. 47-81; Righini Bonelli, Maria Luisa and Tom Settle. 'Egnazio's Danti's 'Great Astronomical

maps were left partly unfinished, when Danti, after the death of Cosimo I, had to leave Florence, to become, first, professor of mathematics at the university of Bologna, and, from 1581, the pontificial mathematician in Rome. He published several treatises on the design of mathematical instruments, the astrolabe and Orsini's 'radio latino'.[31] Besides his Italian translation of Euclid, he also published a commentary on the manual on perspective of Vignola.

Danti's Italian translation of Euclid was based on the Euclidean text, as established by Pena.[32] Danti's preface to Euclid was nothing more than a paraphrase of Pena's 'De usu optices', although, as his other publications show, Danti was well acquainted with the mathematical instrument design and perpective tradition of contemporaneous sixteenth century Italy.

> Perspective is one of the more important [arts], because without it, none of the liberal arts could be perfectly understood. Thus, with great reason, it might be said that it brings ligt and splendor to all the sciences, like the sun gives light to the stars. … Also, to everyone should it be known to what extent and how perspective enriches Geography, because it alone shows the way to reduce in a plane, oval or circular, or in several other ways, the space of the complete earth, and of the particular provinces … And not less help does it offer to Astronomy, because we know with certainty the size of the stars, and the position of the heavens, by which we know that the Moon is lower, and Saturn higher than the Sun, and lower than the fixed stars in the eight sphere. It also shows the distance from one Heaven to another, and from one star to another, and the reason why it happens that the stars appear larger in one place of the Heaven than in another. … And leaving aside the advantages and usefulness it offers, to what extent it is necessary to infinite mechanical arts, in particular to Architecture and all the other arts of disegno, as is well known to your most noble members of this Academy [the Accademia del Disegno of Perugia], I shall only say how I cannot but wonder how it is possible that this science of perspective is so low esteemed by learned men.[33]

Quadrant'', *Annali dell' Istituto e Museo di Storia della Scienza* 4 (1979): 3-13; Danti, Egnazio and Stefano Buonsignori. *Le tavole geografiche della Guardaroba Medicea di Palazzo Vecchio in Firenze/The geographical panels in the Medici Guardaroba of Palazzo Vecchio in Florence*. Firenze: Benucci Editore, 1995.

[31] For example, see Danti, Egnatio. *Trattato del radio latino: Istrumento giustissimo & facile più d' ogni altro per prendere qual si voglia misura, & positione di luogo, tanto in Cielo come in Terra*. Roma: Apresso Marc' Antonio Moretti & Iacomo Brianzi, 1586; Danti, Egnatio. *Trattato dell' uso, e fabbrica dell' astrolabio di M. Egnatio Danti del' ord. di S. Domenico. Con il Planisferio del Roias*. Firenze: Giunti, 1578.

[32] Danti's indebtedness to Pena's 'De usu optices' has already been noted in Frangenberg, Thomas. 'Egnatio Danti's Optics: Cinquecento Aristotelianism and the Medieval Tradition.' *Nuncius* 1 (1988): 3-38.

[33] 'la Prospettiva tiene uno de' primi luoghi; poiche senz' essa niuna dell' arti liberali puote perfettamente essere intesa. Onde con gran ragione si puo dire, che come il Sole da luce alle stelle, cosi essa apporti luce, & splendore à tutte le scienze. … A ciascuno anco facilmente puote essere noto quanto, & quale ornamento arrechi la prospettiva alla Geografia, poiche ella sola ci mostra il modo di ridurre in piano, in forma o ovata, o circulare, & in diverse altre maniere proportionevolmente il sito di tutta la terra insieme, & delle provincie particulari & ci fa havere perfetta cognitione delle distanze de' luoghi facendoci conoscere chiaramente non solo la proportione, & convenienza di sito, che ha una regione con l' altra, ma con il Cielo ancora. Et non minore aiuto dà alla Astronomia essendo cagione, che sappiamo al certo la grandezza delle stelle, & la positura de' Cieli, & conosciamo mediante essa la Luna essere piu bassa, Saturno piu alto, che il Sole, & piu basso delle stelle fisse, che sono locate nella ottava sfera. Ci fa anco conoscere veramente la distantia, che è dall' un Cielo, & l' altro, & dall' una all' altra stella, & ci rende la ragione da che avvenga che le stelle ci appariscono maggiori in un luogo, che in un altro del Cielo; le quali sono tutte cose, che ciascuno doverrebbe disiderare di sapere. … E lasciando da banda il raccostare il giovamento, & l' utile, che ella arreca, anzi quanto sia necessaria à infinite arti mecaniche, & particularmente alla Architettura, & à tutte le altre arti del disegno, si come à voi nobilissimi Accademici è notissimo, diro solo, che non posso se non meravigliarmi grandemente come possa essere, che appresso le persone scientiate, & dotte questa scienza della Prospettiva sia havuta in cosi poca stima' Danti, Egnatio. *La prospettiva di Euclide*. Fiorenza: Giunti, 1573, quoted from 'Proemio'.

Thus, Pena was drawing upon a well-established perspective tradition that claimed perspective to be important to a whole range of practical mathematical arts, important to dialling, map-making, astronomical measurement, painting, architecture, surveying. What else to conclude from this than the ubiquity of perspective?[34] Consequently, included into what sixteenth century mathematical practitioners understood to be optics were projection techniques, to make a map or a painting, and intimately connected to the mathematical instruments developed and used in the context of measurement in all the practical arts just mentioned. It is a peculiar characteristic of the sixteenth century that manuals on astrolabes or sundials were considered optics. In one of the next chapters, there will be an opportunity to develop further the content of this cultural transformation of the field of optics. Here, I will focus on its consequences for sixteenth century thoughts on one what was a major focus of medieval perspective, that is vision.

In antiquity, vision was considered explained when contact between the organ of sight, the eye, and the visible object was established.[35] This contact was established by requiring, either, that the eye emitted something (for example, pneuma for the Stoics and Galen, invisible light for Plato), or, that the eye received something (for example, 'eidola' or 'simulacra' for the atomists). These theories are respectively known as extramission and intromission. Extramission was particularly defended by the mathematicians, Euclid and Ptolemy, whose mathematical account of perception rested upon visual rays, forming a visual pyramid, proceeding from the eye. As is well known, the 11th century Arab mathematician Alhazen integrated the physical, physiological, mathematical and psychological aspects into one grand synthesis, based on intromission. Alhazen's intromission theory was transmitted to Roger Bacon, providing the starting point for the medieval perspectiva to take shape.[36] In Lindberg's account, the suggestion is quite strong that, once Alhazen's intromission theory appeared on the scene, the extramission theory – marginal exceptions notwithstanding – lost its appeal. However, the fate of extramission was more complicated, precisely, because of the mathematical practitioners' appropriation of optics.

First, due to the influence of al-Kindi's 'De radiis stellarum', Bacon allowed extramission to play a small but important part in his overall intromission synthesis. For Bacon, the visual power is not only a recipient, but also an agent, a source of species, which ennobles the medium and the species of the visible object, making them capable of stimulating sight.[37] Tachau has argued that the context of Bacon's admission of visual radiation was eventually astrological. Bacon believed that 'the study of light and its effects brought together and explained how the *Microcosmos* that is a human being and the universe, the *Macrocosmos*, interact', or in Bacon's words, 'especially sight, the species which comes to the stars and to which the species of the stars come in order to produce sight'.[38] As will be argued below in this chapter, these astrological concerns were very

[34] See, most cogently argued, Bennett, Jim. 'Projection and the Ubiquitous Virtue of Geometry in the Renaissance.' In *Making Space for Science: Territorial Themes in the Shaping of Knowledge*, edited by Crosbie Smith and Jon Agar, 27-38. London: Macmillian Press Ltd, 1998.

[35] Lindberg, *Theories of vision*, pp. 1-17.

[36] Ibid., pp. 58-86, 107-16.

[37] Ibid., p. 115.

[38] Tachau, Katherine H. 'Et maxime visus, cuius species venit ad stellas et ad quem species stellarum veniunt: Perspectiva and Astrologia in Late Medieval Thought.' *Micrologus* 5 (1997): 201-24, pp. 223-4; 'et maxime visus, cuius species venit ad stellas et ad quem species stellarum veniunt'. Roger Bacon, De multiplicatione specierum, Chapter I, Part 5, line 78-9. Translation in Lindberg, David C. *Roger Bacon's Philosophy of Nature: A Critical Edition, with English Translation, Introduction, and Notes, of De Multiplicatione Specierum and De Speculis Comburentibus*. Oxford: Clarendon Press, 1983, pp. 74-5.

much present among sixteenth century mathematicians. However, even in a less obvious astrological context, namely in Cesariano's commentary on Vitruvius (1520), a similar belief in this interaction 'through light and optics' is found in the representation of a visual pyramid which reaches to the stars, while the light radiation of the sun reaches to the earth. (Figure 2.1)

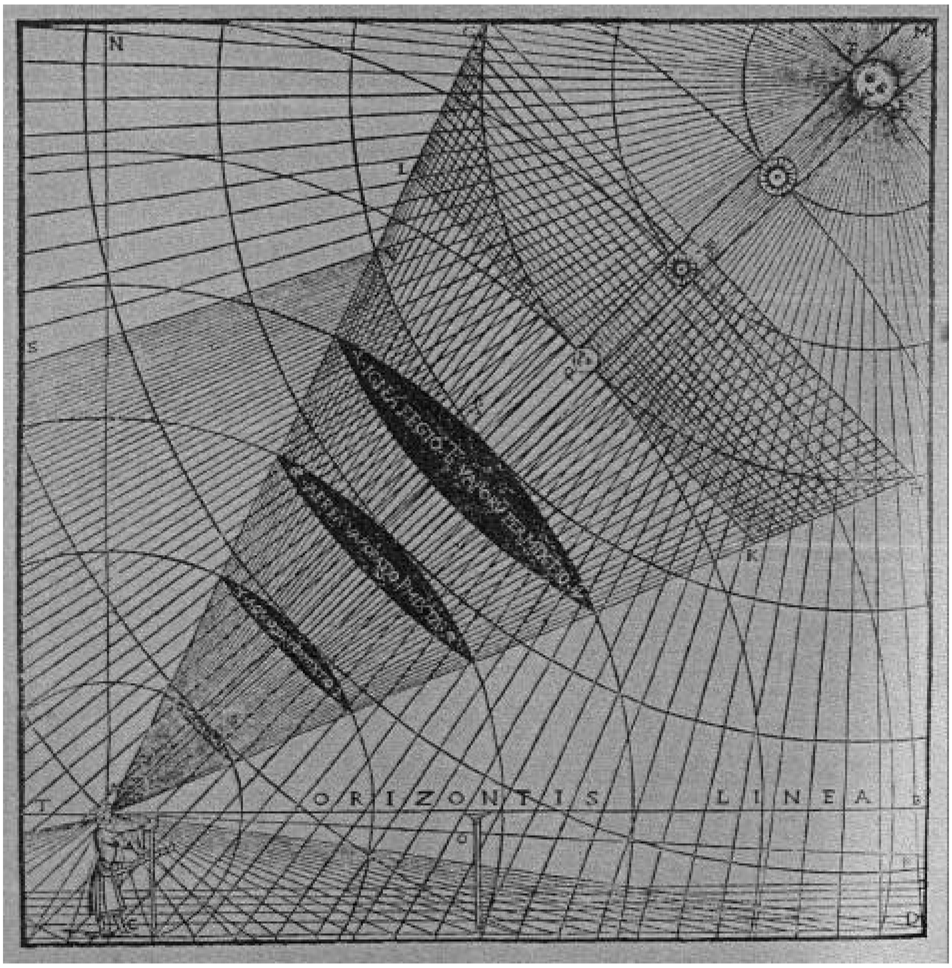

Figure 2. 1

Second, even when such astrological concerns were not present among mathematical practitioners, intromission was not the obvious choice. As shown, the scope and concept of optics for mathematicians in the sixteenth century was highly informed by their involvement with the practical mathematical arts, including painting. For example, the abundant manuals on indirect measurement, that is the measurement of the height of towers, the depth of a well, the distance between two stars, considered their subject to be 'measurement by eye'. These practical mathematical arts, including painting, were grounded in optical theory, more particularly, in the optical theory of Alhazen.[39] However, their endorsement of the optical theory of Alhazen did not necessary entail the adoption of Alhazen's intromission theory of vision. The reason is in Alhazen's optics itself, more particularly, in his account of the certification of sight.

According to Alhazen, vision was a two-stage process of 'aspectus' and 'intuitio'.[40] The 'aspectus' is the first glance that yields only a superficial perception of an object. Next follows the 'intuitio' that gives a 'certified' impression of an object. The difference between the two stages or categories was explained in optical terms. According to Alhazen, the visual pyramid consisted only of those rays that fall perpendicularly on the eye and proceed without refraction to the glacial humor. The non-perpendicular rays, although accounted for, contribute little to vision, because they are refracted and, thus weakened. However, from all these rays, only the central ray, or the axis of the pyramid, falls perpendicularly on the interface between the glacial and vitreous humor, and, thus, goes unrefracted into the optic nerve. Again, since refraction weakens, this central ray is the strongest. Perception through it is the clearest. The 'intuitio' brings points of the visible object, by successive eye movements, or scanning, under the central ray. Thus perceived, it allows vision without any perceptual error, or the certification of sight.

Thus, Alhazen's synthesis preserved the Euclidean pyramid of visual rays. However, in the Euclidean 'Optics', the size of an object was only a function of its visual angle at the eye. Two objects seen under the same visual angle are the same size. In Alhazen's theory of the certification of sight, the size of an object is also a function of its distance measured along the central ray. It allows making a difference of size estimate between two objects seen under the same visual angle, but at a different distance from the eye. The triangulation procedures of indirect measurement are based on Alhazen's theory of the certification of sight, as they make use of the distance measured along the central visual ray in its measurements by formation of similar triangles.

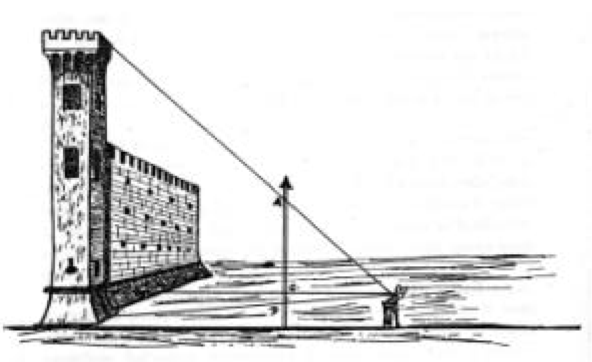

Figure 2. 2

For example, in Alberti's 'Ludi matematici', the height of a tower is measured 'a occhio', by forming a similar triangle by intersecting with a stick the triangle formed by the visual rays sighting the base and the top of the tower.[41] The height taken along the stick is proportional to the height of the tower. In Euclid's visual angle theory, both are the same size. Taking into account Alhazen's measurement of the distance along the visual ray allows to make a difference of size between the two heights and to calculate the unknown height of the tower. (Figure 2.2) Any measurement by sight, however

[39] Camerota, Filippo. 'Misurare 'per Perspectiva': Geometria Pratica e Prospectiva Pingendi.' In *La Prospettiva: Fondamenti Teorici ed Esperienze Figurative dall' Antichità al Mondo Moderno*, edited by Rocco Sinisgalli, 293-308. Firenze: Edizioni Cadmo, 1998.

[40] Lindberg, *Theories of Vision*, p. 84-5.

[41] Alberti, Leon Battista. 'Ludi Rerum Mathematicarum.' In *Opere Volgari*, edited by Cecil Grayson, 131-73. Bari: Laterza e Figli, 1973, pp. 135-6.

complicated with respect to an ordinary stick the instruments used in the procedure might become, is based on the same optical principle.

Alhazen's optical theory of the certification of sight is also at the base of the procedures or projection techniques for representation, including painting. When Alberti wrote the first treatise to discuss perspective, his 'On Painting' (1435), he defined a painting or representation as the intersection of the visual pyramid, similar to the stick which intersected the visual pyramid in his procedure of the indirect measurement of a tower.[42] Also, in agreement with Alhazen, he defined the 'centric ray', the axis of the visual pyramid, through wich vision is clearest.[43] Again, measurements of the objects, also in the picture plane, are based on considerations of proportionalities and similar triangles, thus, the same principles as in his 'Ludi matematici'.

> You have seen how any lesser triangle may be proportional to a greater, and remember that the visual pyramid is made up of triangles. So all we have said about triangles may be transferred to the pyramid, and we may be sure no quantities of the surface that are equidistant from the intersection of the pyramid, undergo any change in the painting; for those equidistant quantities are equal, at any equidistant intersection, to those proportional to them. From this it follows that if the quantities that make up the outline of a surface are not changed, there occurs no change in that outline in the painting. And so it is clear that any intersection of the visual pyramid equidistant from the surface seen is proportional to that surface.[44]

Since Alhazen had stripped away all refracted rays from his 'intuitio' category of vision, mathematical practitioners, basing their procedures of measurement and representation on this theory, did not feel obliged to adopt his intromission theory of vision. To any mathematical practitioner only interested in visual rays as a means to measure, to develop an instrument incorporating triangulation techniques or to invent a projection technique that produces a two-dimensional representation, the question of the direction of these rays might be quite irrelevant. In his 'On painting', Alberti claimed the question of extramission versus intromission, of which he was well aware, to be irrelevant, for his purposes.[45] A similar attitude can be found as late as the second half of the sixteenth century in Barbaro's 'La Prattica della Perspettiva' (1568).[46]

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[42] Alberti, Leon Battista. *On Painting*. Translated by Cecil Grayson. Martin Kemp ed. London: Penguin Books, 1991, p. 48.

[43] Ibid., pp. 43-4.

[44] Ibid., p. 51.

[45] 'among the ancients there was considerable dispute as to whether these rays emerge from the surface or from the eye. This truly difficult question, which is quite without value for our purposes, may here be set aside'. Ibid., p. 40.

[46] 'Imperoche da Greci è detta Optica, da Latini, Prospetto: & per questo nome non intendono uno semplice vedere, ma uno avvertito, & considerat vedere. Percioche il semplice vedere non è altro, che naturalmente ricevere nella virtu del vedere la forma, & la simiglianza della cosa veduta. Ma lo auuertito, & considerato vedere, oltra il semplice, & naturale ricevimento della forma, ha la consideratione, & la investigatione del modo del vedere … Hora non accade, che noi in questo luoco riuochiamo quella quistione, che si suol fare. Se il vedere si fa mandando i raggi dall' occhio alla cosa veduta, o pure riceuendogli mandati all' occhio della cosa veduta: Perche in qualunque modo la cosa si stia, non possono non havere luoco le regole, & i precetti nostri'. Barbaro, Daniele. *La Pratica della Perspettiva*. Edited by Roberto Fregna and Giulio Nanetti. Vol. 8, *Biblioteca di Architettura Urbanistica Teoria e Storia*: Arnaldo Forni Editore, 1980, p. 6. Thus, Barbaro enhanced Alhazen's theory of the certification of sight and, at the same time, considered the question of extramission versus intromission irrelevant for his purposes.

Second, for mathematical practitioners, not surprisingly in light of Alhazen's 'intuitio' category of vision, it was perfectly possible, though not necessarily, to endorse an extramission theory of vision. Thus, when the question of vision was decided, it was not always in favor of intromission. For example, Jamnitzer, who Ramus met in Nürnberg, in the same year, the first's 'Perspectiva corporum regularium' appeared, argued, in this book, that perspectiva is 'an art which teaches the quality, kind and nature of lines and the flux which is projected to and fro from our sight to other things'.[47] No doubt, this is an extramission theory of vision, presumably Platonic, which might not surprise when considering Jamnitzer's focus was on the Platonic regular bodies.

Against this background of Alhazen's 'intuitio' category of vision as the basis of mathematical practice, and Pena's engagement with the mathematical practitioners and the practical utility of optics, it might come as less a surprise that, nothwithstanding he was well aware of Alhazen's intromission synthesis, presumably most directly through the work of Witelo, Pena fully endorsed the extramission theory of the Euclidean optics he was editing.

> Witelo was a man not inferior to Euclid in knowledge and erudition, as his works show, but he had this weakness, common to all ages, to have preconceived opinions, which present themselves as demonstrations. Thus, he states that vision takes place by reception of rays. However, this is not more necessary, than if you would say that it happens by emission. In this book, Euclid clearly teaches that vision happens by rays proceeding from the eyes to the visible objects.[48]

Similar to the tradition of mathematical practitioners Pena was drawing upon, he shifted between irrelevance and Euclidean extramission. However, this hardly reflected Ramus' point of view. In the 'Opticae' of 1606, Ramus and Reisner clearly endorsed intromission.[49] Thus, how should Pena's endorsement of extramission be understood? Was it simply a matter of more and better knowledge of Alhazen and Witelo gained, after editing Alhazen and Witelo, that made a shift from extramission to intromission, possible or necessary for Ramus and Reisner? Or is something else going on? In his edition of Euclid (1573), following Pena not only in his preface, but also commenting on Pena's preface in his additions to the Euclidean theorems, Danti also endorsed Euclidean extramission, balancing it with, as should be by now familiar, pointing to the irrevelance of the question. Danti made clear that he was speaking as a mathematician.

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[47] 'ein kunst die da lehret / von eigenschafft / art und natur / der Linien und Strom so von unserem gesicht auff andere ding hin und wider geworffen werden'. Wenzel Jamnitzer, Perspectiva corporum regularium, quoted from the 'Vorrede'. Edited in Flocon, Albert. *Jamnitzer, Orfèvre de la Rigueur Sensible: Etude sur la Perspectiva Corporum Regularium*. Paris: Gutenberg Reprints, 1981.

[48] 'Vir fuit Vitellio doctrina & eruditione non inferior Euclide, ut ejus monumenta monstrant, sed quae communis omniu temporum labes fuit, opiniones habuit anticipatas, quas pro demonstrationibus saepe obtulit: cujusmodi illud est, visionem fieri receptione radiorum, quod tamen non magis necessarium est, quàm si emissione fieri dicas. Et Euclides hoc libello apertè docet aspectum fieri per radios properantes ab oculis ad res visas.' Pena, 'De usu optices', p. 151. There is a certain reluctance on the part of the commentators to admit, if noted at all, that Pena tends more toward Euclidean extramission than Witelo's intromission. For example, Hallyn has argued that 'Pena juge que la théorie euclidienne de l' extramission des rayons par les yeux est tout aussi possible'. Hallyn, Fernand. 'Jean Pena et l' Éloge de l' Optique.' In *Autour De Ramus: Texte, Théorie, Commentaire*, edited by Kees Meerhoff and Jean-Claude Moisan, 217-32. Paris: Nuit Blanche Editeur, 1997, p. 226.

[49] 'Visio fit specie visibili extrinsecus in oculum recepta'. Risnerus, *Opticae Libri Quatuor*, p. 125.

> As most important foundation, Euclid assumes that the visual rays proceed from the eye and go towards to visible object, and not that the rays proceed from the visible object to find the eye. This has been shown to be true supra by the author of this commentary. Because I intend to deal with this question more diffusively whether sight takes place by rays sent by the eye to the visible object, or rather that the eye receives those rays, which are sent to it by the visible object, here, it is enough for me to have only touched upon the opinion of Euclid, to which, as we see clearly, the complete Peripatetic school is opposed. Nonetheless, we, *as mathematicians*, assume the principles of Euclid and we should follow his opinion, and the opinion of Plato, his master, to which adhere all the mathematicians of antiquity … Whether or not this opinion [extramission] is true or false, is not or little important to the operations of perspective, because whether the rays proceed from the eye to the visible object, or the rays do not proceed, but are received as sent by the visible object, the one as well as the other supposition serve in the same way to the demonstrations of the theorems of perspective.[50]

When Danti came about to go deeper into the question of vision, in his commentary on Vignola's 'Due regole' (1583), he endorsed an intromission theory, within an Aristotelian natural philosophical framework.[51] Is it enough to point out, as Frangenberg has done, that, in the first case, he is only commenting on Euclid, thus following his opinion, while, in the second case, when not limited by the demands of a commentary, he put forward his own intromission view?[52] In my opinion, the differences of opinion, and the ambiguity that was its consequence, is due to the epistemological and sociological border between mathematicians and natural philosophers. As mathematicians, Danti and Pena could well endorse Euclidean extramission, which, due to the theory of the certification of sight, was acceptable within the mathematical framework. There was no need to adopt natural philosophical positions. Eventually, this allowed declaring the solution to the natural philosophical question of vision to be irrelevant to mathematicians, as long as they were speaking as mathematicians, without crossing the border to become natural philosophers.

To conclude, for sixteenth century mathematical practitioners, extramission was not dead. It had its rationale precisely in Alhazen's optics, often considered the end of intromission theories of vision. Moreover, eventually, the problem of vision was often considered not to be relevant to mathematicians. Thus, within the context of sixteenth century mathematicians' appropriation of optics, and, as a consequence, the stronger identification of optics with the practice of perspective and measurement, as also Pena's preface 'De usu optices' shows, the problem of vision, so centrally important to the concerns of medieval optics, was no longer a major issue.

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[50] 'Euclide per principallissimo fondamento della prospettiva presuppone, che i raggi visuali escono dall' occhio, et vanno alla cosa veduta, e non dalle cose vedute escono i raggi, et vanno à trovar l' occhio, il che suffitientemente di sopra si è manifestato esser vero dall' autore della precedente dichiaratione, et perche io intendo con migliore occasione trattare diffusamente questa questione, se il vedere si fa da i raggi mandati dall' occhio alla cosa veduta, o pur l' occhio riceve quegli, che dalla cosa vista le sono mandati, mi basterà qui haver tocco solo qual sia l' opinione d' Euclide, alla quale se bene veggiamo opporsi tutta la squola Peripatetica, noi nondimeno *come Matematici* supponendo i Principij d' Euclide, deviamo seguire la sua opinione, e di Platone suo maestro, alla quale aderiscono tutti i Matematici antichi … La quale o sia vera o sia falsa; nulla o poco importa alle operazioni della prospettiva, percioche o eschino i raggi visuali dall' occhio alla cosa veduta, o non eschino, ricevendo egli quei che dalla cosa vista gli sono mandati; tanto l' una come l' altra suppositione serve nell' istesso modo alle dimstrazioni de' Theoremi della prospettiva.' Danti, *La prospettiva di Euclide*, pp. 7-8, my italics.

[51] Frangenberg, 'Egnatio Danti's Optics', pp. 12-20. See Danti, E. *Le Due Regole Della Prospettiva Practica: A Reproduction of the Copy in the British Library*. Alburgh: Archival Facsimiles Limited, 1987, in particular, pp. 9-11.

[52] Frangenberg, 'Egnatio Danti's Optics', p. 12.

2.2. Atmospheric Refraction in Pena's 'De usu optices'

For Ramus, stressing the utility of geometry, geometry was the 'art of measuring well'.[53] Along the same lines, as Ramus and Reisner in the first proposition of the first book of their 'Opticae' pointed out, optics was the 'art of seeing well', this is 'to judge the truth and falsehood of the visible things accurately and carefully'.[54] Again, herewith, Ramus and his students drew upon the tradition of mathematical practitioners, with their stress on measurement and the design of mathematical instruments. Also, the 'art of seeing well' did not involve a study of the problem of vision, insofar the boundary from mathematics to natural philosophy was not crossed.

However, the definition of optics as 'the art of seeing well' did entail an ingress into natural philosophy, in particular, into cosmology.[55] The ingress was prepared by mathematical practitioners, in particular Gemma Frisius and Petrus Apianus, and appropriated by Pena, who, again, showed him very well aware of such developments in practical mathematics. In his preface, Pena stressed, in particular, the usefulness of optics to astronomy and physics. He argued for the dissolution of the celestial spheres, a central belief of Aristotelian cosmology, by pointing to the absence of astronomical refraction, because Gemma Frisius had been unable to discover it with his state-of-the-art instrument, the 'radius astronomicus et geometricus'. That this was a crossing of the boundary between mathematics and natural philosophy was well realized. It might be argued that the main reason to write the 'De usu optices', was precisely in showing the usefulness of optics to natural philosophy, and, consequently, to elevate the status of optics to a mathematical discipline with cosmological impact. Although Danti left astronomical refraction out of his paraphrase of Pena's 'De usu optices', he very well picked up Pena's message that it was surprising that natural philosophers, different from mathematical practitioners, ignored optics, while it could decide on questions of natural philosophy and cosmology.

> How, then, could a natural Philosopher without perspective understand, and know perfectly, the movement, the rest, the place, the size, and the quality of the natural things, of which all his speculations consist?[56]

Of course, the phenomenon of refraction, and atmospheric refraction, was well known in antiquity and the Middle Ages, but, with its appropriation in the sixteenth century, it gained a new meaning. First, I will give an overview of the study of the cause of refraction in antiquity and the Middle Ages. Next, we will turn to its cosmological significance in the sixteenth century.

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[53] Hooykaas, *Humanisme, Science et Réforme*, pp. 58-9.

[54] 'Optica est ars bene videndi. Optica suo fine definitur, qui est bene videre, id est, de veritate & fallacia visibilium accurate & exquisite judicare'. Risnerus, Fredericus. *Opticae Libri Quatuor Ex Voto Petri Rami*. Casselis: Excudente VVilhelmo VVesselio, 1606, p. 3. Edited by Volgraff, J. A., ed. *Risneri Opticam Cum Annotationibus Willebrordi Snellii*. Vol. 3, *Werken Uitgegeven Vanwege De Rijksuniversiteit Te Gent*. Gandavi: In Aedibus Plantini, 1918, p. 3.

[55] For this section on Gemma Frisius, Jean Pena and (the absence of) atmospheric refraction, I am much indebted to Barker, Peter. 'Jean Pena (1528-58) and Stoic Physics in the Sixteenth Century.' *The Southern Journal of Philosophy* 23, Supplement (1985): 93-107; Hallyn, Fernand. 'Jean Pena et l' Éloge de l' Optique.' In *Autour De Ramus: Texte, Théorie, Commentaire*, edited by Kees Meerhoff and Jean-Claude Moisan, 217-32. Paris: Nuit Blanche Editeur, 1997; Lerner, Michel-Pierre. 'Le Problème de la Matière Céleste après 1550: Aspects de la Bataille des Cieux Fluides. *Revue d' Histoire des Sciences* 42 (1989): 255-80.

[56] 'Come potrà adunq; il Filosofo naturale senza la Prospettiva intendere, & conoscere perfettamente il moto, la quiete, il sito, la grandezza, et qualità delle cose naturali intorno alle quali consiste tutta la sua speculatione?' Danti, *La prospettiva di Euclide*, quoted from the 'Proemio'.

Ptolemy's account of refraction is based on his awareness of a systematic relationship between reflection and refraction. Ptolemy followed the Euclidean extramission theory, that is vision takes place along visual rays proceeding from the eye to the visible object.[57] These visual rays are only altered when they strike a surface, as happens in reflection and refraction, and, consequently, are deflected along another path. In both cases, the physical explanation, based on a sketchy mechanical analogy between the propagation of a visual ray and projectile motion, is the same.[58] While the reflecting surface impedes the passage of the ray or the projectile entirely, the refracting surface does this only partially. Ptolemy assumes a relationship between the density of the medium and the speed of the projectile or the visual ray. Thus, light is thought of as a projectile moving through media with different densities. When the visual ray hits the refracting surface, when going from a rarer to a denser medium, it loses speed and is diverted towards the normal. The greater the density of the new medium, the greater the loss of speed, and, consequently, the more it is deflected towards the normal. Reformulated, in reflection, the density-difference between the two media is too great to allow the traversal of the visual ray.

In light of the discussion of atmospheric refraction, it is important to note that the mechanical analogy is only an analogy to Ptolemy. Thus, Ptolemy assumes that the dynamic effect at the interface of the two media is instanteneous. Once the ray is deflected, it continues its straight path in the new medium. The path of the visual ray is not continually curved, as the path of a real projectile is in a resisting medium. Alhazen developed Ptolemy's physical explanation of refraction by providing a more detailed mechanical analogy lacking in Ptolemy's 'Optics'.[59]

> If one takes a thin board and fastens it over a wide opening, and if he stands opposite the board and throws an iron ball at it forcefully and observes that the ball moves along the perpendicular to the surface of the board, the board will yield to the ball; or, if the board is thin and the force moving the ball is powerful, the board will be broken [by the ball]. And if he [then] stands in a position oblique with respect to the board and at the same distance as before and throws the ball at the same board, the ball will be deflected by the board (unless the latter should be excessively delicate) and will no longer be moved in its original direction, but will deviate toward some other direction.
>
> Similarly, if one takes a sword and places a rod before him and strikes the rod with the sword in such a way that the sword is perpendicular to the surface of the rod, the rod will be cut considerably; and if the sword is oblique and strikes the rod obliquely, the rod will not be cut completely, but perhaps partially, or perhaps the sword will be deflected. And the more oblique the sword [and its motion], the less forcefully it acts on

[57] Smith, A. Mark. *Ptolemy's Theory of Visual Perception: An English Translation of the Optics, with Introduction and Commentary*. Vol. 86 (2), *Transactions of the American Philosophical Society*. Philadelphia: The American Philosophical Society, 1996, pp. 17-21. Compare Lejeune, Albert. *Euclide et Ptolémée: Deux Stades de l' Optique Géométrique Grecque*. Louvain: Bibliothèque de l' Université Bureaux du 'Recueil', 1948.

[58] Ibid., pp. 37, 42-3.

[59] For a discussion of Alhazen and the cause of refraction, see Lindberg, David C. 'The Cause of Refraction in Medieval Optics.' *The British Journal for the History of Science* 4 (1968): 23-38, reprinted in Lindberg, David C. *Studies in the History of Medieval Optics*. London: Variorum Reprint, 1983, pp. 25-9; Smith, A. Mark. *Descartes' Theory of Light and Refraction: A Discourse on Method*. Vol. 77, *Transactions of the American Philosophical Society*. Philadelphia: The American Philosophical Society, 1987, pp. 49-51; Sabra, A.I. 'Explanation of Optical Reflection and Refraction: Ibn Al-Haytham, Descartes, Newton.' In *Actes Du Xe Congrès International d' Histoire des Sciences, Ithaca-26 Viii-2 Ix, 1962*, 551-54. Paris: Hermann, 1964, reprinted in Sabra, A.I. *Optics, Astronomy and Logic: Studies in Arabic Science and Philosophy*. Aldershot Broofkield: Variorum, 1994.

> the rod. And there are many other similar things, from which it is evident that motion along the perpendicular is stronger and easier and that the oblique motion which approaches the perpendicular is [stronger and] easier than that which which is more remote from the perpendicular.[60]

Since this mechanical analogy actually concerns impact on a body, instead of transmission through a medium, it showed vividly, what already was assumed by Ptolemy, that refraction is a surface phenomenon. Second, the analogy showed, as Alhazen was convinced, that the passage along the perpendicular to a surface is the strongest and the easiest, or, mechanically, a blow perfectly along the perpendicular is the most effective. The nearer an oblique motion is to the perpendicular, the stronger and easier it is. If the blow is obliquely, it will be deviated from its original line of impact. When the passage is from a rarer to a denser medium, it is deflected toward the normal. Since motion along the perpendicular is the easiest and the strongest, Alhazen argued, what actually happens when light is deflected toward the normal, is that it is deviated toward the path of least resistance or an easier and stronger path.

Of course, it is not quite clear how this mechanical analogy could explain why light should be deviated toward a direction of easier or stronger passage. Although presented as a mechanical analogy, it introduced an animistic or teleological element into the explanation of the cause of refraction.[61] The ray was thought of as either choosing a direction of easier passage or as preserving its ease of passage uniformly when passing from one medium to another. Through the reading of Alhazen, this explanation of the cause of refraction was transmitted to the medieval perspectivistic tradition.[62] As concerns the cause of refraction, medieval optics left Alhazen's heritage unchanged. The cause of refraction was still thought of in the same way, when Ramus and Reisner wrote their 'Opticae' and, even, when Snellius annotated it at the beginning of the seventeenth century.[63] Thus, Alhazen's mechnical analogy, and its consequences, was still in use, when atmospheric refraction became a cosmologically significant subject in the sixteenth century.

Ptolemy had derived atmospheric refraction from his general principles of refraction.[64] When the visual ray passes from a denser to a rarer medium, it will be deviated away from the normal. Thus, when a visual ray passes from the denser terrestrial atmosphere to the rarer celestial ether, the visual ray will be bent away from the normal. Consequently, celestial bodies will appear closer to the observer's zenith than they actually are. The closer to the observer's zenith a celestial body is, the lesser will be the refraction, until it diminishes to nothing at the observer's zenith. Consequently, at horizon level, atmospheric refraction will be the greatest. To determine

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[60] Alhazen, Opticae Thesaurus, VII.21, in Lindberg, David C., ed. *Opticae Thesaurus Alhazeni Arabis Libri Septem, Nuncprimùm Editi. Eiusdem Liber De Crepusculis et Nubium Ascensionibus. Item Vitellionis Thuringopoloni Libri X*. Vol. 94, *The Sources of Science*. New York London: Johnson Reprint Corporation, 1972, p. 241. Quoted from Lindberg, 'The Cause of Refraction in Medieval Optics', pp. 26-7.

[61] Ibid., pp. 32-3.

[62] Ibid., pp. 29-34; Smith, *Descartes' Theory of Light and Refraction*, pp. 51-6.

[63] Volgraff, J. A. 'Pierre de la Ramée en Willebrord Snel van Royen (1580-1626)'. In *Risneri Opticam*, edited by J. A. Volgraff, 1b-31b, in particular, pp. 28b-29b; Volgraff, J. A. 'Snellius' Notes on the Reflection and Refraction of Rays.' *Osiris* 1 (1936): 718-25; Hallyn, Fernand. 'Kepler, Snellius En De Lichtbrekingswet.' *Academiae Analecta: Mededelingen van de Koninklijke Academie voor Wetenschappen, Letteren en Schone Kunsten van België: Klasse der Wetenschappen* 56, no. 2 (1994): 121-34, in particular, pp. 131-4.

[64] Smith, *Ptolemy's Theory of Visual Perception*, p. 64.

atmospheric refraction, values for the distance of the celestial body and for the height of the atmosphere are needed. Since Ptolemy was not able to determine the height of the atmosphere, it was, unfortunately, impossible for him to determine atmospheric refraction.[65] Medieval perspectivists, for example, Pecham and Witelo, also assumed atmospheric refraction.[66]

Since Alhazen's mechanical analogy entailed that refraction is a surface phenomenon, light of the celestial bodies was considered to be refracted once at the interface between the ether and the atmosphere. It was not until Oresme in his 'De visione stellarum' (1340-1351) that light was thought of as travelling along a curve through a medium of uniformily varying density, as the atmosphere is.[67] It was generally assumed that there indeed was a second phase of refraction within the second medium, where, due the medium's resistance, because of its density, light lost power or speed continously.[68] However, Pecham was the only exception who had suggested that light might diverge from a rectilinear path, when it passes through a single medium of varying density, but no mention is made of it, when Pecham dealt with atmospheric refraction.[69] Oresme's 'De visione stellarum' was generally unknown, even to Kepler. It was not until the second half of the seventeenth century that, first, Hooke, and, later, Newton and Flamsteed, took into account that light travels along a curved path in their account of atmospheric refraction.[70]

Thus, when Gemma Frisius (1508-1555), physician and mathematician of Louvain, took up the subject of atmospheric refraction in his 'De radio astronomico et geometrico' (1545), he ridiculed the view of those, unfortunately unidentified, who assume a non-uniform density of the air, and, thus, that light would travel along a curved path in the earth's atmosphere.[71] Gemma developed

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[65] The first determination of the height of the atmosphere appeared in the 'Liber de crepusculis' of the 11th century Arab astronomer Ibn Mu'adh. In the sixteenth century this work was very popular and published several times. However, it was thought to be an original work of Alhazen by, among others, Ramus and Reisner. The height of the atmosphere was derived from considerations on twilight, considered to be caused by the reflection of sunlight from vapors that rise from the earth alone, and not by refraction. From a value for the solar depression angle and the earth's radius, by a simple geometrical argument, the height of the atmosphere could be calculated to be 50 miles. However, there was no mention of atmospheric refraction in this work. See Sabra, A.I. 'The Authorship of the Liber De Crepusculis: An Eleventh-Century Work on Atmospheric Refraction.' *Isis* 58 (1967): 77-85, reprinted in Sabra, *Optics, Astronomy and Logic*; Goldstein, Bernard R. 'Refraction, Twilight, and the Height of the Atmosphere.' *Vistas in Astronomy* 20 (1976): 105-7, and Goldstein, Bernard R. 'Ibn Mu'adh's Treatise on Twilight and the Height of the Atmosphere.' *Archive for History of Exact Sciences* 17 (1977): 97-118, reprinted in Goldstein, Bernard R., ed. *Theory and Observation in Acient and Medieval Astronomy*. London: Variorum Reprints, 1985.

[66] John Pecham, Perspectiva Communis, III. 12-13. See Lindberg, David C. *John Pecham and the Science of Optics*. Translated by David C. Lindberg. Madison, Milwaukee London: The University of Wisconsin Press, 1970, pp. 222-9. Witelo, Opticae Thesaurus, X. 49-54. See Lindberg, *Opticae Thesaurus*, pp. 444-9.

[67] Burton, Danny Ethus. 'Nicole Oresme's on Seeing the Stars (De Visione Stellarum): A Critical Edition of Oresme's Treatise on Optics and Atmospheric Refraction, with an Introduction, Commentary, and English Translation.' Ph. D., Indiana University, 2000, in particular, pp. 40-2.

[68] Smith, *Descartes's Theory of Light and Refraction*, pp. 51-3.

[69] Ibid., p. 52.

[70] Whiteside, D. T. 'Kepler, Newton and Flamsteed on Refraction through a 'Regular Aire': The Mathematical and the Practical.' *Centaurus* 24 (1980): 288-315.

[71] Gemma Frisius. *De radio astronomico & geometrico liber*. Antverpiae: apud Greg. Bontiu, and Lovanii: apud Petrum Phalesium, 1545, ff. 29r-29v. 'Nevertheless, in order that one not think that they had not seen these things, they also add some new cause, such as the non-uniform [density] of the air which causes images (simulacra) of things [to appear]'. Translation in Goldstein, Bernard R. 'Remarks on Gemma Frisius's De Radio Astronomico et Geometrico.' In *From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences Presented to Asger Aaboe*, edited by J. L. Berggren and B. R. Goldstein, 167-80. Copenhagen: University Library, 1987, p. 173.

an instrument, the 'radius astronomicus et geometricus', based on Levi ben Gerson's Jacob staff, an instrument widely used in the Renaissance for surveying and navigation purposes.[72] The instrument consisted of a longitudinal staff equipped with a traverse staff. Measurement was based on the construction of similar triangles. The traversal staff and the segment of the longitudinal staff made, respectively, the base and the height of a triangle of which the visual ray of the measurer formed the third side. This triangle was similar to the one decribed by the visual rays with the unknown size as its base. Simple triangulation procedures then allowed determining the unknown size, for example, the breadth of a building or the height of a mountain. While Levi ben Gerson's original Jacob staff came with traversal staffs of different lenghts, Gemma's 'radius' had only one graduated crosspiece, equipped with sliding pinnules, to make effectively traversal staffs of different lenghts, to be read off along the scale.[73]

The 'radius' was also used to measure the size of celestial bodies and the distance between stars. From these measurements, Gemma concluded that there was no atmospheric refraction. To this end, he gave two arguments. First, he referred to the moon illusion, that is the apparent enlargement of the moon and the sun when they are viewed near the horizon.

> Whoever wished to destroy this impression (phantasia) can do so easily with the Radius: let him measure the diameter of the Moon emerging from the horizon when it is full, in southern signs [of the zodiac] or at any other time, and then measure its diameter on the same night when it reaches culmination. When he finds that the diameter does not differ by even a minute from that found at first, he can surely and without doubt believe that the density of the air in no way changes the size of the stars. For although the luminaries seem larger near the horizon, when they are measured with the instrument no difference is perceived. Though it is true that images of things which appear in air that is denser seem larger, in fact they do not become larger as one can see from ordinary experience.[74]

The reference is to Ptolemy's 'Almagest'. In this work, Ptolemy explained the apparent enlargement, assuming it to be real, by refraction caused by vapors in the air through which the moon is viewed.[75] He concluded that the enlargement is due to the moon being seen through a

[72] Ibid., 167-71; Roche, John J. 'The Radius Astronomicus in England.' *Annals of Science* 38 (1981): 1-32, pp. 3-18; For its use in navigation, see Mörzer Bruyns, W.F.J. *The Cross-Staff: History and Development of a Navigational Instrument*. Zutphen: Walburg Pers, 1994; for its use in indirect measuring and surveying, see Camerota, Filippo. *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*. Vol. 14, *Biblioteca Della Scienza Italiana*. Firenze: Giunti Gruppo Editoriale, 1996, pp. 141-2; Turner, Anthony. *Early Scientific Instruments Europe 1400-1800*. London: Sotheby's Publications, 1987, pp. 20-1.

[73] Goldstein, 'Remarks on Gemma Frisius', p. 169.

[74] 'At qui hanc phantasiam facile evertere volet Radii artificio, is Lunae primum ab inferis emergentis diametrum metiatur cum plena est, in signis borealibus vel alio quovis tempore: ac deinde cum coeli culmen ascenderit, rursum eadem nocte eandem diametrum accipiat. Quam si nullo minuto a priori differentem videat, credat indubiè, densitatem aëris nihil mutare re versa stellarum magnitudines. Quanquam enim phantasiam nobis maioris quantitatis exhibeant luminaria circa Finitorem constituta, revera tamen per instrumeta quantumvis magna, nulla percipietur differentia. Crassiora enim fiunt in aëre densiori rerum simulachra, ideoque maiora videntur, & revera non evadunt maiora, ut cuivis per experientiam discere licebit quotidie'. Gemma Frisius, *De radio astronomico*, f. 30r. Translation in Goldstein, 'Remarks on Gemma Frisius', p. 173.

[75] Ptolemy, Almagest, I.3. See Smith, *Ptolemy's Theory of Visual Perception*, p. 151; Sabra, A.I. 'Psychology versus Mathematics: Ptolemy and Alhazen on the Moon Illusion.' In *Mathematics and its Application to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett*, edited by Edward Grant and John E. Murdoch, 217-47. Cambridge: Cambridge University Press, 1987, reprinted in Sabra, *Optics, Astronomy and Logic*.

denser medium, just as the images of objects lying in water are enlarged. Ptolemy's explanation is wrong, because, as he showed him well aware in his 'Optics', water is a denser medium than air and, thus, this situation cannot be compared to the moon in a rarer medium, the ether, seen by an observer in a denser medium, the earth's atmosphere. In fact, in this case, the size of the moon should be diminished.[76] Gemma denied that there was any real difference of the size of the moon when it was viewed at the horizon or at zenith. Therefore, there is no atmospheric refraction.

Second, Gemma Frisius also denied that, if measured with the 'radius', there was any real difference in the distances between the stars, often attributed to atmospheric refraction, when toward the horizon or when toward the zenith. Therefore, there is no atmospheric refraction.

> For, though the distances between stars near the horizon appear to be greater than when they are high in the sky, nevertheless, when they are measured with the Radius, they do not differ at all.[77]

From Gemma Frisius' conclusion of the absence of atmospheric refraction, Pena took his argument to attack the Aristotelian cosmology in his 'De usu optices'.[78] Aristotelian cosmology taught that the heavens were made of celestial spheres. The celestial spheres consisted of the space envelopping the orbs that carried each planet around the center of the world. The celestial spheres were made of a substance, ether, which was assumed to be rarer than air. Consequently, it was possible to attack Aristotelian cosmology, and its celestial spheres, seperating the planets from each other with their impenetrable surfaces, by denying the existence of a celestial matter completely different from the air of the earth's atmosphere.[79]

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[76] The 'Almagest' is only one of the three occasions when Ptolemy gave an explanation of the moon illusion. There is one very obscure passage in the 'Planetary Hypotheses', where he seems to imply that the perceptual differences arise from differences in distances. In his 'Optics', his explanation is based on the statements that, (1) the relative difficulty of looking upwards, which makes the moon appear farther away than when viewed ordinally or horizontally, (2) objects that subtend the same angle at the eye appear to be smaller when they seem to be nearer, (3) therefore, the moon will appear smaller toward the zenith than toward the horizon. The cause of the difference in apparent distances might be in (1) the viewing posture, sometimes confirmed by modern psychological research, or (2) in a reasoning based on the relation between the brightness and size of an object and atmospheric perspective, in particular that (2a) dimmer objects appear larger than brighter ones of the same size lying at the same distance, and (2b) atmospheric perspective, that is objects whose color is dimmer appear to be farther away than objects of a brighter hue. Since the moon seen toward the horizon seems dimmer than when viewed toward zenith, it must appear larger at the horizon than at zenith. Nowadays, the moon illusion is usually attributed to psychological factors, although some research also claims physiological factors to be important. A cleary formulated psychological explanation of the moon illusion was already invoked by Alhazen in his 'Optics'. See Smith, *Ptolemy's Theory of Visual Perception*, p. 151; Sabra, 'Psychology versus Mathematics', pp. 218-27; Ross, Helen E., and George M. Ross. "Did Ptolemy Understand the Moon Illusion?" *Perception* 5 (1976): 377-85. For a recent discussion of the moon illusion, from two different points of view, see Kaufman, Lloyd, and Irvin Rock. 'The Moon Illusion.' *Scientific American* 207 (1962): 120-30; Enright, J. T. 'The Moon Illusion Examined from a New Point of View.' *Proceedings of the American Philosophical Society* 119 (1975): 87-107.

[77] 'Nam & stellarum distantiae circa Finitorem longè maiores apparent, acceptae tamen per Radium, non differunt ab iis quae in sublimi apparent'. Gemma Frisius, *De radio astronomico*, f. 30r. Translation in Goldstein, 'Remarks on Gemma Frisius', p. 173.

[78] Pena's indebtness to Gemma Frisius has been particularly stressed by Hallyn, 'Jean Pena', p. 222.

[79] For the background to this argument, see Grant, Edward. 'Celestial Orbs in the Latin Middle Ages.' *Isis* 78 (1987): 153-73; Rosen, Edward. 'The Dissolution of the Solid Celestial Spheres.' *Journal of the History of Ideas* 46 (1985): 13-31; Jardine, Nicholas. 'The Significance of the Copernican Orbs.' *Journal for the History of Astronomy* 13 (1982): 168-94; Grant, Edward. 'Celestial Matter: A Medieval and Galilean Cosmological Problem.' *Journal of Medieval and Renaissance Studies* 13 (1983): 157-86; Lerner, Michel-Pierre. *Le Monde des Sphères II: La Fin du*

First, Pena argued that celestial matter was not different from the air of the earth's atmosphere. In Pena's opinion, there was no other substance between heaven and earth than ordinarily air, or 'pneuma', a concept, as Barker has shown, Pena derived from Stoic cosmology, which, traditionally, had not accepted the Aristotelian distinction between heaven and earth.[80] If celestial matter was ether, as the Aristotelians wanted it to be, different in density from the air of the earth's atmosphere, there should be atmospheric refraction. However, as Pena learned from Gemma Frisius, in disagreement with Witelo, no atmospheric refraction was found.

> Witelo claims that the celestial matter is more radiant, more rarefied and more limpid than the transparency of the air, consequently, that the air is different from the celestial matter. From this, he goes at pains to demonstrate that the distance between two stars appearing at the horizon, measured with an instrument, is different than when the same stars go through zenith. I would readily concede this, if Gemma, in his explanation of the radius astronomicus, had not taught that the distances between the stars on whatever altitude, measured with his instrument, always appear to be the same. Thus, the art of optics teaches that the medium between us and the globes of the fixed stars is air.[81]

Second, according to convential Aristotelian cosmology, the light of the fixed stars would not only have to pass the ether composing the spheres of the heavens and the earth's atmosphere, both of different densities, but, also, a sphere of fire composing the outermost celestial sphere, again with a different density. Again, Pena invoked Gemma's argument from the absence of atmospheric refraction to deny the existence of such a sphere of fire.[82] Consequently, the denial of the existence of a sphere of fire undermined the Aristotelian theory of comets, and the Milky Way, which until the sixteenth century, were grouped together with other 'meteorological' phenomena, like the rainbow and halo's, discussed in Aristotle's 'Metereologica'.

In convential Aristotelian cosmology, comets were considered sublunary phenomena. They were exhalations rising from the earth or created by the motion of heavenly bodies that decomposed the terrestrial element of fire in the sphere below the moon. In his 'Astronomicus Caesareum' (1540), Apian agreed that at least some comets could indeed be accounted for in Aristotelian terms, but not the comet that had recently appeared in the skies in 1531. Apian found out that the

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Cosmos Classique. Paris: Les Belles Lettres, 1997, pp. 3-20; Randles, W. G. L. *The Unmaking of the Medieval Christian Cosmos, 1500-1760: From Solid Heavens to Boundless Aether*. Aldershot: Ashgate, 1999, pp. 1-63; Donahue, William H. *The Dissolution of the Celestial Spheres, 1595-1650*. New York: Arno Press, 1981, pp. 21-63.

80 Barker, 'Jean Pena', pp. 99-101. See also Barker, Peter. 'Stoic Contributions to Early Modern Science.' In *Atoms, Pneuma, and Tranquillity: Epicurean and Stoic Themes in European Thought*, edited by Margaret J. Osler, 135-54. Cambridge: Cambridge University Press, 1991, pp. 138-44; Barker, Peter, and Bernard R. Goldstein. 'Is Seventeenth Century Physics Indebted to the Stoics?' *Centaurus* 27 (1984): 148-64, pp. 153-5.

81 '… qua coporis coelestis nitorem rariorum & limpidiorem asserit aëris perspicuitate, unde sequitur, aëra à coelo distingui: Id enim ex eo demonstare nititur, quòd duarum stellarum àb Horizonte, emergentium distantia per instrumentum capta, diversa appareat ab ea quam habent eaedam stellae per verticem transeuntes: quod facilè conderem, nisi Gemma in explicatione radii Astronomici doceret distantias syderu' in qualibet altitudine positorum per instrumentum observatas, easdem semper apparere. Ergo docet Optica ars quicquid medium est inter nos & fixarum stellarum globos, aëra esse.' Pena, 'De usu optices', p. 142.

82 'Cùm ergo necesse sit, si supra aëra ullus ignis sit, stellas per refractionem semper & in qualibet coeli parte cerni, excepto tantum puncto verticis: stelle autem nullam ejusmodi refractionem admittant, ut antea ex Gemmae observationibus ostendimus: quis no videt ex opticis oraculis concludi, nullam ignis regionem inter aëra & Lunae orbem interjici?' Pena, 'De usu optices', p. 149.

tail of the comet always pointed away from the sun. Moreover, he argued that this tail was caused by the light of the sun.[83] Taking up Apian's finding of antisolarity in his 'De radio astronomico et geometrico' (1545), Gemma hypothesized that the tail of the comet is due to the refraction of the solar rays, which makes these rays converge.[84] Pena's account is again strongly indebted to Gemma Frisius. From Gemma's introduction of refraction into the explanation, he deduced that these comets resemble burning crystal spheres, because only transparent bodies, 'clear like the purest glass', are able to form such pyramids of refraction, according to optical theory.[85] Thus, for Pena, Apian's finding of antisolarity followed from the nature of the comets.

This argument alone should have been convincing that comets are no sublunary phenomena, since terrestrial fires hardly are able to focus rays in the way just described. However, Pena added another argument, directly based on the distance of the comet from the earth. This argument was not based on parallax, a mathematical technique already applied to the study of comets by Toscanelli and Regiomontanus, but which had not delivered results incompatible with Aristotelian doctrine.[86] Bringing in the perception of the viewer, Pena's argument was based on the proposition that, if bodies move with equal speed, as is the case for heavenly bodies, according to Pena, those that appear to move most slowly are the farthest away.

> But since the motion of comets is sometimes slower and sometimes faster than the motion of the Moon, one concludes that certain comets move in the great space above the Moon, on the grounds that among things which move with equal speed, those which appear to move more slowly are further away.[87]

In convential Aristotelian cosmology, the Milky Way was considered of the same origin as the comets. Aristotle considered the Milky Way to be the slow burning of a belt of dry exhalations raised into the zone of fire below the sphere of moon and ignited by the movement of the stars. Aristotle's opinion was in fact rather unpopular. Few medieval and sixteenth century authors considered the Milky Way to be a sublunar phenomena; most authors considered it to be a zone of closely-spaced stars, or a denser part of the sphere of stars.[88] Pena did not give his opinion about the nature of the Milky Way, but he argued against the locus of the Milky Way. In this

[83] Apianus, Petrus. *Astronomicum Caesareum*. Ingolstadii, 1540, book 2, chapter 15. For a discussion, see Barker, Peter. 'The Optical Theory of Comets from Apian to Kepler.' *Physis* 30 (1993): 1-25, pp. 6-10.

[84] Gemma Frisius, *De radio astronomico*, f. 33r. See Barker, 'The optical theory of comets', p. 10.

[85] 'Necesse ergo est Cometen esse corpus perspicuum & diaphanum vitti instar perlucidum: ab iis enim tantùm corporibus fieri refractionum pyramidas docet Optice.' Pena, 'De usu optices', p. 150. Translation and discussion in Barker, 'Jean Pena', p. 97; see also Barker, 'The optical theory of comets', pp. 11-2.

[86] Jervis, Jane L. *Cometary Theory in Fifteenth-Century Europe*. Vol. 26, *Studia Copernicana*. Wroclaw Warszawa Krakow Gdansk Lodz Ossolineum: The Polish Academy of Sciences Press, 1985, pp. 43-85; 93-120; see also Barker, Peter, and Bernard R. Goldstein. 'The Role of Comets in the Copernican Revolution.' *Studies in History and Philosophy of Science* 19 (1988): 299-319, in particular, pp. 303-13.

[87] 'Sed ex Cometae motu interdum tardiore quàm sit lunae motus, interdum citatiore, colligit Cometarum quosdam longo supra lunam spatio sese versare, eò quòd aequabili celeritate delatorum, que tardiùs ferri videantur, longiùs distent.' Pena, 'De usu optices', p. 151. Translation and discussion in Barker, 'Jean Pena', p. 98.

[88] Jaki, Stanley, L. 'The Milky Way before Galileo.' *Journal for the History of Astronomy* 2 (1971): 161-67.

case, he used the argument from the absence of parallax, because, if the Milky Way was an Aristotelian sublunar phenomenum, it should have a parallax greater than the Moon's.[89]
As will become evident in the chapter 4, Pena's discussion of (the absence of) atmospheric refraction, as an argument against the existence of the Aristotelian spheres, and his discussion of the nature and locus of comets, in the same context, was, more than any other, less original, element of his 'De usu optices' influential. Also, as it will turn out in chapter seven, Galileo was acquainted with these optical discussions, and Pena's 'De usu optices', in particular, and this proved influential in his development of the telescope in the first decade of the seventeenth century. For the moment, it can be concluded that, with Pena's 'De usu optices', optics became a key discipline to solve questions traditionally belonging to the domain of natural philosophy.
However, Pena was drawing highly upon a tradition of mathematical practitioners. He was drawing upon Gemma Frisius' measurements with the 'radius astronomicus et geometricus' to reject the Aristotelian spheres. As concerns comets, the focus of research of mathematical practitioners shifted from their astrological significance to their importance for natural philosophy.[90] The optical account of comets, initiated by Apianus, placed comets in the supralunar realm, before the same could be concluded from parallax. When eventually parallax arguments where used, the arguments were based on a technique developed by Toscanelli and Regiomontanus. In the hands of mathematical practitioners, medieval optical knowledge was allowed to change the look and constitution of the world. When in the preface to his 'Dioptrice' (1611), Kepler published Galileo's letters to him about his telescopic discoveries after reviewing Pena's 'De usu optices', the rhetoric was obvious. Galileo's telescope was the heir of Pena's and the mathematical practitioners' use of optics to uncover the stuff the world was made of.[91]

2.3. Catoptrics, Natural Magic, and Burning Mirrors

Another aspect of optics in which Ramus and his students were particularly interested, was catoptrics, that is the study of mirrors. According to Ramus' biographer Nancel, Ramus' library did not only contain books and manuscripts, but also instruments, models, and mirrors. 'Witness also the countless kinds of optical instruments and *mirrors*, exhibited under glass, agreeable to look at, and giving pleasure when you understand how they work', he wrote, and, again, complaining about the plundering and loss of Ramus' library, 'so many clever inventions, *so many marvellous mirrors*, so many globes and machines, prepared so laboriously over so many years and at such great cost, pillaged, removed and now lost for ever, within the space of scarcely

[89] 'Quid enim de iis statuemus, qui lacteum circulu componut ex calida & sicca exhalatione accensa ex motu plurimarum maximarumque stellarum in eo circulo positarum: quae exhalatio cum sub citimo & lunari orbe sit, sub eisdem tamen stellis perpetuo appareat? Quam opinionem Optica ars ex parallaxi omnino damnat, & falsam judicat. Nam si Luna parallaxin habet, quanto majorem habebit lacteus circulus, si sub Lunae orbe fiat?' For discussion, see Barker, 'Jean Pena', p. 98; Hallyn, 'Jean Pena', p. 226.

[90] For an account of this shift of focus, see Schechner, Sara J. *Comets, Popular Culture, and the Birth of Modern Cosmology*. Princeton: Princeton University Press, 1997, pp. 104-29.

[91] 'doceamque hac Optices parte, quam Dioptricen apellamus, ejusque subjecto, Perspicillis nos de rerum Natura longè admirabilissima brevi temporis spacio dedicisse; adeò quidem, ut puerilia videri possint, quaecunque hactenus Optices benificio detecta ex PENA produximus.' Johannes Kepler, Dioptrice, in Caspar, Max, and Franz Hammer, eds. *Gesammelte Werke*. Vol. 4. München: C. H. Beck'sche Verlagsbuchhandlung, 1941, p. 341.

one hour'.[92] Ramus' interest in mirrors is reflected in Pena's 'De usu optices'. One of the uses of optics is that it allows revealing the forgery of so-called magicians involved in catoptromancy and divination, which were so omnipresent in sixteenth century popular culture.[93] The study of catoptrics shows that their tricks are based on nothing else but knowledge of optics, in particular of how mirrors work, sometimes used inside a camera obscura, or dark room, in projecting images 'in the air'.

> This part of optics, which is called catoptrics, teaches to make a mirror, which does not retain the images of objects, but reflects them in the air. Witelo has written about its composition, and (if it pleases God) we will say someting about it when we explain catoptrics. Thus, should one prohibit cunning women to fool the eyes of men with this mirror, by making them believe they see ghosts raised from death, while they see the image of some hidden child or statue in the air outside the mirror? Because what is most certain, what seems to exceed all faith, is that, if a cylindrical mirror is placed inside a room closed from all sides, and if a mask, or a statue, or whatever else, is placed outside this room, so that there is a fissure in the window or in the door of this room, through which the rays from the mask penetrate [into the room] to the mirror, then the image of the mask, placed outside the room, will be observed inside the room hanging in the air, and, since the reflections from these mirrors are highly deformed and show a misshapen image of a beautiful thing, how hideous and terrible will the image seem of a mask prepared to arouse horror and consternation.[94]

With this reference to catoptrical tricks, Pena was drawing upon a medieval tradition of magic, still very much alive, and certainly not marginal to mainstream developments, in the sixteenth century. It is important to understand that magic was intimately linked with technology and mathematics rather than being its antagonist.[95] In fact, magic was a craft-like activity, or a technology, as its aim was to accomplish things that were considered as going against the natural course of nature. However, although deviating from the normal course of nature, magic should not be regarded as achieving something supernatural. In an Aristotelian framework, the natural and the normal were identical. Notwithstanding magic achieved something going against the

[92] 'Testes opticorum et catoptricorum exhibitae vitro species innumerae, ut aspectando plausibiles, sic intelligendo gratissimae. … tot compositiones artificiosae, tot specula mirifica, tot globi, tot automata [tot comparata laboribus et sumptibus, tot annorum constructa spatiis, vix intra unius horae intervallum sublata et] furtim exportata disperierunt.' From Nancel's 'Petri Rami Vita', translation in Sharratt, 'Nicolaus Nancelius', p. 202-3, my italics.

[93] Fot the general context, see Delatte, A. *La Catoptromancie Grecque et Ses Dérivés*. Liège; Paris: Imp. H. Vaillant-Carmanne; Librairie E. Droz, 1932, pp. 61-86.

[94] 'Docet enim ea Optica pars, quae Catoptrice dicitur, speculum componere, quod objectorum imagines non in se retineat, sed in aëre rejiciat: de cujus compositione & Vitellio scripsit, & nos aliquid dicemus (favente Deo) cùm Catoptrica explicabimus. Quid ergo prohibet mulieres versutas hoc speculo, hominum oculos ludificare, ut evocatos manes mortuorum se videre existiment, cùm tamen aut pueri aut statuae alicujus delitescentis simulacrum in aëre extra speculum videant? Nam quod certissimum quidem est, fidem tamen omnem videtur excedere, Si Cylindricum speculum in cubiculo undecunque clauso statuatur, extra autem cubiculum ponatur larva, aut statua, aut quidlibet aliud, ita tamen ut in fenestra vel ostio cubiculi sit rimula aliqua, per quam radii à larva in speculum irrumpa't, imago larvae extra cubiculum positae, intra cubiculum cernetur in aëre pendens. & cùm reflexiones à speculis illis nonnihil deformes sint, ut rei speciosae deformem imaginem ostentent, quàm terra & terribilis videbitur imago larvae ad horrorem & consternationem comparatae?' Pena, 'De usu optices', p. 157.

[95] Eamon, William. 'Technology as Magic in the Late Middle Ages and the Renaissance.' *Janus* 70 (1983): 171-212; Eamon, William. 'Technology and Magic.' *Technologia* 8 (1985): 57-64; Hansen, Bert. 'Science and Magic.' In *Science in the Middle Ages*, edited by David C. Lindberg, 483-506. Chicago London: The University of Chicago Press, 1978, in particular, pp. 484-5, 495.

natural or normal course of nature, it could still make use of natural laws and processes. In the end, magic was based on a belief in the apparently limitless technological power of man. For example, Giovanni Fontana (who will be discussed below), a physician, mathematician and instrument/machine designer of the beginning of the fifteenth century, regarded his technological inventions as some kind of magic and himself as a magician, even a sorcerer. In his treatise on military machinery, 'Bellicorum instrumentorum liber', he showed a kind of magic lantern designed to project images of demons, apparently to terrify the enemy.[96] (Figure 2.3) There seemed to have been no clear-cut distinction between where technology ended and magic started.

Figure 2.3

[96] Giovanni Fontana, Bellicorum instrumentorum liber, Bayerische Staatsbibliothek (München), Cod. Icon. 242, f. 70r. Reproduced in Battisti, Eugenio, and Giuseppa Saccaro Battisti. *Le Macchine Cifrate di Giovanni Fontana: con la Riproduzione del Cod. Icon. 242 della Bayerische Staatsbibliothek di Monaco di Baviera e la Decrittazione di esso e del Cod. Lat. Nouv. Acq. 635 della Bibliothèque Nationale di Parigi*. Milano: Arcadia Edizioni, 1984, p. 140.

From the Middle Ages to the sixteenth century, there is a shift towards technology becoming more mathematical than magical. However, this certainly did not end the magical tradition. In fact, often the opposite was achieved. Instead of mathematics driving out the magical, mathematics, and the mathematical arts based on it, became regarded as some kind of magic itself, as is reflected in titles of books, such as John Wilkins' 'Mathematicall Magick' of the 1630s.[97] Moreover, in the sixteenth century, magic, at least polemically, became identical with technology itself, as 'white' or 'natural magic' was opposed to 'black' magic, making use of demons, evil spirits and the like. One of its most well-known exponents, Giambattista Della Porta (who will be discussed below) defined natural magic as 'the practical part of natural philosophy, which produceth her affects by the mutual and fit application of one natural thing unto another', and he polemically opposed it to another kind of magic, which he called sorcery.

> There are two sorts of Magick: the one is infamous, and unhappie, because it hath to do with foul spirits, and consists of Inchantments and wicked Curiosity; and this is called Sorcery; an art which all learned and good men detest; neither is it able to yeeld any truth of Reason or Nature, but stand meerly upon fancies and imaginations, such as vanish presently away, and leave nothing behinde them … The other Magick is natural; which all excellent wise men do admit and embrace, and worship with great applause …[98]

Della Porta's 'Magia naturalis' (1589) was a book that dealt with a range of Renaissance crafts and technologies from winemaking, gardening and the making of perfume to the making of optical instruments and their use for entertainment. Against this background of the tradition of natural magic, Pena's 'De usu optices', as is evident from his last quote, wanted to get rid of sorcery and demons, but, also, to show how mirrors could amuse and entertain, in a tradition of natural magic, for those who understand their mathematics, in casu their optics. Apparently, optics as the 'ars bene videndi' could also be turned on its head, using optics not only to correct vision, but also, once the mathematics are understood, to use it to deceive vision for amusement.

> What should someone fear who has learned from optics to construct a mirror, in which one and the same thing is seen one hundred times or in which many images of seductive dancers are seen; who understands to place a mirror so that in it you see those things which happen in the streets and houses of strangers? who knows that there certainly is a place, at which, if you look into a concave mirror, you will see but your eye? who knows that a mirror from plane mirrors can be constructed so that, he who looks into it, sees his image flying? Tell me, he who understand these things from optics, does he not easily recognize the prestige of the Thracian women? Does he not distinguish forgery and imposture from the truly physical things?[99]

[97] Zetterberg, J. Peter. 'The Mistaking of 'the Mathemathicks' for Magic in Tudor and Early Stuart England.' *Sixteenth Century Journal* 11 (1980): 83-97, in particular, p. 89.

[98] Porta, John Baptista. *Natural Magick*. Edited by Derek J. Price, *The Collector's Series in Science*. New York: Basic Books, Inc., 1957, p. 1.

[99] 'Quid enim reformidabit is qui ex Opticis didicerit, speculum construi posse, in quo unus & idem videat sui centum aut eo plures imagines choreas ducentes? Qui intelligat speculum ita collocari posse, ut in eo videas ea quae fiunt & in vicis & in alienis aedibus? Qui sciat certum esse locum, & quo si inspicias speculum concavum, tuum oculum tantummodo visurus sis? Qui sciat speculum è planis speculis ita construi posse, ut qui se in eo aspiciat, suam imaginem volantem videat? Cedo, qui ista ex Opticis intelliget, nonné mulierum Thasselicarum praestigias facilè agnoscet? Nonné fucum & imposturam à rebus verè physicis distinguet?' Pena, 'De usu optices', p. 158.

The same kind of optical and catoptrical trick culture pervaded treatises on perspective of the sixteenth century. Mirror anamorphoses, that are deformed pictures that are seen correctly when viewed in a cylindrical or conic mirror, appeared in perspective manuals only in the seventeenth century. For example, Niceron, who considered optical laws a kind of natural magic, discussed and illustrated them in his 'La perspective curieuse' (1638).[100] (Figure 2.4)

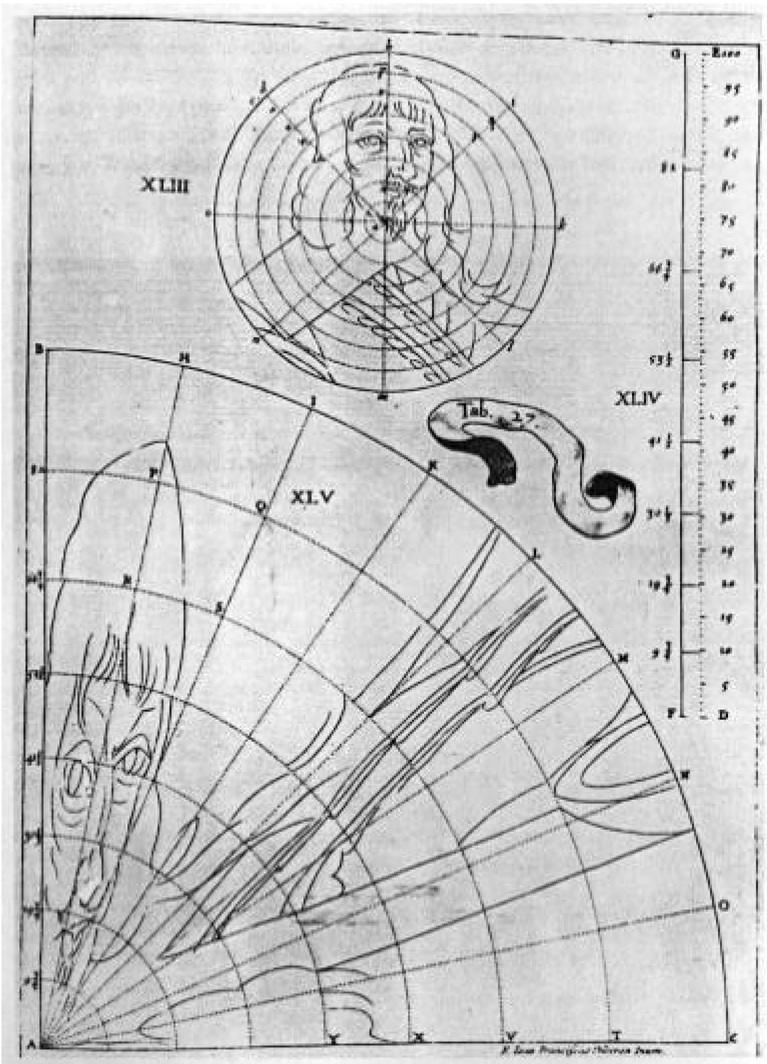

Figure 2.4

[100] Baltrusiatis, Jurgis. *Anamorphic Art*. Translated by W.J. Strachan. Cambridge: Chadwyck-Healey Ltd, 1977, pp. 131-69. For other reproductions of mirror anamorphoses, see Leeman, Fred, and Elffers, Joost, and Schuyt. *Hidden Images: Games of Perception – Anamorphic Art – Illusion from the Renaissance to the Present*. New York: Harry N. Abrams Inc., 1976; Musée des Arts Décoratifs Paris and Rijksmuseum Amsterdam. *Anamorphoses: Jeu de Perspective/Anamorfosen: Spel met Perspectief*. Köln: M. Du Mont Schauberg, 1975. See also Massey, Lyle. 'Anamorphosis through Descartes or Perspective Gone Awry.' *Renaissance Quarterly* 50 (1997): 1148-89.

However, optical tricks with mirrors, which are not anamorphoses in a technical sense, were around in the sixteenth century. In his commentary to Vignola's 'Le due regole' (1583), Danti discussed 'how to make pictures which cannot be seen by the eye if not reflected in a mirror'.[101] This optical game, which, as Danti indicated, was hardly new at the time, consisted of making a painting on triangular pieces of wood, turned away from the observer, which however could be observed by its reflection in a plane mirror, correctly positioned above the original painting. (Figure 2.5)

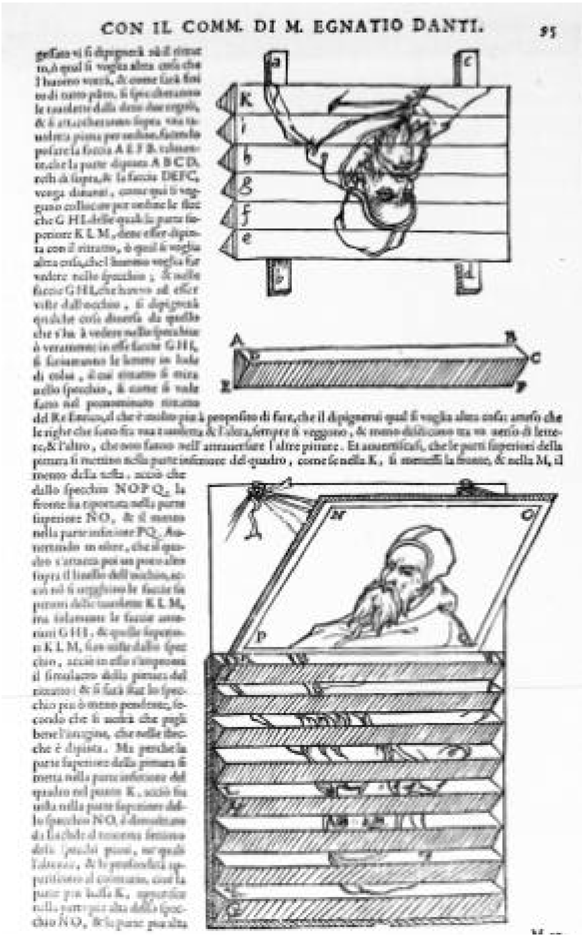
CON IL COMM. DI M. EGNATIO DANTI. 95

Figure 2. 5

[101] 'Come si facciano quelle pitture che dall' occhio non possono essere viste se non riflesse nello specchio'. Danti, E. *Le Due Regole Della Prospettiva Practica: A Reproduction of the Copy in the British Library*. Alburgh: Archival Facsimiles Limited, 1987, p. 55. Most likely on the basis of Danti's instructions, the painter Ludovico Buti (1555-1611) made such an optical game, formerly attributed to Niceron, now in the Museum of the History of Science in Florence, showing two faces, one directly, one reflected in a mirror, for the Grand Duke Ferdinando I in 1593. See Zanieri, Stefania. 'Un Gioco Ottico di Ludovico Buti al Museo di Storia Della Scienza di Firenze.' *Nuncius* 15 (2000): 665-70. Compare Truci, Isabella. 'Le Anamorfosi di Jean François Niceron all' Istituto e Museo di Storia della Scienza di Firenze.' *Annali dell' Istituto e Museo di Storia della Scienza* 1 (1976): 57-64.

Contrary to mirror anamorphoses, perspectival anamorphoses, that are deformed pictures only seen correctly from a lateral viewpoint, were known in the sixteenth century. There is even an early example of an anamorphic object in the work of the fifteenth century painter and mathematician Piero della Francesca, but most examples have come to us from the sixteenth century, for example Leonardo's anamorphic eye, the German 'Vexierbilder' of Dürer's pupil Schön, and, presumably most famous, the anamorphic skull in Holbein's 'Ambassadors'.[102] The geometry of perspectival anamorphoses is equivalent to the geometry for creating an ordinary picture in perspective.[103] Both could be created with a geometrical method, known as the distance point method. In this construction, the distance between the distance point and the central vanishing point determined the ideal viewing distance and point of an observer.

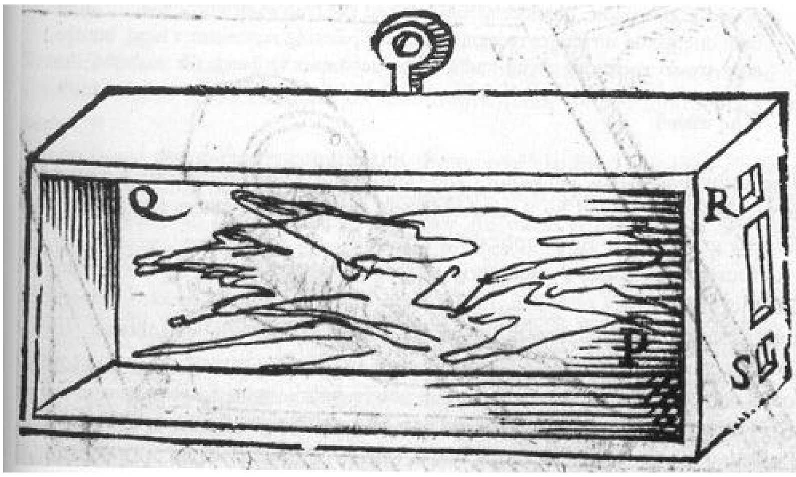

Figure 2.6

Compared to the construction of an ordinary perspective picture, to make an anamorphic picture, the painter only needed to bring the distance point closer to the central vanishing point and to heighten the horizon, or the central vanishing point. Thus, an anamorphic picture determined an ideal lateral viewing point close to the picture plane. Only when viewed from this ideal lateral viewing point, the picture would look 'right'. For example, Danti used an instrument, attributed to Tomasso Laureti, which forced the observer to take a lateral point of view close to the picture plane in order to view the anamorphic picture.[104] (Figure 2.6)

[102] For Leonardo's anamorphoses, see Veltman, Kim H. *Studies on Leonardo Da Vinci I: Linear Perspective and the Visual Dimensions of Science and Art*. München: Deutscher Kunstverlag, 1986, pp. 143-169; for Piero's anamorphic objects, see Andersen, Kirsti. 'The Mathematical Treatment of Anamorphoses from Piero Della Francesca to Niceron.' In *History of Mathematics: States of the Art - Studies in Honor of Christoph J. Scriba*, edited by Joseph W. Dauben, Menso Folkerts, Eberhard Knobloch and Hans Wussing, 3-28. San Diego: Academic Press, 1996, p. 7; for the German "Vexierbilder", see Baltrusaitis, *Anamorphic Art*, p. 11. The anamorphic skull in Holbein's 'Ambassadors' is widely discussed, see, for example, Hallyn, Fernand. 'Holbein: La Mort en Abyme.' *Gentse bijdragen tot de kunstgeschiedenis* 25 (1979): 1-13.

[103] Andersen, The Mathematical Treatment of Anamorphoses', pp. 10-3.

[104] Danti, *Le Due Regole della Prospettiva Practica*, p. 96.

However, Danti's illustration also shows that the geometrical equivalence between an 'ordinary' picture and an anamorphosis was not always perceived in the sixteenth century. Danti formulated the correct rule that the anamorphic picture has to be lengthened more when the observer's eye is closer, and less, when the eye is further away. Thus, while he understood the function of the distance point, he did not understand that an anamorphic picture was created by a central projection, involving a vanishing point, because he based his construction on a parallel projection.[105]

That a central projection, i.c. the distance point method, was needed to make an anamorpic picture, was recorded by the painter Cigoli, a friend of Galileo, in his 'Trattato pratico di prospettiva', a treatise which never went to print until recently.[106] (Figure 2.7)

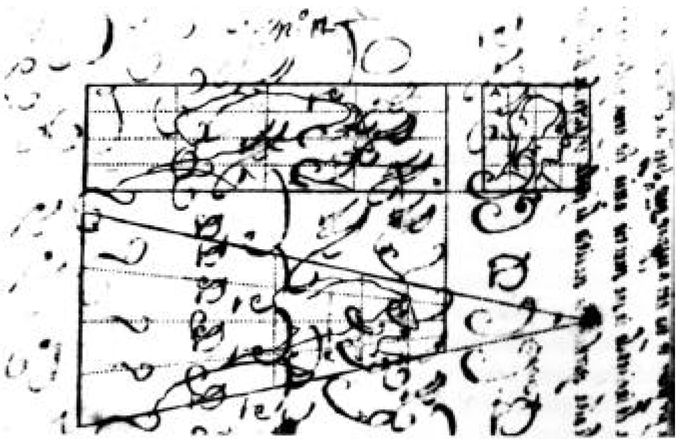

Figure 2.7

Cigoli also invented an instrument to project an ordinary picture into an anamorphosis, which would prove very influential in the French circle of Niceron involved with anamorphoses.[107] Cigoli's 'instrumentum catholicum' was an adaptation of one of Dürer's famous perspective instruments, but it was now used to create an anamorphic picture, either by recording an object on a tilted picture frame or by projecting an ordinary picture, positioned in the picture frame, unto a non-perpendicular plane.

[105] Ibid., p. 96. It's widely discussed why this mathematical equivalence went unnoticed until the seventeenth century, together with the related question why anamorphoses were made, and whether they should be regarded a counter movement in painting. See Naitza, Salvatore. 'Anamorfosi e Legittimità Prospettica tra Rinascimento e Barocco.' *Annali delle Facoltà di Lettere Filosofia e Magistero dell' Università di Cagliari* 23 (1970): 175-239, criticizing Baltrusaitis' 'surrealistic interpretation' of anamorphoses, and Bessot, Didier. 'La Perspective de Nicéron et Ses Rapports avec Maignan.' In *Geometria E Atomismo Nella Scuola Galileiana*, edited by Massimo Bucciantini and Maurizio Torrini, 147-69. Firenze: Leo S. Olschki, 1992.

[106] Profumo, Rodolfo, ed. *Trattato Pratico di Prospettiva di Ludovico Cardi detto il Cigoli: Manoscritto Ms 2660a del Gabinetto dei Disegni d delle Stampe degli Uffizi*. Roma: Bonsignori Editore, 1992, p. 159.

[107] Cigoli, *Trattato*, pp. 149-69. For discussion, see Camerota, Filippo. 'Dalla Finestra allo Specchio: La 'Prospettiva Pratica' di Lodovico Cigoli alle Origini dei Disegni e delle Stampe degli Uffizi.' Ph. D., Università degli Studi di Firenze, 1985-1986.

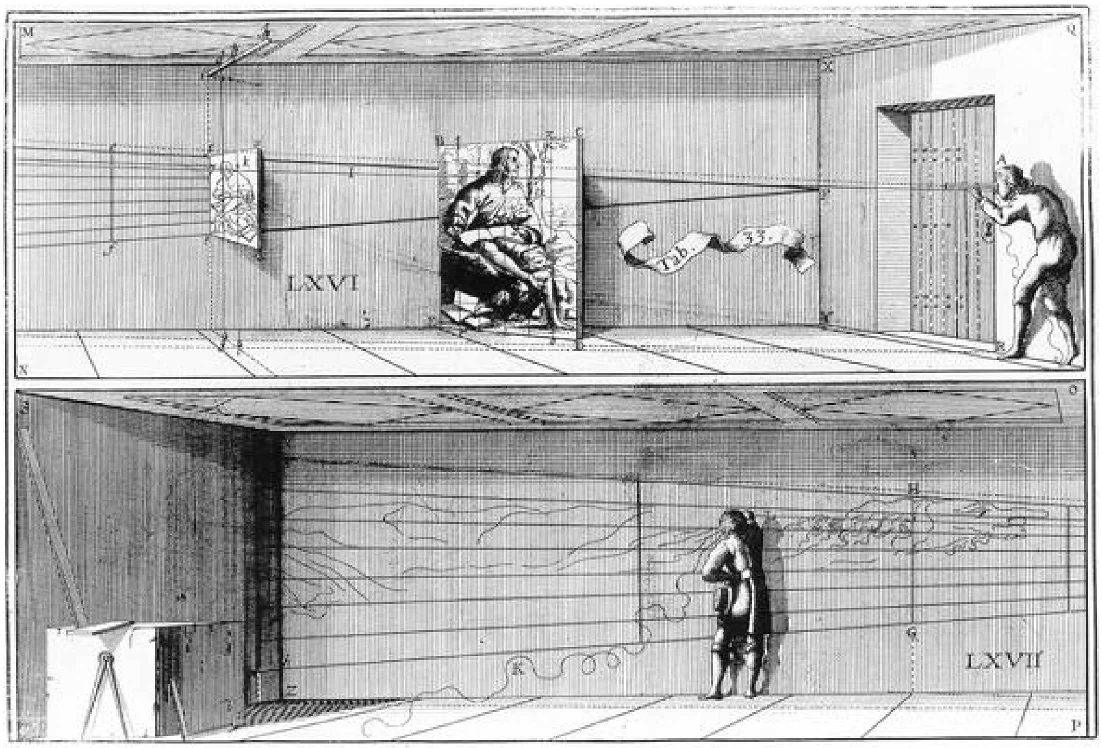

Figure 2.8

On engravings of this instrument to project anamorphic images, for example Niceron's, it is evident that in an anamorphosis left and right are reversed, similar to the left-right reversal which apparently takes place when an object is viewed in a mirror.[108] (Figure 2.8) Thus, perspectival anamorphoses function like mirrors, and are an integral part of the optical and catoptrical trick culture of the sixteenth century, which is often limited to natural magic.

In this tradition of natural magic, Roger Bacon was a highly important figure as a legendary point of departure and a personified symbol of the Renaissance magus.[109] It was not so much what Bacon had actually achieved what mattered, although many of the things he had written made him a very convenient figure for building a legend, but how he was perceived in the sixteenth century. In the sixteenth century, Bacon gained a reputation as a powerful magician. This reputation as a magician was not so much based on the use of demonic or forbidden magic, from which Bacon had always tried to distance himself, but on his grasp of mathematics. One of the most convincing advocates of this image of Bacon was the English mathematician Recorde.

> … many thynges seme impossible to be done, whiche by arte may very well be wrought. And whan they be wrought, and the reason therof not understande, than say the vulgare people, that those thynges are done by negromancy. And hereof came it that fryer Bakon was accompted so greate a negromancier, whiche never

[108] Camerota, Filippo. 'Il Giardino Anamorfico.' In *Il Giardino delle Muse: Arti e Artfici nel Barocco Europeo*, edited by Maria Adriana Giusti and Alessandro Tagliolini, 255-72. Firenze: Edifir Edizioni Firenze, 1995, p. 260.

[109] Molland, A. G. 'Roger Bacon as Magician.' *Traditio* 30 (1974): 445-60.

> used that arte (by any coniecture that I can fynde) but was in geometrie and other mathematicall sciences so experte, that he coulde dooe by theim suche thynges as were wonderfull in the syght of most people.[110]

The achievement of optical marvels played a major part in this image of Bacon magician. Two of these optical marvels stood out as exemplary in the sixteenth century. First, the construction of an optical instrument that showed things that happened elsewhere and/or far away.

> Great talke there is of a glasse that he made in Oxforde, in whiche men myght see thynges that were doon in other places, and that was judged to be done by power of evyll spirites. But I knowe the reason of it to be good and naturall, and to be wrought by geometrie (sythe perspective is a parte of it) and to stande as well with reason as to see your face in a common glasse. But this conclusion and other dyvers of lyke sorte, are more mete for princes, for sundry causes, than for other men, and ought not to bee taught commonly.[111]

For the moment, it is less important what kind of optical instrument, a camera obscura, a lens, a concave mirror, or any combination thereof, Recorde had in mind. That such optical marvels were attributed to Bacon is no coincidence. Bacon had given more than enough reason for it in his own writings, most prominently in his 'Epistola de secretis operibus artis et naturae et de nullitate magiae', which contained references to such optical marvels. For example,

> Glasses so cast, that things at hand may appear at distance, and things at distance, as hard at hand: yea so farre may the designe be driven, as the least letters may be read, and things reckoned at an incredible distance, yea starres shine in what place you please.[112]

The second optical marvel connected with Roger Bacon was the construction of burning mirrors. On several occasions, Bacon had mentioned his own experience with the construction of burning mirrors and his interest in their construction by his contemporary, Petrus Peregrinus.

> I have mentioned that this type of congregation [of rays] can be made by reflection and that a mirror has been made, a model and sign, as it were, of this wonder of nature, so that the possibility of such a work may be seen. But it was with great expenses and labour that it was made, for its contriver was set back by the sum of 100 Parisian pounds and he [Peregrinus] worked at it for many years laying aside study and other necessary occupations. … Certainly if the citizens of Acre [Aconenses] and those Christians who are beyond the sea had twelve such mirrors they could drive the Saracens from their land without any shedding of blood, and there would be no need for the lord king of France to set out with an army to take that land.[113]

In the same passage from the preface of his 'The path to knowledg' (1551), from which the references to Bacon and his optical marvels were taken, Recorde also referred to burning mirrors, in a context which, also, explains the provenance of Bacon's reference to militarian utility.

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[110] Recorde, Robert. *The pathway to knowledg, containing the first principles of geometrie*. London, 1551, p. 7.

[111] Recorde, *The pathway to knowledg*, p. 8.

[112] Roger Bacon, Epistola de secretis operibus artis et naturae ed de nullitate magiae, p. 40. Translation in *Frier Bacon his discovery of the miracles of art, nature, and magick. Faithfully translated out of Dr Dees own copy, by T. M. and never before in English*. London, 1659, p. 20.

[113] Roger Bacon, Opus tertium, 36. Quoted and translated in Molland, 'Roger Bacon as magician', p. 454.

> But to retourne agayne to Archimedes, he dyd also by arte perspective (whiche is a parte of geometrie) devise such glasses within the town of Syracusae, that dyd bourne their ennemies shyppes a great way from the towne, whyche was a mervaylous politike thynge. And if I shulde repete the varietees of such straunge inventions, as Archimedes and others have wrought by geometrie, I should not onely excede the order of a preface, but I should also speake of such thy nges as can not well be understande in talke, without somme knowledge in the principles of geometrie.[114]

Burning mirrors of paraboloid shape became a major focus of research in the Renaissance. As evident from Recorde's quote, it was linked with the figure of Roger Bacon magician, but even more with Archimedes and the legendary story that he built a giant burning mirror with which he burned the ships of the enemy and, consequently, defeated them at Syracuse.[115] The use of the figure of Archimedes in this context shows very clearly, what has been said above, that mathematics, as the basis of instruments, became the dominant component in sixteenth century natural magic. In the sixteenth century, Archimedes was regarded a mathematician, but not yet the mathematician he would become in seventeenth century perception, as a founder of the new physics. For sixteenth century mathematicians, Archimedes, shaped after their own self-image, primarily was a designer of ingenious instruments.[116] That the design of optical instruments, with reference to the legendary story of the burning mirror at Syracuse, was a highly visible component of this image, might also be gained from the fact that seventeenth century optical instrument makers were using the image of Archimedes for advertisement purposes.[117]

There had been no particular attention to the study of burning mirrors of a paraboloid shape in the medieval optical tradition. The parabolic burning mirror, and its focal properties, had been thoroughly discussed in antiquity, by Diocles' 'On burning mirrors', the anonymous Bobbio fragment, and, already triggered by the legend of Archimedes' burning of ships at Syracuse, which seems to have come into existence in the second century AD, by Anthemius of Tralles.[118] It was further developed by Arabic mathematicians, al-Kindi, and his circle, Ibn Sahl and

[114] Recorde, *The pathway to knowledg*, p. 7.

[115] For the legendary story of Archimedes, see Knorr, Wilbur. 'The Geometry of Burning Mirrors in Antiquity.' *Isis* 74 (1983): 53-73, pp. 53-5; Simms, D. L. 'Archimedes and the Burning Mirrors of Syracuse'. *Technology and Culture* 18 (1977), 1-24; Simms, D.L. 'Archimedes and Burning Mirrors.' *Physics Education* 10 (1975): 517-21; Schneider, Ivo. 'Die Entstehung der Legende um die Kriegstechnische Anwendung von Brennspiegeln bei Archimedes.' *Technikgeschichte* 36 (1969): 1-11; Schneider, Ivo. *Archimedes: Ingenieur, Naturwissenschaftler und Mathematiker.* Darmstadt: Wissenschaftliche Buchgesellschaft, 1979, pp. 96-7; Simms, D. L. 'Galen and Archimedes: Burning Mirror or Burning Pitch?' *Technology and Culture* 32 (1991): 91-96.

[116] Laird, W.R. 'Archimedes among the Humanists.' *Isis* 82 (1991): 629-38. For a typical case study of Guidobaldo del Monte (discussed below) in this respect, see also Bertoloni Meli, Domenico. 'Guidobaldo Del Monte and the Archimedean Revival.' *Nuncius* 7 (1992): 3-34; Henninger-Voss, M. 'Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge.' *Isis* 91 (2000): 233-59, pp. 250-1.

[117] Bryden, D. J., and D. L. Simms. 'Archimedes and the Opticians of London.' *Bulletin of the Scientific Instrument Society* 35 (1992): 11-14.

[118] Toomer, G.J., ed. *Diocles on Burning Mirrors*. Berlin Heidelberg New York: Springer-Verlag, 1976, in particular, pp. 9-25; Cantor, M. 'Über das Neue Fragmentum Mathematicum Bobiense.' *Hermes* 16 (1881): 637-42; Knorr, 'The Geometry of Burning Mirrors in Antiquity', pp. 53-70. On the passages of catoptrics atttributed to Archimedes, compare Knorr, Wilbur R. 'Archimedes and the Pseudo-Euclidean Catoptrics: Early Stages in the Ancient Geometric Theory of Mirrors.' *Archives Internationales d' Histoire des Sciences* 35 (1985): 28-104, and, Rome, A. 'Notes sur les Passages des Catoptriques d' Archimède Conservés par Théon d' Alexandrie.' *Annales de la Societé Scientifique de Bruxelles* 52 (1932): 30-41.

Alhazen in his 'De speculis comburentibus'.[119] Of all this material, only Alhazen's 'De speculis comburentibus' seems to have been regularly available to the thirteenth century, when it served as the basis of Witelo's propositions on the burning mirror of paraboloid shape in his 'Perspectiva'.[120] Notwithstanding the rather modest attention to the parabolic burning mirror in Witelo's 'Perspectiva', the burning mirror figured prominently in the sixteenth century perception of Witelo's 'Perspectiva', as can be derived from the frontispieces of its sixteenth century editions.

Witelo's 'Perspectiva' was published twice in the sixteenth century, once in 1535 (with a reprint in 1551) and once in 1572. The earlier edition was prepared by Georg Tanstetter Collimitius (1482-1535) and his student, Petrus Apianus (1495/1501-1552).[121] After receiving his MA at the University of Ingolstadt, Tanstetter came to Vienna in 1502, where he became the most important figure of the 'second Vienna school' of mathematics, after the first generation of Regiomontanus.[122] He was also the physician of Emperor Maximilian I. His most important publications include editions of late medieval mathematical works, of, among others, Peurbach and Regiomontanus, maps and, maybe most important in his function as physician of the emperor, astrological calendars and works on medical astrology or iatromathematics. Apianus came to the University of Vienna to study with Tanstetter, after a stay at the University of Leipzig.[123] His early publications on geography, cartography and cosmography reflect his close association with the interests of his teacher. When he left the University of Vienna in 1523, he became the professor of mathematics at the University of Ingolstadt, after a short stay in Landshut and Regensburg. In Ingolstadt, he established his own publishing press, so he could make a kind of career for himself in publishing introductory textbooks in the various fields of

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[119] Rashed, Roshdi. *Oeuvres Philosophiques et Scientifiques d' Al-Kindi*. Edited by H. Daiber and D. Pingree. Vol. 1: L' optique et la catoptrique, *Islamic Philosophy, Theology and Science*. Leiden New York Köln: E. J. Brill, 1997, pp. 97-102, 111-25; Rashed, Roshdi. 'A Pioneer in Anaclastics: Ibn Sahl on Burning Mirrors and Lenses.' *Isis* 81 (1990): 464-91, pp. 468-474. Reprinted in Rashed, Roshdi. *Optique et Mathématique: Recherches sur l' Histoire de la Pensée Scientifique en Arabe*. Vol. 388, *Collected Studies Series*. Aldershot, Hampshire; Brookfield, Vermont: Variorum, 1992; Heiberg, J.L., and Eilhard Wiedemann. 'Ibn Al Haithams Schrift über Parabolische Hohlspiegel.' *Bibliotheca Mathematica* 10 (1910): 201-37. On the history of burning mirrors, see also Baltrusiatis, Jurgis. *Le Miroir: Révélations, Science-fiction et Fallacies*. Paris: Elmayan/Le Seuil, 1978, pp. 95-121.

[120] Witelo, Opticae Thesaurus, IX. 39-44, in Lindberg, *Opticae Thesaurus*, pp. 398-403, partly translated in Grant, Edward, ed. *A Source Book in Medieval Science*. Cambridge: Harvard University Press, 1974, pp. 417-8.

[121] *Vitellonis Mathematici doctissimi peri optikes, id est de Natura, ratione et projectione radiorum visus, luminum, colorum atque formarum quam vulgo perspectivam vocant, libri X … nunc primam opera … Georgii Tanstetter et Petri Apiani in lucem aedit*. Nuremberg: Johannes Petreius, 1535.

[122] Graf-Stuhlhofer, Franz. *Humanismus zwischen Hof und Universität: Georg Tannstetter (Collimitius) und sein Wissenschaftliches Umfeld im Wien des Frühen 16. Jahrhunderts*. Wien: WUV-Universitätsverlag, 1996; see also, Stuhlhofer, Franz. 'Georg Tannstetter (Collimitius), 1482-1535: Astronom und Mathematiker.' *Lebensbilder aus dem Bayerischen Schwaben* 13 (1986): 18-33; Stuhlhofer, Franz. 'Georg Tannstetter (Collimitius): Astronom, Astrologe und Leibartz bei Maximilian I und Ferdinand I.' *Jahrbuch des Vereins für Geschichte der Stadt Wien* 37 (1981): 7-49; Graf-Stuhlhofer, Franz. 'Zu den Hofastronomen Kaiser Maximilians: Über das Jahrzehntelange Fortwirken Historischer Irrtümer." *Bibliothèque d' Humanisme et Renaissance* 60 (1998): 413-19.

[123] Wattenberg, Diedrich. *Peter Apianus und sein Astronomicum Caesareum - Peter Apianus and his Astronomicum Caesareum*. Leipzig: Edition Leipzig, 1967, pp. 40-52; see also the different articles in Röttel, Karl, ed. *Peter Apian: Astronomie, Kosmographie und Mathematik am Beginn der Neuzeit, mit Ausstelungskatalog*. Buxheim Eichstätt: Polygon-Verlag, 1997, in particular, for his biography, pp. 9-84.

mathematics.[124] Beside geography and cartography, the main focus of his publications is on astrology (to which category, his work on comets belongs) and mathematical instruments.
The edition of Witelo's 'Perspectiva' fitted Tannstetter's and Apianus' program to provide textbooks for the teaching of all branches of sixteenth century mathematics. The elements of the frontispiece with which they provided their edition shows the sixteenth century interpretation of what was considered optics. (Figure 2.9)

VITELLIONIS MATHEMATICI DOCTISSIMI περὶ ὀπτικῆς, id est de natura, ratione, & proiectione radiorum uisus, luminum, colorum atque formarum, quam uulgo Perspectiuam uocant, LIBRI X.

Habes in hoc opere, Candide Lector, quum magnum numerum Geometricorum elementorum, quae in Euclide nusquam extant, tum uero de proiectione, infractione, & refractione radiorum uisus, luminum, colorum, & formarum, in corporibus transparentibus atque speculis, planis, sphaericis, columnaribus, pyramidalibus, concauis & conuexis, scilicet cur quaedam imagines rerum uisarum aequales, quaedam maiores, quaedam minores, quaedam rectas, quaedam inuersas, quaedam intra, quaedam uero extra se in aere magno miraculo pendentes; quaedam motum rei uerum, quaedam eandem in contrarium ostendant: quaedam Soli opposita, uehementissime adurant, ignemque admota materia excitent: deque umbris, ac uarijs circa uisum deceptionibus, a quibus magna pars Magiae naturalis dependet. Omnia ab hoc Autore (qui eruditorum omnium consensu, primas in hoc scripti genere tenet) diligentissime tradita, ad solidam abstrusarum rerum cognitionem, non minus utilia quam iucunda. Nunc primum opera Mathematicorum praestantiss. dd. Georgij Tanstetter & Petri Apiani in lucem aedita.

Norimbergae apud Io. Petreium, Anno MDXXXV.

Figure 2.9

The frontispiece shows a figure sigthing geometrical bodies, representing perspective, a figure viewing his own face in a mirror, representing the study of reflection and image formation in mirrors, a figure with 'broken' legs, representing the study of refraction, a rainbow, referring to the study of meteorological phenomena, and a burning mirror. In this frontispiece, the burning mirror seems to be a paraboloid ring mirror. In their preface, the editors stressed, in particular, the

[124] Van Ortroy, F. 'Bibliographie de l' Oeuvre de Pierre Apian.' *Le Bibliographe Moderne* 5 (1901): 89-156; see also, Gingerich, Owen. 'Apianus' Astronomicum Caesareum and Its Leipzig Facsimile.' *Journal for the History of Astronomy* 2 (1971): 168-77; Gingerich, Owen. 'Astronomical Paper Instruments with Moving Parts.' In *Making Instruments Count: Essays on Historical Scientific Instruments Presented to Gerard L' Estrange Turner*, edited by R.G.W. Anderson, J.A. Bennett and W.F. Ryan, 63-74. Aldershot Brookfield: Variorum, 1993.

importance of image formation in concave mirrors (the 'wonder of images hanging in the air') and burning mirrors in the vein of natural magic.[125]

The frontispiece of the second edition of 1572, prepared by Ramus' student, Reisner, was inspired by the 1535 frontispiece. (Figure 2.10)

Figure 2.10

[125] 'Habes in hoc opere, Candide Lector, quum magnum numerum Geometricorum elementorum, quae in Euclide nusqua extant, tum vero de proiectione, infractione, & refractione radiorum visus, luminum, colorum, & formarum, in corporibus transparentibus atq speculis, planis, sphaericis, columnaribus, pyramidalibus, concavis & convexis, scilicet cur quaedam imagines rerum uisarum aequales, quaedam maiores, quaedam minores, quaedam rectas, quaedam inversas, quaedam intra, *quaedam vero extra se in aëre magno miraculo pendentes*: quaedam motum rei uerum, quaedam eundem in contrarium ostendant: *quaedam Soli opposita, uehementissime adurant, ignemq admota materia excitent*: de q umbris, *ac varijs circa visum deceptionibus, & quibus magna pars Magiae naturalis, dependet* …'. *Vitellonis Mathematici doctissimi peri optikes,* quoted from the editors' introduction on the first page, my italics. See also Kühne, Andreas. 'Peter Apian als Herausgeber der 'Perspectiva Communis' von Witelo.' In *Peter Apian: Astronomie, Kosmographie und Mathematik am Beginn der Neuzeit, mit Ausstelungskatalog*, edited by Karl Röttel, 233-38. Buxheim Eichstätt: Polygon-Verlag, 1997.

In his preface to the edition, Reisner singles out four elements which were important to the study of optics, with which we are already familiar from Pena's 'De usu optices': (1) the study of the matter, number, order and movement of the celestial bodies, that is the relevance of optics for astronomy, (2) the study of meteorological phenomena, primarily the rainbow, (3) the study of mirrors, image formation, in particular, the images 'in the air' known from the context of natural magic, as well as burning mirrors, and (4) the relevance of optics for painting, architecture and mechanics.[126] The 1572 frontispiece shows the same elements as the 1535 frontispiece, however, without a man sighting geometrical bodies, but adding elephants crossing a bridge. This was reference to a story in book 22 of Livius' 'Ab urbe condita', in which Hannibal deceived the eyes of the Roman enemy, by setting fire to bundles of torch-wood tied to the horns of a herd of oxens. When the chaotically running herd was driven towards the camp of the Roman enemy Fabius, his soldiers thought it to be an enormous army of men approaching and fled. According to Reisner, it not only showed the particular optical phenomenon that light appears to be larger in the dark, or against a dark background, but, more generally, the power of light, making it a fit subject, representing the importance of the study of optics, to be shown on the frontispiece of the edition of Witelo's 'Perspectiva'.[127] In the same classicizing (and military) vein, the burning mirror on the 1572 frontispiece is shown in the context of the legendary story of Archimedes' mirrors burning the ships of the Roman fleet at Syracuse. Thus, although Witelo's 'Perspectiva' had only minor attention for burning mirrors, both sixteenth century editions had burning mirrors figure prominently on the frontpage.

Why did these burning mirrors become so important in Renaissance optics? Basically, there was an interest in burning mirrors in the Renaissance, because of two reasons, (1) instrument design and (2) astrology. I will first deal with the matter of instrument design before turning to astrological concerns in the next section. As might be gathered from the choice of the figure of Archimedes, and the perception of Archimedes in the sixteenth century as a designer of ingenious instruments, it were primarily instrument makers and mathematicians, with a strong interest in instrument design, who dealt with burning mirrors in the Renaissance. Clagett has identified several treatises on conic sections of the fifteenth and sixteenth centuries, dealing primarily with the parabola, and he has noted that, even when purely mathematical concerns were discussed, they centered on optical investigations, and the design of a parabolic burning mirror, in particular.[128] Here, I am not primarily interested in the mathematical content of these treatises as such, but in their authors, who turn out to be instrument designers or makers.

[126] 'Etenim quaecunque hominibus de corporum coelestium materia, numero, ordine, deq; motuum coelstiu' infinita varietate aperta ac patefacta sunt, Optica ferè aperuit & patefecit: meteora: miracula in Iride una praesertim Opticis radiis distincta sunt: falsas opiniones de numero, motu, atque loco elementorum Optica solertia deprehendit & convicit. In vita verò hominum pleraq; daemonu' praestigiis attributa: ut imagines in aëre quocunq; mobiles repraesentare: ut loginquo spatio disjunctum exercitum velut ante oculos intueri: ut classem hostium incendio consumere, opticae artis vi & facultate omnia efficiuntur: ut picturam, architecturam, mechanicam intera taceam, nihil admodum nisi Opticam esse.' Alhazeni Arabis Opticam Praefatio, in Lindberg, *Opticae Thesaurus*, p. a3.

[127] Reisner referred to Livius' 'Ab urbe condita' in proposition 31 of book 2, under the heading 'lux ignea noctu major apparet': 'Sic Hannibalis boves farmentis ad cornua accensis primis tenebris vigilias Fabii terruerunt. Nam haud sec quàm sylvis montibusq; accensis omnia circum virgulta ardere hominum passim discurrentium speciem praebebat, ut Livius 3. decad. 2. lib. testatur.' Risnerus, *Opticae Libri Quatuor ex Voto Petri Rami*, p. 159.

[128] I am much indebted to Clagett, Marshall. *Archimedes in the Middle Ages*. Vol. 4: A supplement on the medieval Latin traditions of conic sections. Philadelphia: The American Philosophical Society, 1980, in particular, pp. 357-8.

Let me first point out that there is no immediate connection between the proliferation of these treatises on parabolic burning mirrors and the revolution in the making of glass mirrors. First, the fifteenth century interest in burning mirrors predated the technological innovation. Glass technology to make mirrors only matured in the sixteenth century. During the Middle Ages, most mirrors were small and, consequently, could be hand-held.[129] They were made either in metal or in glass. On the one hand, the technology to make large sheets of glass did not yet exist, on the other hand, the glass used to make mirrors was far from colourless. In the fifteenth century, in Venice, 'cristallo', a pure and 'whitish' glass was invented. At the beginning of the sixteenth century, Venetian 'cristallo' revolutionized the making of mirrors, when, from Flanders, the tin-amalgam process was introduced.[130] This process consisted in laying a tin-foil on to glass by means of mercury as a cementing medium. Thus, cold metal was applied to glass to avoid the difficulties connected with hot materials. This allowed making larger mirrors.

Second, the treatises on parabolic burning mirrors came with a section on their actual making. This shows that practical concerns were at stake, but, also, these sections show that parabolic burning mirrors were made of metal.[131] Moreover, they would still regularly be made of metal in the seventeenth century, thus, after the technological innovation that allowed making larger mirrors.[132] Even then, it should be doubted that these burning mirrors were effectively parabolized, thus, that the discussion of the parabola in this context, as it was technologically infeasible, was anything more than a matter of mathematical design 'on paper'.[133]

That these fifteenth and sixteenth century treatises on burning mirrors were directed toward their practical design can be gathered, as will become evident, from the fact that (1) they reduced the construction of the parabola to a two-dimensional problem, without reference to the three-dimensional cone, eventually allowing the parabola to be constructed solely on the basis of the knowledge of the focal distance required, and the fact that (2) they stressed the invention of draughting instruments with which a parabolic curve could be drawn.[134] Beside Alhazen's 'De speculis comburentibus', and Witelo's few propositions on the parabolic burning mirror, derivative of Alhazen's work on burning mirrors, a third treatise on the parabolic burning mirror was highly influential in the fifteenth and sixteenth centuries. This was the 'Speculi almukesi compositio', of uncertain authorship, most likely, by a fourteenth century monk, but formerly,

[129] For example, the size of the well known convex mirror in Jan Van Eyck's 'Arnolfini' is rather exceptional, but it appears to have been exaggerated by Van Eyck, maybe by using a convex mirror to paint the scene. See Carleton, David L. 'A Mathematical Analysis of the Perspective of the Arnolfini Portrait and Other Similar Interior Scenes by Jan Van Eyck'. *The Art Bulletin* 64 (1982): 119-24, p. 122.

[130] Schweig, Bruno. 'Mirrors.' *Antiquity* 15 (1941): 257-68, pp. 266-7; Zecchin, Luigi. *Vetro e Vetrai di Murano*. 3 vols. Venezia: Arsenale Editrice, 1990, pp. 3: 165-9, pp. 3: 368-71; Keil, Inge. *Augustanus Opticus: Johann Wiesel (1583-1662) und 200 Jahre Optisches Handwerk in Augsburg*. Edited by Johannes Burkhardt and Theo Stammen. Vol. 12, *Colloquia Augustana*. Berlin: Akademie Verlag, 2000, p. 220-6.

[131] The basis of all these discussions was the section on 'Conditions of a Good Steel' in the 'Speculi almukesi compositio' (discussed below). See Clagett, *Archimedes in the Middle Ages*, pp. 4: 133-7, 154-6.

[132] Keil, *Augustanus Opticus*, p. 238-9.

[133] Even in the eigteenth century, Herschel did not succeed in making mirrors of a perfectly parabolic shape. See Hysom, E.J. 'Tests of the Shape of Mirrors by Herschel.' *Journal for the History of Astronomy* 27 (1996): 349-52.

[134] Clagett, *Archimedes in the Middle Ages*, pp. 4: 357-8.

although wrongly, attributed to Roger Bacon.[135] For example, it exerted its influence on the 'Libellus de seccione mukesi', dated presumably shortly before 1400, of Jean Fusoris.

Jean Fusoris (1365-1436), having received a master of medicine degree, in Paris, was a French instrument maker, foremost of astrolabes, but also of other mathematical instruments for astronomical and astrological purposes.[136] He also showed interest in optics, in particular, burning mirrors, but also, as his annotations to Witelo's 'Perspectiva' and other notes show, catoptrics and image formation of a genre that, in the sixteenth century, would become known as natural magic. For example, Fusoris' manuscripts contain notes on 'how to make a mirror in which many moving images of one object appear', similar to collections of catoptrical notes which were widespread across Europe, for example in England, during the fifteenth century.[137] In his 'Libellus', Fusoris showed how to construct a curve that will reflect all rays parallel to its axis to a given focus at a given distance from a given vertex of the curve. The resulting curve is a parabolic curve, and it has been constructed completely in two dimensions. Fusoris' procedure was, assuming a given focus e at a given focal distance ae from the vertex a, (Figure 2.11)

> Then with line ae indefinitely extended toward d [and toward x], I take ah to be equal to ae. Then I erect from point e a perpendicular eb equal to eh and I draw line hbk. Then I also draw tb parallel to db and let it be as a ray falling on point b of line hbk, which ray, I assert, is reflected to point e. This is evident because the angle of incidence tbk is equal to the angle of reflexion ebh. This is proved, for angle tbk is equal to angle h, by I.20 of Euclid, line hb falling on two parallel lines hd and bt. Now angle h is equal to angle ebh, by I.5 of Euclid, since the two sides eh and eb are equal by hypothesis, and thus the earlier assumption is evident. So, it is therefore evident that the section we seek will pass through point a to point b. But in order that we may also have other points between a and b through which the section or curved line we seek ought to pass, we use the following method. For with any point z designated in line ae, I make ai equal to az. And having erected from point z a perpendicular zq of indefinite length and having placed the immobile foot of a compass in point e, I extend the other foot to point i. Then I draw it (a circular arc) as far as the aforesaid line zq, which it cuts, for example, in point g. Then I say that the section we seek will pass through point g. Whence, with a line drawn from i through g and with a ray mg falling [on g] and being parallel to ea, it will be proved that it (the ray) is reflected to point e in the same way and by the same propositions that it was proved that ray tb was reflected to e. Now just as there has been found between b and a a point g through which the section or curved line we seek ought to pass, so similarly and by the same propositions any intermediary points of any desired number – a hundred or a thousand – will be found until we could traverse from point to point the [whole] curved line which we seek. [138]

[135] Ibid., pp. 99-100.

[136] Poulle, Emmanuel. *Un Constructeur d' Instruments Astronomiques au XVe Siècle: Jean Fusoris*. Paris: Librairie Honoré Champion, 1963, in particular, pp. 1-6.

[137] 'Speculum in quo visu uno multe apparebunt ymagines se moventes constituere'. Bibliothèque Municipale (Dijon), 441(226), f. 206r. The complete passage is transcribed in Rosinska, Grazyna. *Optyka W XV Wieku Miedzy Nauka Sredniowieczna a Nowozytna - Fifteenth Century Optics between Medieval and Modern Science*. Vol. 24, *Studia Copernicana*. Wroclaw Warszawa Krakow Gdansk Lodz: Polska Akademia nauk Instytut historii nauki, oswiaty i techniki zaklad badan kopernikanskich, 1986, p. 151; the corresponding, almost identical, passage in an English manuscript, British Library (London), Egerton 2852, f. 19r, transcribed in Ibid., p. 168.

[138] Fusoris, De seccione mukesi, Bibliothèque de Dijon, Anciens Fonds 441 (266), ff. 204r-205v; transcription in Clagett, *Archimedes in the Middle Ages*, p. 4: 189-90, translation in ibid., p. 197.

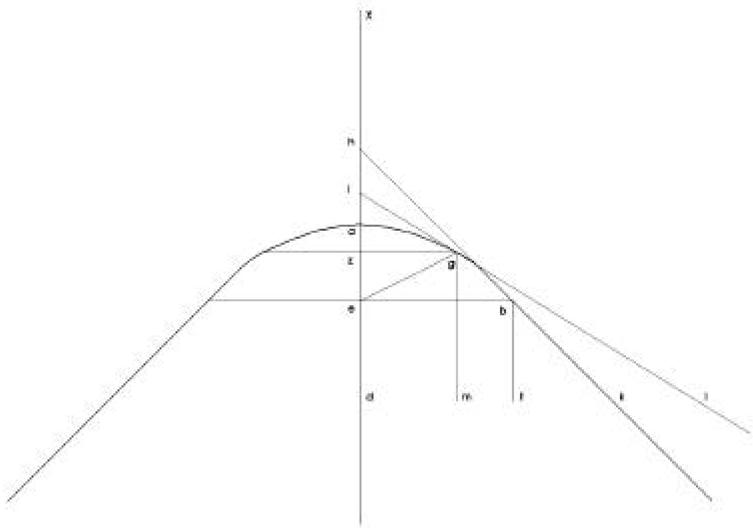

Figure 2.11

On this basis, Fusoris also invented an instrument that allowed him to draw a parabolic curve. However, he also invented a compass-like instrument to trace a parabola, which he presented as an alternative to the method of making a cutting instrument that will hollow out a paraboloidal mirror in the 'Speculi almukesi compositio'. As will become evident, such parabolic draughting compasses will become an important research subject among mathematical practitioners involved with the design of parabolic burning mirrors in the sixteenth century. Fusoris' compass consists of a right triangle adc made of metal.[139] (Figure 2.12)

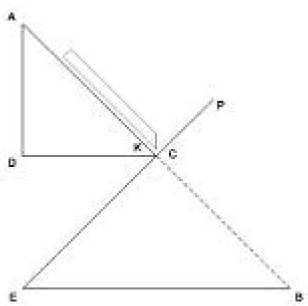

Figure 2.12

[139] Fusoris, De seccione mukesi, ff. 204v-205r; transcription in ibid., p. 4: 188, translation in ibid., p. 195

It is held at points a and d, so that it can be rotated about ad. A plate of steel, iron or bronze is fixed to the surface pl, so that pl is perpendicular to ac. A sharp-pointed stylus is attached by cramps to the side ac. When the surface pl is held fixed, and the triangle adc is rotated about ad, the stylus will trace a parabolic curve on the plate pl. The focal distance of this parabola (from the vertex of the section) will equal ½ac.

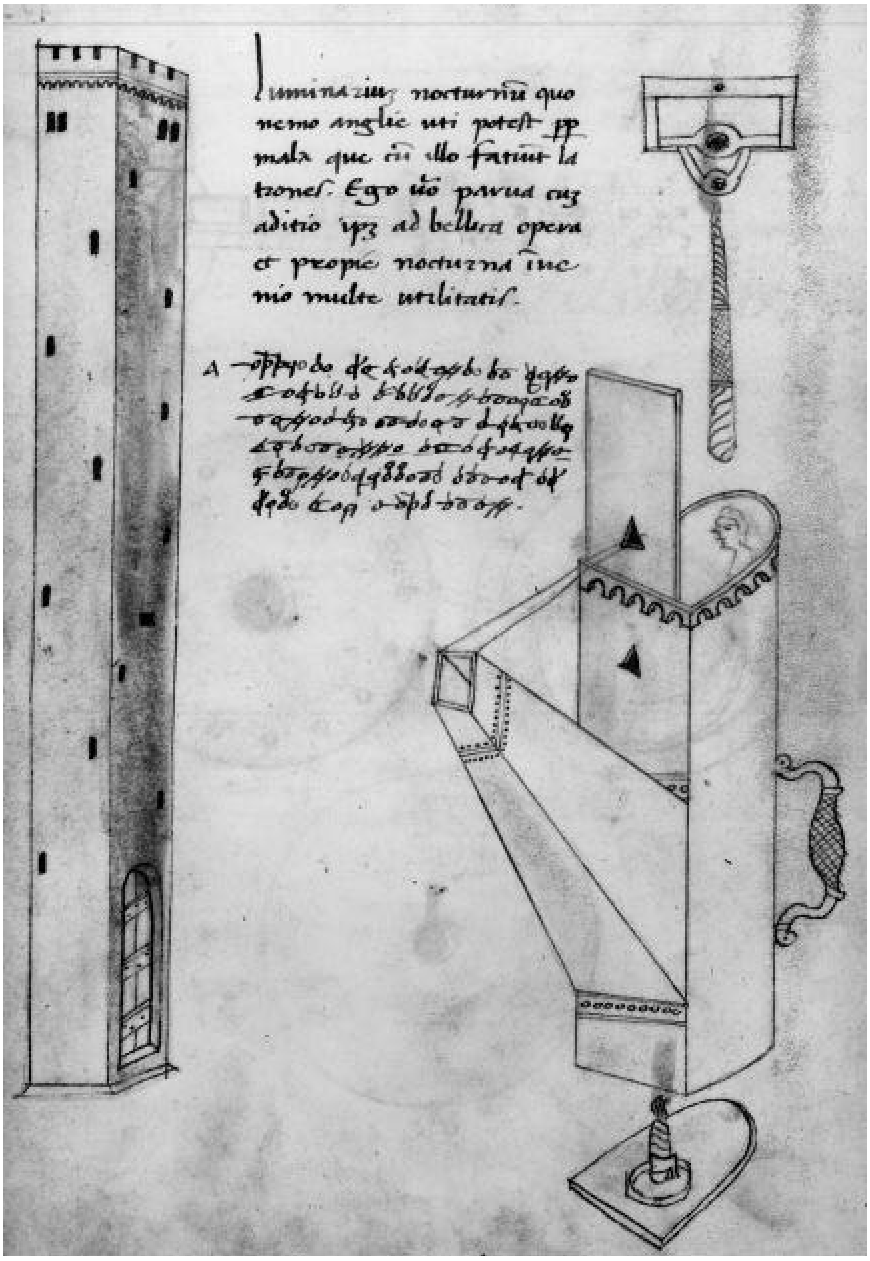

Figure 2.13

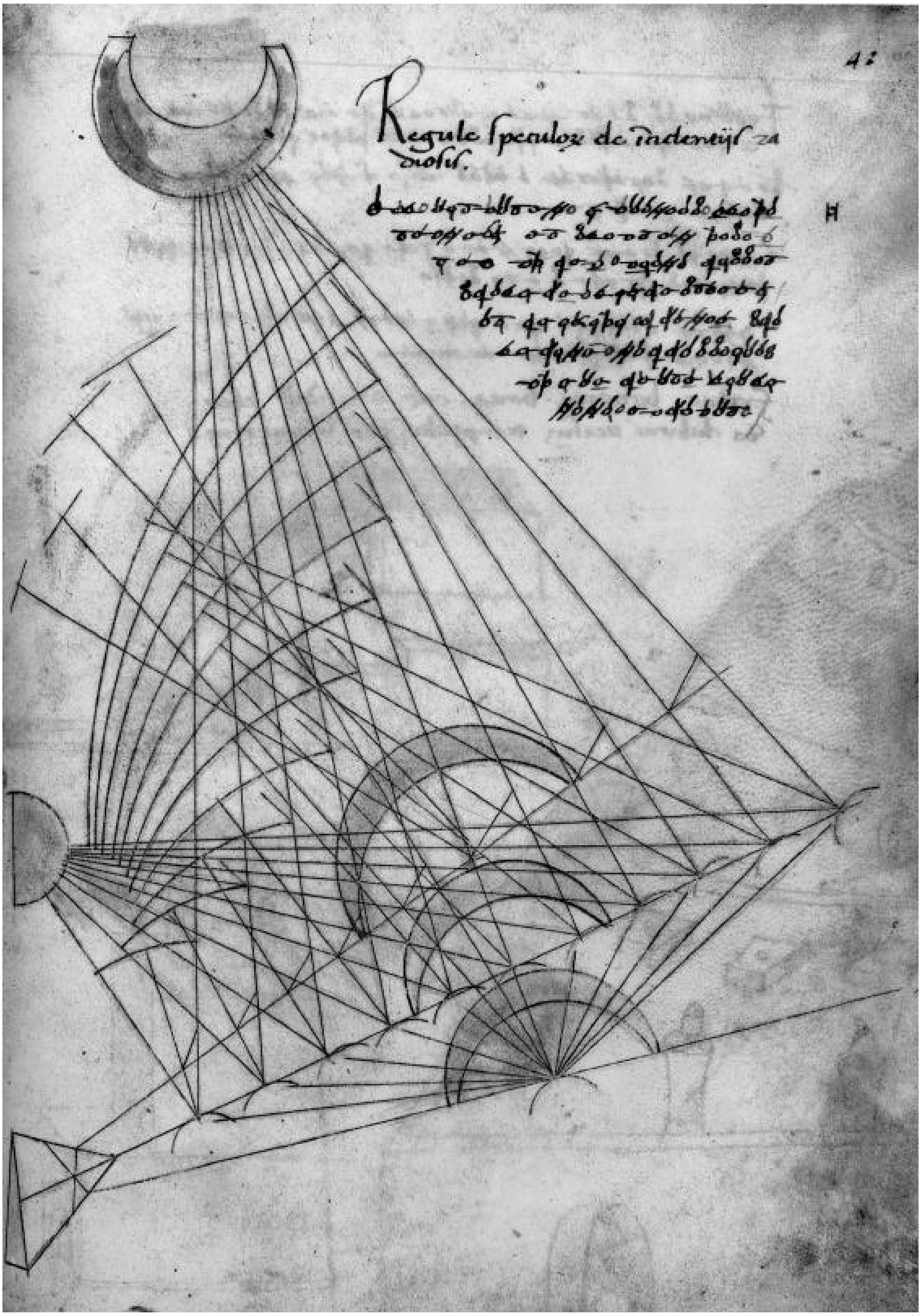

Figure 2.14

Not long after the composition of the 'Libellus' by Jean Fusoris, notes on the parabolic burning mirror were made by a physician and instrument designer, who was presumably an acquaintance of Fusoris, Giovanni Fontana. Born in Venice in the 1390s, Giovanni Fontana (d. 1450s) obtained his medical degree from the University of Padua in 1421, as a student of Biagio Pelacani.[140] Around the same time, he compiled several works on horology. Later, he moved to Udine, where he practiced as a physician, but also compiled a work on mensuration with an instrument of his own invention, the 'Tractatus de Trigono Balistario'. He also left a work on war machinery, the 'Bellicorum Instrumentorum Liber'. As a machine and mathematical instrument designer, he was also interested in optics. In his 'Bellicorum instrumentorum liber', he put optics to use to develop mensuration procedures and to design mirrors and magic lanterns.[141] (Figures 2.13-2.14) As will become evident in a later chapter, he most likely was also the author of an anonymous treatise on optics, known as 'Della prospettiva', which was known to Galileo. However, most important in this context is that Fontana annotated Gerard of Cremona's translation of Alhazen's 'Liber de speculis comburentibus' in a manuscript of translations of Gerard of Cremona owned by Fontana.[142] The content of these notes are less important here, but, as Clagett has shown, they are completely in the established tradition of creating a parabola from a known focal distance and designing instruments to draw and cut the parabola in order to make a parabolic burning mirror.[143] More important is that, again, we are confronted with a mathematical instrument designer who was interested in the optics of burning mirrors as instruments.

During his stay in Italy between 1461 and 1468, Regiomontanus (1436-1476) frequented the circle of students of Biagio Pelacani, as Toscanelli and Alberti, to which also Giovanni Fontana had belonged.[144] Regiomontanus had an early interest in optics. Already in 1458, after taking his MA degree in the previous year at the University of Vienna, he taught on Pecham's 'Perspectiva communis' at the same university. However, Regiomontanus' main contribution to optics arose in the context of his publication programme for the restoration of mathematics, which he launched in Nürnberg in 1474, after his stay in Italy together with his patron Bessarion and a period of teaching in Hungary.[145] According to Regiomontanus himself, he chose to live in Nürnberg, because this city was known for its instrument makers, who could provide him with the best mathematical measuring instruments to conduct his astronomical research.

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[140] Clagett, Marshall. 'The Life and Works of Giovanni Fontana.' *Annali dell' Istituto e Museo di Storia della Scienza* 1 (1976): 5-28: Clagett, Marshall. *Archimedes in the Middle Ages*. Vol. 3: The fate of the medieval Archimedes 1300 to 1565. Philadelphia: The American Philosophical Society, 1978, pp. 239-94; Birkenmajer, Alexander. 'Zur Lebensgeschichte und Wissenschaftliche Tätigkeit von Giovanni Fontana (1395?-1455?).' *Isis* 17 (1932): 34-53.

[141] In the 'Bellicorum instrumentorum liber', ff. 41v and 42r show a burning mirror, combined with a convex mirror; f. 49v shows a lantern to measure the height of a tower at night; f. 70v shows a magic lantern. Reproduced in Battisti and Battisti, *Le macchine Cifrate di Giovanni Fontana*, pp. 82, 87, 99.

[142] Bibliothèque Nationale de France (Paris), MS Lat. 9335, f. 134r. The manuscript is discussed and the marginal notes are attributed to Giovanni Fontana in Clagett, 'The life and works of Giovanni Fontana', pp. 17-9.

[143] Clagett, *Archimedes in the Middle Ages*, pp. 4: 172-4.

[144] For bio-bibliographical information on Regiomontanus, see Zinner, Ernst. *Leben und Wirken des Johannes Müller von Königsberg genannt Regiomontanus.* München: Beck'sche Verlagsbuchhandlung, 1938.

[145] Rose, *The Italian Renaissance of Mathematics*, pp. 90-117.

Regiomontanus' publication programme included the printing of Ptolemy's 'Optics'.[146] This did not succeed during his lifetime, but, as will become evident in the next chapter, the project, and the manuscript, was carried over to, and acquired by, the instrument maker Georg Hartmann. The publication programme also included a list of Regiomontanus' own works to be published. One of these was a treatise 'De speculis ustoriis atque aliis multorum generum ususque stupendi'. Most likely, this referred to a still to be expanded version of Regiomontanus' annotations and version of the 'Speculi almukesi compositio'.[147] Although one of most important changes of Regiomontanus to this treatise, was that he left out its technological section on 'the conditions of good steel' to make a parabolic burning mirror, his annotations were backed up by a practical interest in the design of mirrors. Regiomontanus possessed a paraboloid ring mirror of the type represented on the frontispiece of Apianus' and Tanstetter's edition of Witelo's 'Perspectiva'.[148] It is even most likely that these mathematicians, working with Regiomontanus' heritage, found inspiration in Regiomontanus' paraboloid ring mirror for their frontispiece. Regiomontanus' mirror was of considerable size. The diameter of the bigger section was about 110 cm, the diameter of the smaller section about 60 cm, while its total height was about 60 cm.

The most important of Regiomontanus' annotations to the 'Speculi almukesi compositio' was the one in which he showed how to compute the length of a ray reflected to the focus from a point on the parabola.[149] The underlying assumption was that the distance of the focus, or the point of combustion or congruence to which all rays parallel to the axis of the parabola are reflected, to the vertex (thus, the burning distance) is equal to one fourth of the latus rectum, understood to be the line of direction through the focus and the axial intercept. Moreover, Regiomontanus showed that the axial intercepts of lines of order are proportional to the squares of those lines of order, i.e. ordinates, intercepting the segments. Then, the burning distance can be calculated by taking the root of the sum of the square of the line of order and the square of the segment of the axis between the line of order and the focal point. These conclusions were based on the fourth proposition of the 'Speculi almukesi compositio', but Regiomontanus used them to make a table with values and linear equations relating all relevant values as in a two-dimensional plane.

This tradition of mathematical practitioners focussing on parabolic burning mirrors continued into the sixteenth century. It was particularly taken up in Italy by mathematical practitioners belonging directly or indirectly to the circle of Galileo. In 1551, Oronce Fine (1494-1555) published his 'De speculo ustorio'. Fine was the predecessor of Jean Pena at the Collège Royal in Paris, where he was appointed professor of mathematics in 1531.[150] He gained a reputation as mathematical practitioner, instrument designer, cartographer, engraver and illustrator of books. His most important publication was his 'Protomathesis', first published in 1531, consisting of four books on practical arithmetics, practical geometry, including a discussion of mathematical

[146] Ibid., p. 105.

[147] Clagett, *Archimedes in the Middle Ages*, pp. 4: 174-5.

[148] This mirror is known from a letter of Regiomontanus to Roder. See Rosinska, *Fifteenth Century Optics*, p. 187.

[149] Clagett, *Archimedes in the Middle Ages*, pp. 4: 179-80, 222-4, 228-9.

[150] For bio-bibliographical information on Fine, see Hillard, D., and E. Poulle. 'Oronce Fine et l' Horloge Planétaire de la Bibliothèque Sainte-Geneviève.' *Bibliothèque d' Humanisme et Renaissance* 33 (1971): 311-51; Ross, Richard P. 'Oronce Fine's Printed Works: Additions to Hillard and Poulle's Bibliography.' *Bibliothèque d' Humanisme et Renaissance* 36 (1974): 83-85; see also Johnson, A. F. 'Oronce Fine as an Illustrator of Books.' *Gütenberg-Jahrbuch* 3 (1928): 107-9; Ross, Richard P. 'Oronce Fine's De Sinibus Libri II: The First Printed Trigonometric Treatise of the French Renaissance.' *Isis* 66 (1975): 379-86.

instruments as the geometrical quadrant, the 'squadra' and the Jacob's staff for indirect measuring, cosmography and gnonomics, including a discussion of sundials. The 'Protomathesis' was translated in Italian by Cosimo Bartoli, with a dedication to Guidobaldo del Monte, published together with an Italian translation of the 'De speculo ustorio' by Ercole Bottrigaro, in 1587.[151] Clagett has shown that Fine's 'De speculo ustorio' is an unoriginal, even mathematically inferior version of the 'Speculi almukesi compositio'.[152] However, Fine is a fine example of a mathematical practitioner placing the discussion into an optical and instrumental context.

First, Fine added four simple optical postulates to his discussion. The first defined a solar ray as a mathematical line; the second stated the law of equal angles for plane mirrors; the third expanded this law to convex and concave mirrors; the fourth stated that a burning mirror which will reflect the incident solar rays to one point of combustion produces the quickest and most intense combustion of all burning mirrors, and that the parabolic burning mirror is such a mirror.[153] Second, Fine expanded the description of how to make such a parabolic burning mirror. Not only did he include a description of the instrument to hollow out the paraboloidal surface and a paraphrase of the section on 'the conditions of good steel', which he found in the 'Speculi almukesi compositio', he also added another procedure by which a metal mirror is cast from a paraboloidal mold and he included different techniques of polishing the mirror's surface.[154]

However, Fine's 'De speculo ustorio' is most important for its influence on Della Porta's 'Magia naturalis' (1589). Giambattista Della Porta (1535-1615), born in a noble Neapolitan family, wrote a first version of his 'Magia naturalis', as a young man, in 1558, possibly as a consequence of his involvement with Ruscelli's Accademia Segreti, which was set up in his home-town Naples.[155] Later, Della Porta would establish a similar academy of his own, the Academia dei Secreti. The task of such academies was to investigate the 'secrets of nature' by experience, including optical marvels, as discussed above, which brought magicians close to artisans.[156] As such, Della Porta became the primary advocate of natural magic of the second half of the sixteenth century. As shown above, Della Porta's natural magic was far from alien to the concerns of mathematical practitioners. In this light, it does not come as a surprise that Della Porta's 'Magia naturalis' was drawing upon sources like Fine's 'De speculo ustorio' in its discussion of the parabolic burning mirror. A second, much expanded version, of his 'Magia naturalis' appeared in 1589.

Chapters 14 to 17 of Della Porta's 'Magia naturalis', concerning how to draw a parabolic section and the making of the mirror itself were taken from Fine's proposition 8 and 9 of his 'De speculo

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[151] Fine, Oronce. *Opere di Orontio Fineo del Delfinato, divise in cinque parti; aritmetica, geometria, cosmografia, e orivoli; tradotte da Cosimo Bartoli, Gentilhuomo, & Academico Fiorentino; et gli specchi, tradotti dal Caualier Ercole Bottrigaro, Gentilhuomo Bolognese.* Venetia: Presso F. Franceschi, 1587. Ercole Bottrigaro (1531-1612) was a Bolognese humanist, who besides a translation of Finé's 'De Specchio Ustorio', also edited Ptolemy's 'Geography'. On Ercole Bottrigaro, see the Ghisalberti Alberto M., ed. *Dizionario biografico degli italiani.* Roma: Istituto della Enciclopedia Italiana, 1960, pp. 491-5.

[152] Clagett, *Archimedes in the Middle Ages*, pp. 4: 321-31. See also Ross, Richard P. 'Oronce Fine's De Speculo Ustorio: A Heretofore Ignored Early French Renaissance Printed Treatise on Mathematical Optics.' *Historia Mathematica* 3 (1976): 63-70.

[153] Fine, *Opere di Orontio Fineo*, ff. 3r-3v.

[154] Ibid., proposition 9, ff. 16v-17v.

[155] Eamon, William. *Science and the Secrets of Nature: Books of Secrets in Medieval and Early Modern Culture*. Princeton, New Jersey: Princeton University Press, 1994, pp. 195-203.

[156] Ibid., pp. 210-21.

ustorio'. For example, in proposition 8, Fine outlined the geometry involved in drawing a parabolic section when the focal distance is known.[157] (Figure 2.15)

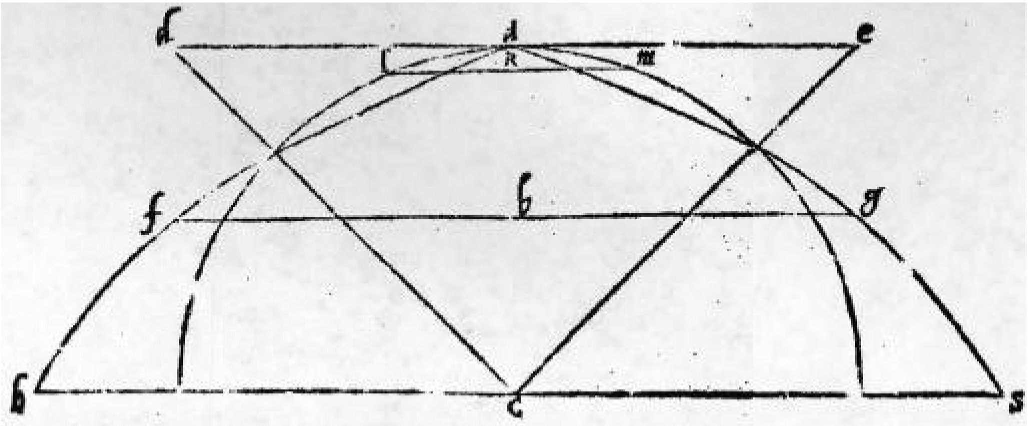

Figure 2.15

The focal distance ab is doubled to produce the axis ac of the section. This doubled distance is also taken to be the base of the right-angled cone from which the parabolic section is taken. Thus, a perpendicular de is constructed to ac, so that da and ae are each equal to ac. When the lines dc and ce are drawn, a right-angled (angle dce), isosceles triangle is constructed. This triangle is rotated about dc to produce the right-angled cone, with dc as the axis of the cone and ce as the radius of the cone's base. A perpendicular fg to abc through b is erected, with fg equal to de, and bf and bg each equal to abc. Also, a perpendicular hi to abc through c is erected, with hc and ci equal to dc. Consequently, a line connecting hfagi will produce a parabola with focal distance ab, with vertex a and focus b. Della Porta repeated this construction almost verbatim in chapter 15, in the section about how 'to make a glass of a parabolic section', before, as Fine, describing a diminished, section of the parabola that would still focus the rays in b, maybe more practical from the point of view of actual contruction, and the instrument to draw a parabolic section.[158] (Figure 2.16)

[157] Fine, *Opere di Orontio Fineo*, ff. 13v-14r.

[158] 'Let the distance be known how far we would have the glass to burn, namely, AB ten foot; for were it more, it could hardly be done: double the line AB, and make ABC, the whole line will be AC: from the point A, draw a right line DA, and let DA and AE be equal one to the other, and cut at right Angles by AC, but both of the must be joined to the quantitaty AC, as DCE, which in C make a right triangle, DCE. Therefore the Triangle DCE is a right angled Triangle, and equal sides: and were this turned about the Axis CD, until it come to its own place whence it parted, there would be made a right angled Cane, EDNC, whose parabolic section will be ABC: the right line DC will be the axis of the Cane, and CE will be the semidiameter of the basis of the Cane: Through the point C you must draw a line parallel to DE, and that is HI of the length of CE and CD; and by the point B draw another parallel to the said line ED, which is FBG; and let BG and BF be both of them equal to AC: so FG shall be the upright side, and HI the basis of the Parabolic section: If therefore a line will be drawn through the points HEAGI, that shall be a Parabolic section.' Quoted from the English translation in Porta, John Baptista. *Natural Magick*. Edited by Derek J. Price, *The Collector's Series in Science*. New York: Basic Books, Inc., 1957, p. 372.

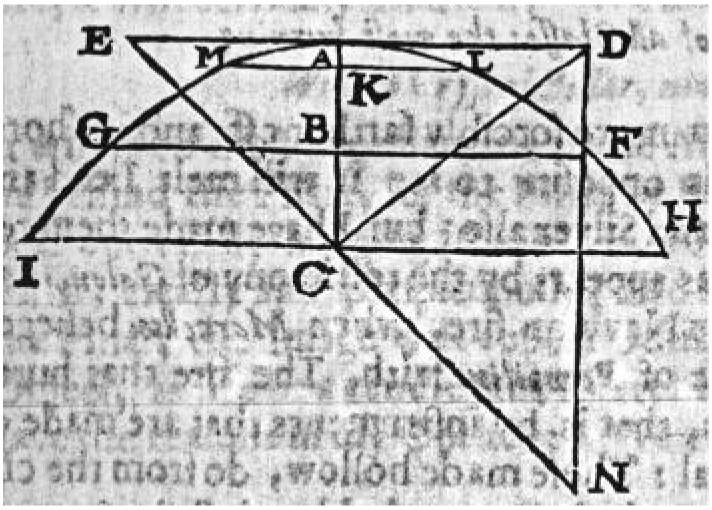

Figure 2.16

In this construction, Della Porta also borrowed Fine's mistake to consider the radius of the right-angled cone from which the parabola is produced to be double the focal distance of the parabola.[159]

However, Della Porta's involvement with the parabolic burning mirror was not limited to the copying of works on the subject by mathematical practitioners. In 1580, Della Porta's patron, the Cardinal d' Este, sent him to Venice to make or obtain a parabolic burning mirror.[160] Looking for guidance to construct a parabolic burning mirror, he turned to Jacomo Contarini, presumably not only to provide the means, but also the skills. On 29 November 1580, Della Porta wrote to his patron that Contarini had spent a day and most of a night at the Arsenal with him supervising an attempt by one of the Arsenal craftsmen to cast a parabolic mirror. Jacomo Contarini (1536-1595) was the Provveditore of the Venetian Arsenal.[161] He owned an important collection of books, manuscripts and mathematical instruments, but he was also involved in the design of instruments himself. Moreover, he was an important member of a network of Italian mathematicans, from Daniele Barbaro and Francesco Barozzi to Guidobaldo del Monte and Galileo.[162]

It is no coincidence that Della Porta sought guidance from Contarini. There is some evidence to suggest that Contarini was interested in the design of parabolic burning mirrors. Contarini was highly interested in drawing compasses. In a manuscript of Contarini, there is a letter preserved which was presumably sent to Contarini by Francesco Barozzi, who, well informed about conic

[159] Clagett, *Archimdes in the Middle Ages*, p. 4: 331.

[160] Rose, Paul Lawrence. 'Jacomo Contarini (1536-1595), a Venetian Patron and Collector of Mathematical Instruments and Books.' *Physis* 18 (1976): 117-30, pp. 125-6.

[161].Ibid., pp. 117-30. See also Benzoni, Gino, and Tiziano Zanato, eds. *Storici e Politici Veneti del Cinquecento e del Seicento.* Milano Napoli: Riccardo Ricciardi, 1982, p. 502.

[162] On Contarini's collection of instruments and his library, see Magani, Fabrizio. 'Il collezionismo a Venezia al tempo del soggiorno di Galileo'; Zorzi, Marino. 'Le biblioteche a Venezia nell' età di Galileo'. In *Galileo Galilei e la Cultura Veneziana*, 137-90. Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1995; See also Tafuri, Manfredo. *Venezia e il Rinascimento: Religione, Scienza, Architettura*. Torino: Giulio Einaudi Editore, 1985, pp. 185-212.

sections through the work of Apollonius and Pappus, published on conic sections in his 'Admirandum Illud Geometricum Problema' (1586).

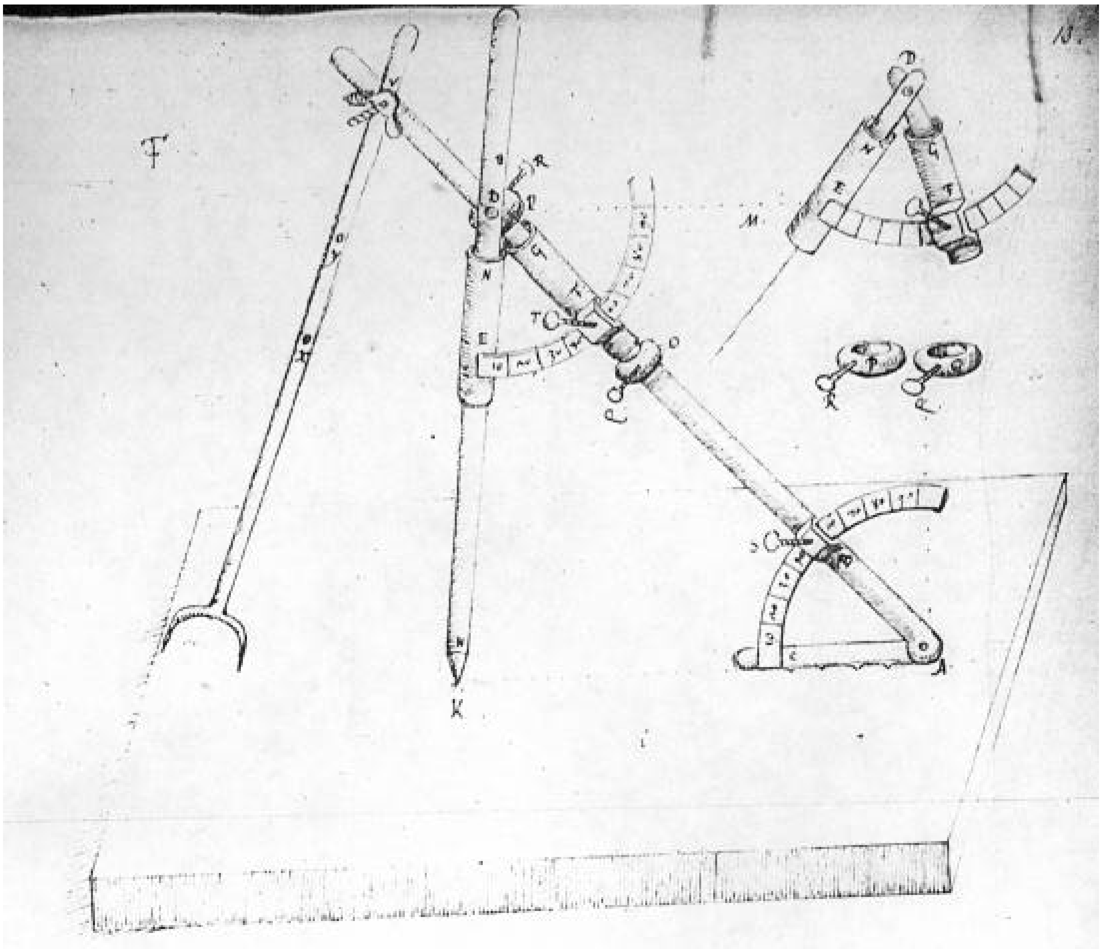

Figure 2.17

The letter dealt with a compass that allowed drawing the different conic sections, the parabola, the hyperbola and the ellips, by variation of the inclination of the axis of the cone AD with respect to the plane of projection. (Figure 2.17) The cone is generated by rotation of the leg DK about the leg AD. The latter leg of the compass slides in and out the tube NE, so that it always touches a piece of paper with its point K, which, consequently traces, the desired conic section. The letter that accompanied the drawing of the compass points out that the instrument could be used to control the curvature of mirrors.

> From this, Your Excellency [Contarini] can see how easy it is to draw the curvatures to make concave mirrors of whatever cavity you wish, parabolic, hyperbolic or oval, with this same instrument.[163]

[163] 'Da qui V. S. puo vedere con quanta facilita potro dissegnare le sagome per fare li specchij cavi ò voglia di cavita Parabolica, ò Hiperbolica, o ovale con l' istesso instrumento'. Bodleian Libary (Oxford), MS Canon. Ital. 145, ff. 12-

Although Contarini was, no doubt, a mathematical practitioner interested in the design of parabolic burning mirrors, the introduction of the theory of conic sections of Apollonius and Pappus in the second half of the sixteenth century by, among others, Barozzi and Commandino, also was the beginning of mathematicians' concerns with the parabolic burning mirror moving away from optics. For our purposes, it is interesting to note that instrument makers and mathematical practitioners interested in instrument design placed burning mirrors high on the agenda in the fifteenth and sixteenth centuries. It explains why the sixteenth century editors of Witelo's 'Perspectiva' considered it necessary to single out the burning mirror on their frontispieces to an extent Witelo's thirteenth century 'Perspectiva' did not seem to allow.
Of particular importance seems to be the era around 1550. As has been shown, while the discussions of the parabolic burning mirror had always been more with an eye towards the optics, and the design of a parabolic mirror, than towards the mathematics, Fine emphasized the optical context of the parabolic burning mirror, by introducing general optical principles in his work on the parabolic burning mirror, for example, by suggesting that the parabolic burning mirror was only one case of a more general study of burning mirrors. Of similar importance was that, while the practical mathematicians often, as in the case of Fusoris, had also discussed image formation in mirrors, reminiscent of sixteenth century natural magic, the mid-sixteenth century, exemplary in the case of Della Porta, saw the fusion of both aspects of catoptrics, that is, the study of burning mirrors and the study of image formation. In chapter 4, it will be argued that this fusion, contrary to what one might expect, only came about in the middle of the sixteenth century and that it had important consequences which eventually allowed the development of the telescope. However, first, we will turn to a second reason why burning mirrors became so important in the sixteenth century. Burning mirrors were important to the study of light, and the multiplication of species. This discussion became embedded within astrological and alchemical concerns.

3. John Dee, the Study of Light and Mirrors, and Astrology

3.1. John Dee's 'Mathematicall Praeface' (1570) and the Influence of Ramus

Pena's 'De usu optices' was highly influential among sixteenth century mathematical practitioners. This influence was not limited to Danti's Italian translation of Pena's version of Euclid's 'Optica et Catoptrica', with its paraphrase of Pena's introduction. Although it has been suggested that Dee's 'Mathematicall Praeface', in general, found its inspiration in Ramism, among other sources, it has not been noted, to the best of my knowledge, that its section on 'perspective' was highly derivative of Pena's 'De usu optices'. Shortly after his graduation from the University of Cambridge in 1548, John Dee (1527-1608/9) went for the second time, after a first short visit in 1547, to Louvain.[164] Until 1550, when he left Louvain, he studied with Louvain's premier mathematical practitioners and instrument makers, Gemma Frisius, Gerard Mercator, Caspar Van Der Heyden and Antonio Gogava. On 20 July 1550, he arrived in Paris to teach Euclid. In his autobiographical 'Compendious rehearsall' (1592), Dee noted that his

13, edited in Camerota, Filippo. *Il Compasso di Fabrizio Mordente: Per la Storia del Compasso di Proporzione*. Firenze: Leo S. Olschki, 2000, pp. 75-7, 245-6.

[164] For an overview of Dee's life, see Roberts, Julian, and Andrew G. Watson. *John Dee's Library Catalogue*. London: The Bibliographical Society, 1990, pp. 75-8.

lectures attracted the attention of the leading mathematicians of Paris, Oronce Fine, Petrus Ramus, and, also his student Pena.[165] Once acquainted, Ramus and Dee corresponded, so it is most unlikely that Dee would not have been aware of Pena's preface, while he was avidly collecting sixteenth century editions of Euclid. Anyway, he also acquired Danti's Italian translation of 1573.[166]

After his return to England in 1551, Dee tried to establish himself as a mathematical practitioner, in particular by providing geographical advice in the context of the English discovery voyages.[167] While, as will become evident in the next section, Dee showed a growing tendency toward the occult in the next years, this is hardly apparent in his 'Mathematicall Praeface' of 1570. Roberts and Watson have described it as 'an essentially retrospective work – a statement in Ramist dress of what had been his principles and practice as a teacher of mathematics and navigation in the nineteen years following his return from Paris'.[168] Although there was more stress on the theoretical significance of mathematics, presumably influenced by Proclus' commentary on Euclid, in Dee's 'Mathematicall Praeface', boasting mathematics practical utility was an important line of thought in this introduction, which was written in the vernacular, primarily intended for artisans and mathematical practitioners.[169] After introducing the two main divisions of mathematics, arithmetic and geometry, Dee enumerated no less than nineteen derivative disciplines, some of them most familiar, like astronomy and music, others, with apparently obscure names of Dee's own invention, like 'trochilike, which demonstrateth the properties of all circular motions: simple and compound', or 'helicosophie, which demonstrateth the designing of all spirall lines: in plaine, on cylinder, cone, sphare, conoïd, and spharoid: and their properties'.[170]

There have been assigned several, sometimes presented as exclusive, sources for Dee's 'Mathematicall Praeface' besides the influence of Ramus. This is hardly surprising in light of, what was said above, that Ramus' concept of mathematics and optics was itself not completely original, as it was derived from, or similar to, mathematical practitioners' introductions to perspective treatises or the sixteenth century Vitruvian commentary tradition. Beside the fact that Dee was able to draw upon an already established English practical mathematical tradition, in the person of Dee's already quoted predecessor Robert Recorde, he was well acquainted with both perspective treatises and the Vitruvian commentary tradition. One of the nineteen derivative disciplines of the 'Mathematicall Praeface' was 'zographie', practiced by the 'Mechanicall Zographer (commonly called the Painter)', which Dee defined as

[165] John Dee, 'The Compendious Rehearsal'. In *Autobiographical Tracts of Dr. John Dee, Warden of the College of Manchester*, edited by James Crossley. Manchester, 1851, p. 8. Discussed in French, Peter J. *John Dee: The World of an Elizabethan Magus*. London: Routledge & Kegan Paul, 1972, p. 31.

[166] Roberts and Watson, *John Dee's Library Catalogue*, pp. 29-30.

[167] Taylor, E. G. R. *Tudor Geography, 1485-1583*. London: Methuen & Co. Ltd., 1930, pp. 75-139; Taylor, E. G. R. *The Mathematical Practitioners of Tudor & Stuart England*. Cambridge: University Press, 1970, pp. 18-21.

[168] Roberts and Watson, *John Dee's Library Catalogue*, p. 10.

[169] On this more theoretical background to Dee's 'Mathematicall Praeface', see Allen G. Debus' introduction to Dee, John. *The Mathematicall Praeface to the Elements of Geometrie of Euclid of Megara (1570)*. New York: Science History Publications, 1975, pp. 8-12. Compare Proclus' commentary to Euclid in Morrow, Glenn R., ed. *Proclus: A Commentary on the First Book of Euclid's Elements*. Princeton: Princeton University Press, 1970.

[170] Dee, *Mathematicall Praeface*, quoted from 'Groundplat'.

> an Arte Mathematicall, which teacheth, and demonstrateth, how the Intersection of all visuall Pyramides, made by any playne assigned, (the Centre, distance, and lightes, beyng determined) may be, by lynes, and due propre colours, represented.[171]

Thus, although he did not possess a copy of Alberti's 'De pictura' (1435), he gave the Albertian definition of painting, assigning it the name 'zographie'. Dee considered drawing to be important to architects, goldsmiths and weavers. Moreover, also another discipline, 'anthropographie', that is 'the description of the Number, Measure, Waight, Figure, Situation, and colour of every diverse thing, conteyned in the perfect body of MAN', assumed knowledge of perspective, as it was based on the manuals teaching how to draw the human body, with respect of the human proportions, as established, exemplary, in Dürer's 'Vier Bucher von menschlicher Proportion' (1528).[172] Also, Dürer's 'Underweysung der Messung' (1525) was an important source for Dee. His own little treatise on perspective, preserved in manuscript, seems to have been largely based on Dürer. Similar to Dürer, it introduced perspective as of practical utility for all those involved in the 'arts of measuring', before showing how to construct the typical checkerboard-floor.[173]
Likewise, Dee's debt to the Vitruvian commentary tradition was firmly established by Frances Yates.[174] He owned several sixteenth century editions and commentaries on Vitrivius' 'De architectura', including Daniele Barbaro's, and, also, Alberti's work on architecture. From this tradition, Dee took the view of the architect well versed in the other mathematical disciplines, or, as Dee quoted Alberti, 'an Architect (sayth he) ought to understand languages, to be skilfull of Painting, well instructed in Geometrie, not ignorant of Perspective, furnished with Arithmetike, have knowledge of many histories, and diligently have heard Philosophers, have skill of Musike, not ignorant of Physike, know the aunsweres of Lawyers, and have Astronomie, and the courses Caelestiall, in good knowledge'.[175] The architect was not a mechanician, because

> thought, the Architect procureth, enformeth, & directeth, the Mechanicien, to handworke, & the building actuall, of house, Castell, or Pallace, and is chief Iudge of the same: yet, with him selfe (as chief Master and Architect,) remaineth the Demonstrative reason and cause of the Mechaniciens worke … by Geometricall, Arithmeticall, Opticall, Musicall, Astronomicall, Cosmographicall (& to be brief) by all the former Derived Artes Mathematicall, and other Naturall Artes, hable to be confirmed and stablished.[176]

Thus, it can not be denied that in the second half of the sixteenth century, with Dee, the Italian Renaissance, in particular the Vitruvian ideas about architecture, the tradition of perspective and the 'zographie'-equivalent of the Italian 'disegno', as we have seen it used in Danti's preface to his Italian translation of Euclid's 'Optica et Catoptrica', had arrived in England. However, Yates has exaggerated her argument when she claimed that the Vitruvian ideas of architecture became the organizing principle of the 'Mathematicall Praeface'. Although it certainly suggested a wider

[171] Ibid., f. d.ijv.

[172] Ibid., f. c.iiijr.

[173] 'et aliorum omnium, qui arte mensurandi'. Bodleian Library (Oxford), Cotton Vitellius C. VII, ff. 15r-24v, f. 15r. For the construction of the checkboard-floor in perspective, see f. 18r-19r.

[174] Yates, Frances A. *Theatre of the World*. London: Routledge & Kegan Paul Ltd., 1969, pp. 20-41.

[175] Dee, *Mathematicall Praeface*, ff. d.iijr-d.iijv.

[176] Ibid., f. d.iijr.

application in drawing together all mathematical disciplines, the influence of the tradition of perspective and the Vitruvian commentary tradition was foremost the introduction of drawing and architecture themselves among the 'derivative mathematicall disciplines'.

The general organizing principle of Dee's 'Mathematicall Praeface' was the typical Ramist chart that became highly popular in the second half of the sixteenth century.[177] These charts were based on the splitting of a subject, mostly in dichotomies and by use of brackets, going from the general to the particular. Ultimately, the whole of reality could be grasped in the Ramist chart, making it into a kind of encyclopedic vision. Ong has pointed out that these Ramist charts, by setting out an order on the printed page, had an element of spatial visualization in them, as such, making visible the encyclopedia of knowledge.[178] Regardless whether these charts were intimately linked with the advent of printing, as Ong has claimed, or had precedents in the schematic lay-outs of the manuscript tradition, following Yates' thesis, the diffusion of Ramism made the charts popular items in sixteenth century books.[179] Dee's 'Groundplat', organizing all the 'Sciences and Artes Mathematicall' that was discussed in the 'Mathematicall Praeface', was definitely of Ramist inspiration. (Figure 2.18) Thus, while the 'practical utility of mathematics' was of a more general tendency than specifically Ramist, Ramus' charts provided the visualization of this 'practical utility of mathematics' likewise boasted by Ramus and Dee.

The influence of Ramism, in particular of Pena's 'De usu optices' was much more specific in Dee's discussion of 'Perspective', that is optics.[180] It contained all the elements with which we are by now familiar from the discussion of Pena's preface. Optics was presented as a key discipline containing knowledge essential to understand all other mathematical disciplines. Dee acknowledged that neither the draftsman nor the architect could cope with knowledge of optics. Moreover, he touched upon the importance of optics for the other mathematical disciplines and natural philosophy, however, without going into detail with the convential excuse of lack of space and time.[181] He went into more detail concerning the use of perspective to attain 'perfect knowledge of Astronomicall Apparences', using Euclidean examples which are also familiar from Pena's 'De usu optices', for example, 'of thinges being in like swiftnes of moving, to thinke the nerer, to move faster: and the farder, much slower'.[182] Next, he mentioned the convential meteorological phenomena, the rainbow and halo's, and the phenomena of refraction. Finally, Dee stressed the catoptrical phenomena in the vein of natural magic. He referred to a 'marveilous Glasse' of his friend Pickering, and, again, the 'image in the air' is encountered.

> Yea, so much, to scare, that, if you, being (alone) nere a certaine glasse, and proffer, with dagger or sword, to foyne at the glasse, you shall suddenly be moved to give backe (in maner) by reason of an Image,

[177] On Ramism, in particular, the Ramist charts, in England, see Jacobus, Lee A. *Sudden Apprehension of Knowledge in Paradise Lost*. The Hague Paris: Mouton, 1976, pp. 60-7; see also Oldrini, Guido. 'Le Particolarità del Ramismo Inglese.' *Rinascimento* 25 (1985): 19-80.

[178] Ong, *Ramus, Method and the Decay of Dialogue*, pp. 75-91, 307-17.

[179] Ibid., p. 307; Yates, Frances A. *The Art of Memory*. Harmondsworth: Penguin Books, 1969, p. 230.

[180] Dee, *Mathematicall Praeface*, ff. b.jr-b.jv.

[181] 'I speake nothing of Naturall Philosophie, which, without Perspective, can not be fully understanded, nor perfectly atteined unto. Nor, of Astronomie: which, without Perspective, can not well be grounded: Nor Astrologie, naturally Verified, and avouched.' Dee, Mathematicall Praeface, b.j verso.

[182] Ibid., f. b.jv.

appearing in the ayre, betwene you & the glasse, with like hand, sword or dagger, & with like quicknes, foyning at your very eye, likewise as you do at the Glasse. Straunge, this is, to heare of: but more mervailous to behold, than these my wordes can signifie. And neverthelesse by demonstration Opticall, the order and cause thereof, is certified: even so, as the effect is consequent.[183]

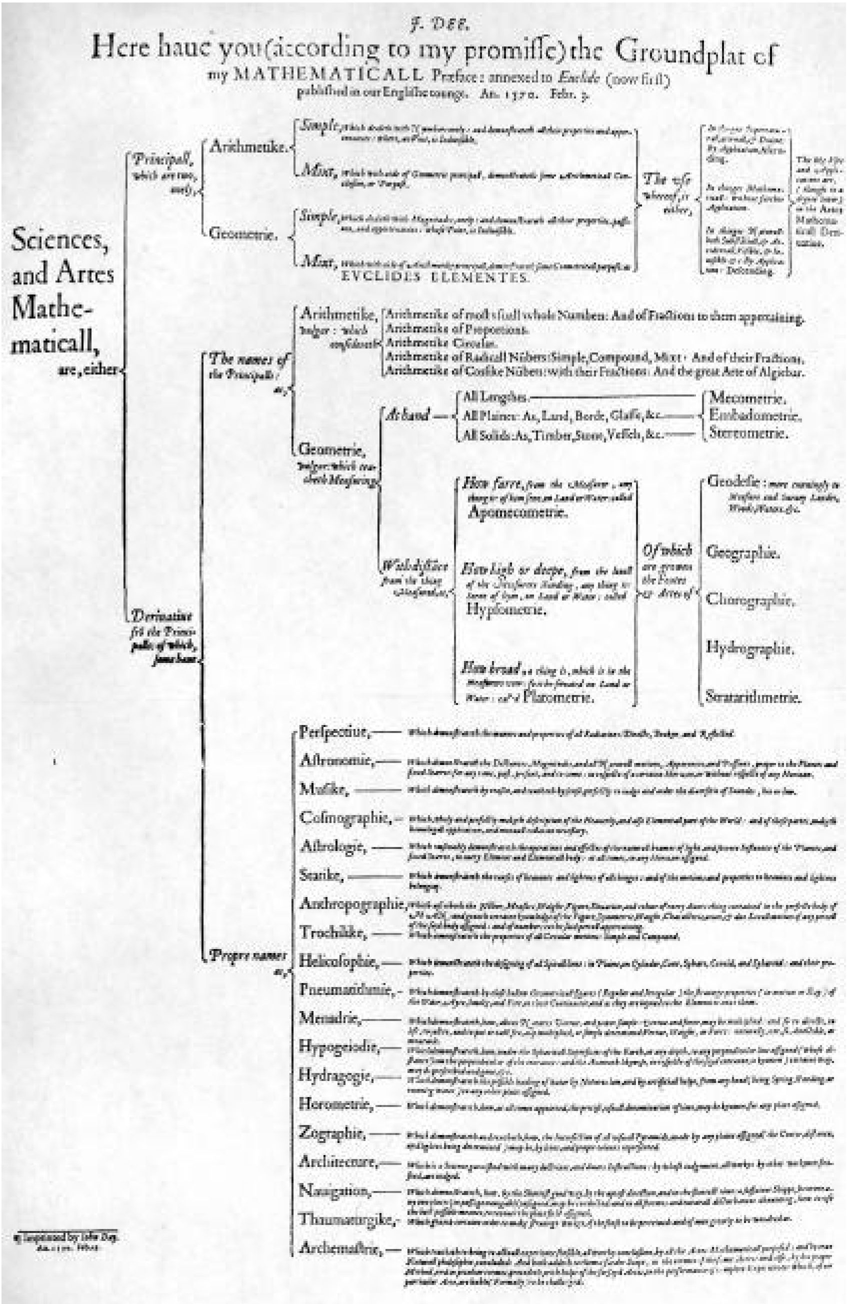

Figure 2.1 8

[183] Ibid., f. b.jv.

Again, under the heading of Thaumaturgike, which is 'that Art Mathematicall, which giveth certaine order to make straunge workes, of the sense to be perceived, and of men greatly to be wondred at', Dee referred to automata, which he had seen with Oronce Fine in Paris, speaking heads, flying wooden doves and, of course, optical marvels, in particular images 'in the air'.

> And by Perspective also straunge thinges, are done. As partly (before) I gave you to understand in Perspective. As, to see in the Ayre, a loft, the lyvely Image of an other man, either walkyng to and fro: or standyng still. Likewise, to come into an house, and there to see the lively shew of Gold, Silver or precious stones: and comming to take in your hand, to find nought but Ayre.[184]

Dee referred to the explanation of apparent marvels by optics in Claudio Celestino's 'De his quae mundo mirabiliter', published under the auspices of Fine in 1542, together with what was one of the earliest printed editions of Roger Bacon's 'Epistola de secretis operibus artis et naturae', already cited here for the same references of optical marvels.[185] This work presumably influenced Dee, having the opportunity to discuss it with Fine when visiting in 1550, but, also, Pena, Fine's predecessor, who is said to have taken his classes at the Collège Royal, and, as already pointed out, referred to similar optical marvels in his 'De usu optices'. Dee himself was strongly influenced by Bacon, even writing a now lost work about him in the 1550s.[186] Thus, it might not come as a surprise that, in his 'Mathematicall Praeface', under the heading of 'stratarithmetrie', or the military sciences, Dee hinted at the optical marvel of telescopic vision.

> The Herald, Pursevant, Sergeant Royall, Capitaine, or who soever is carefull to come nere the truth herein, besides the Iudgement of his expert eye, his skill of Ordering Tacticall, the helpe of his Geometricall instrument: Ring, or Staffe Astronomicall: (commodiously framed for cariage and use). He may wonderfully helpe him selfe by perspective Glasses. In which, (I trust) our posterity will prove more skillfull and expert, and to greater purposes, then in these dayes, can (almost) be credited to be possible.[187]

Thus, given Dee's connection with Ramus and Pena, and their meeting in 1550, and the fact that Dee's section on perspective contained the same elements as Pena's 'De usu optices', in particular, the hint at the utility of optics for natural philosophy and the references to catoptrical marvels, Dee's section on perspective was most likely modeled after Pena's 'De usu optices'. Nevertheless, part of the similarities might be explained by drawing upon similar sources, for example, the mathematical practioners' textbooks on perspective or the influence of Oronce Fine on optical marvels and the role of the perception of Roger Bacon in this context.

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[184] Ibid., f. A.iv.

[185] Caroti, Stefano. 'Nicole Oresme, Claudio Celestino, Oronce Fine e i 'Mirabilia Naturae'.' *Memorie Domenicane* 8-9 (1977-78): 355-410, pp. 355-7.

[186] Dee mentioned this work in the letter of dedication to Mercator, prefacing the 'Propaedeumate aphoristica'. 'The Mirror of Unity, or Apology for the English Friar Roger Bacon; in which it is taught that he did nothing by the aid of demons but was a great philosopher and accomplished naturally and by ways permitted to a Christian man the great works which the unlearned crowd usually ascribes to the acts of demons'. Translation in Shumaker, Wayne, and J. L. Heilbron. *John Dee on Astronomy, Propaedeumate Aphoristica (1558 and 1568), Latin and English*. Berkeley Los Angeles London: University of California Press, 1978, p. 117.

[187] Dee, *Mathematicall Praeface*, f. b.jr.

Dee's 'Mathematicall Praeface' was highly influential. Limiting the discussion to the optical marvels, they resurfaced in the work of his student, Thomas Digges, and William Bourne, who had access to Dee's collection of mirrors. For example, Digges wrote about these optical marvels in his 'An arithmeticall militare treatise, named Stratioticos' (1579).

> Ans such was his Foelicitie and happie successe, not only in these Conclusions, but also in the Optikes and Catoptrikes, that he was able by Perspective Glasses duely scituate upon convenient Angles, in such sorte to discover every particularitie in the Countrey rounde aboute, wheresoever the Sunne beames mighte pearse: As sithence Archimedes, (Bakon of Oxforde only excepted) I have not read of any in Action ever able by meanes natural to performe ye like. Which partly grew by the aide he had by one old written booke of the same Bakons Experiments, that by straunge adventure, or rather Destinie, came to his hands, though chiefelye by conioyning continual laborious Practise with his Mathematical Studies.[188]

All familiar elements, the reference to the optical marvel of telescopic vision, to Roger Bacon and to Archimedes, are present in this quote. In chapter 6, we will discuss and show how to interpret this and similar contemporary quotes, most of them gathered by De Waard and Van Helden.[189] Moreover, it will be shown what made them possible in the second half of the sixteenth century, because they are far from obvious from the point of view of medieval optics, notwithstanding the guise of the genius of Archimedes and Bacon in which they were dressed by mathematicians.

First, we will turn to the second reason why burning mirrors became an important research topic among sixteenth century mathematical practitioners. Different from Pena, Dee stressed the utility of optics, in particular catoptrics, to astrology in his 'Mathematicall Praeface'. Dee was, in particular, thinking of parabolic burning mirrors, which were consequently only apparently absent from his overview of optics in the 'Mathematicall Praeface'. Moreover, Dee defined optics as the study of the multiplication of species, not strictly limited to the study of light.

> It [Perspective] concerneth all Creatures, all Actions, and passions, by Emanation of beames perfourmed. Beames, or naturall lines, (here) I meane, not of light onely, or of colour (though they, to eye, give shew, witnes, and profe, wherby to ground the Arte upon) but also of other Formes, both Substantiall, and Accidentall, the certaine and determined active Radiall emanations.[190]

Again, Dee was highly influenced by the work of his admired Roger Bacon, and, with him, he took optics, as the study of the multiplication of species, to be the ground of the description of the complete physical world. Above in the discussion of vision in the sixteenth century, it was already pointed out that there was an astrological background to Bacon's concerns with the multiplication of species.[191] This astrological background came to the fore in the sixteenth century. Dee's astrological physics, based on an optical model, has been well discussed by Shumaker and Heilbron, Clulee, Vanden Broecke and Szulakowska, and our discussion will be

[188] Digges, Thomas. *An arithmeticall militare treatise, named Stratioticos*. London, 1579, pp. 189-90.

[189] Van Helden, Albert. *The Invention of the Telescope*. Vol. 67, *Transactions of the American Philosophical Society*. Philadelphia: The American Philosophical Society, 1977, pp. 29-34; De Waard, C. *De Uitvinding der Verrekijkers: Een Bijdrage tot de Beschavingsgeschiedenis*. 's-Gravenhage: De Nederl. Boek- en Steendrukkerij, 1906, pp. 73-7.

[190] Dee, *Mathematicall Praeface*, f. b.jr.

[191] See also Molland, George. 'Roger Bacon and the Hermetic Tradition.' *Vivarium* 31 (1993): 140-60, pp. 148-60.

much indebted to these studies.[192] However, we will also point out the possible importance for Dee's optics of his link with sixteenth century Venice, in particular, Ausonio.

3.2. Burning Mirrors, the Multiplication of Species, Astrology and Alchemy

Among the thirteenth century perspectivists of the Latin West, Roger Bacon (ca. 1220-1292) was the primary advocate of the doctrine of the 'multiplication of species'.[193] It was the core of what would become the standard explanation of vision, perception and cognition. According to this theory any visible object generates or 'multiplies' species, also called 'forms', 'images' or 'likenesses', of light and color in the transparant medium. Thus, it is important to understand that these species are not material emanations. In turn, the species in this medium generate further species in the medium continguous to it. The result is a continuous multiplication of species.

> But a species is not body, nor is it moved from one place to another; but that which is produced [by an agent] in the first part of the air [or other medium] is not separated from that part, since form cannot be separated from the matter in which it is unless it should be mind; rather, it produces a likeness to itself in the second part of the air, and so on. Therefore, there is no change of place, but a generation multiplied through the different parts of the medium; nor is it body which is generated there, but a corporeal form that does not have dimensions of itself but is produced according to the dimensions of the air; and it is not produced by a flow from the luminous body, but by a drawing forth out of the potentiality of the matter of the air.[194]

Moreover, these species proceed from every point of the visible object's surface in all directions. They travel along straight lines, which allows a geometrical analysis of their propagation. The object's accidents are conveyed by the species to the eye of the viewer upon which they are impressed. Of less importance here for our purposes, next, the species are multiplied in the internal senses, until they result, by a process of abstraction, into the intelligible species. Thus, the multiplication of species was Bacon's basic model of vision, perception and cognition, and, it should be stressed, the multiplication of species in the medium, that is, their rectilinear propagation along rays, could be subjected to geometrical analysis. Much of Bacon's 'De multiplicatione specierum' was devoted to showing how radiation could be geometrically analyzed, borrowing the mathematical laws of radiation from Greek and Arabic sources.[195]

[192] Shumaker and Heilbron, *John Dee on Astronomy*, pp. 50-73; Clulee, Nicholas H. 'Astrology, Magic, and Optics: Facets of John Dee's Early Natural Philosophy.' *Renaissance Quarterly* 30 (1977): 632-80; Clulee, Nicholas H. *John Dee's Natural Philosophy: Between Science and Religion*. London New York: Routledge, 1988, pp. 39-74; Vanden Broecke, Steven. 'Dee, Mercator, and Louvain Instrument Making: An Undescribed Astrological Disc by Gerard Mercator (1551).' *Annals of Science* 58 (2001): 219-40, pp. 226-9; Szulakowska, Urszula. 'Geometry and Optics in Renaissance Alchemical Illustration: John Dee, Robert Fludd and Michael Maier.' *Cauda pavonis* 14 (1995): 1-12; Szulakowska, Urszula. *The Alchemy of Light: Geometry and Optics in Late Renaissance Alchemical Illustration*. Leiden Boston Köln: Brill, 2000, pp. 12-70.

[193] Lindberg, *Theories of vision*, pp. 112-14; Tachau, *Vision and Certitude in the Age of Ockham*, pp. 3-26.

[194] Roger Bacon, Opus maius, V.1, distinction 9, chapter 4, translation in Grant, *A Source Book in Medieval Science*, p. 394. Discussed in Lindberg, David C. *Roger Bacon's Philosophy of Nature: A Critical Edition, with English Translation, Introduction, and Notes, of De Multiplicatione Specierum and De Speculis Comburentibus*. Oxford: Clarendon Press, 1983, p. lxiii.

[195] Ibid., p. lxxi.

However, for Bacon, the visible species is only one instance of a more general category of species. Species denote the effect of any agent, or the likeness of any object, emanating from the object, whether or not a percipient being is present. In this sense, the meaning of species is clearly one of the force or power by which any object acts on its environment.

> Every efficient cause acts through its own power, which it exercises on the adjacent matter, as the light (lux) of the sun exercises its power on the air (which power is light [lumen] diffused through the whole world from the solar light [lux]). And this power is called 'likeness', 'image' and 'species', and is designated by many other names, and it is produced both by substance and by accident, spiritual and corporeal. … This species produces every action in the world, for it acts on sense, on the intellect, and on all matter of the world for the generation of things.[196]

Thus, Bacon attributed all natural causation to the multiplication of species. Henceforward, all natural causation would be subjected to the same geometrical laws applied by perspectiva to the visible species, in particular, concerning the propagation of light.

The doctrine of the multiplication of species was not completely original with Bacon. Bacon seems to have been deeply influenced by a treatise of al-Kindi on astrology, known in its Latin translation 'De radiis'.[197] Al-Kindi argued that everything depended on the disposition of the stars, because they sent their rays into the world. Moreover, everything in the world produced rays in the manner of the stars. This seems to be the source of the central doctrine of Bacon's 'De multiplicatione specierum' that everything in the world is a source of radiation, thus, that the world itself is a vast network of forces. Al-Kindi's optical work, 'De aspectibus', also known to Bacon, showed how to mathematically investigate radiation.[198] It is not clear whether al-Kindi himself thought this mathematical investigation also applicable to the rays of his 'De radiis', but reading both as pair might have suggested as much to Bacon. The first in the Latin West to take up the suggestion, provided by the work of al-Kindi's 'De radiis', was Grosseteste, who, within the framework of his neoplatonic cosmogony of light, developed al-Kindi's claim of universal radiation into the stronger claim that all physical causation can be reduced to such radiation.[199]

> A natural agent … multiplies its power from itself to the recipient, whether it acts on sense or on matter. This power is sometimes a likeness, and it is the same thing whatever it may be called; and the agent sends the same power into sense and into matter, or into its own contrary, as heat sends the same thing into the sense of touch and into a cold body … But the effects are diversified by the diversity of the recipient, for when this power is received by the senses, it produces an effect that is somehow spiritual and noble; on the other hand, when it is received by matter, it produces a material effect. Thus the sun produces different effects in different recipients by the same power, for it cakes mud and melts ice.[200]

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[196] Roger Bacon, Opus maius, IV, distinction 2, chapter 1, translation in Grant, *A Source Book in Medieval Science*, p. 393. Discussed in Lindberg, *Theories of Vision*, p. 113.

[197] Lindberg, *Roger Bacon's Philosophy of Nature*, pp. xliv-xlv.

[198] Ibid., pp. xlv-xlvi.

[199] Ibid., pp. xlix-liii. See also Lindberg, *Theories of Vision*, pp. 94-102.

[200] Robert Grosseteste, Concerning lines, figures and angles. In *Die philosphische Werke des Robert Grosseteste, Bischofs von Lincoln*, edited by Ludwig Bauer in *Beiträge zur Geschichte der Philosophie des Mittelalters* 9 (1912): 59-65, p. 60, translation in Grant, *A Source Book in Medieval Science*, p. 385-6. Discussed in Lindberg, *Roger Bacon's Philosophy of Nature*, p. lii.

However, the thorough application of the associated geometrical optics to this universal 'multiplication of species', only slightly developed by Grosseteste, was left to Bacon, in particular, in his 'De speculis comburentibus'. It should be emphasized that the multiplication of species avoided that action at a distance, which was held impossible, would be allowed in a theory of causation. For example, without the multiplication of species in the medium, as outlined above, vision would require an action at a distance and, more generally, the transmission of influences through the medium would be an occult phenomenon. One of the most obvious examples of what is apparently an action at a distance is the sun's ability to kindle fire on earth. Consequently, in his 'De speculis comburentibus', Bacon attempted to link his doctrine of the multiplication of species to a mathematical model of the propagation of light, using burning mirrors, and the radiation of light through small pinholes, which, also, apparently involved action at a distance, as test cases.[201] Bacon mainly discussed spherical burning mirrors (to be discussed below in chapter 4), leaving only a minor treatment of the parabolic burning mirror. He was aware that this mirror brings all solar rays to one focus, but he presented no mathematical analysis, admitting that he did not know how to calculate the locus of the point of combustion.[202]
In the fourteenth and fifteenth centuries the ontology of species in the medium was a widely discussed problem. A full outline of the problem would lead us too far, but I will briefly discuss here four cases, Ockham, Oresme, Biagio Pelacani, and Martin Hamerlin, which are more or less representative and, above all, interesting as a background to the Parisian, Viennese and Paduan situation and the discussion of burning mirrors by, for example, the already discussed Oronce Fine and Regiomontanus, and Ettore Ausonio in the following chapters. The fourteenth century English Franciscan, William of Ockham (1280/90-1349), denied the existence of any species.[203] For Ockham, intuitive cognition, understood to provide direct knowledge of an object's existence would be impeded by species if they would exist. Thus, for Ockham, there is an unmediated apprehension of an object. All that is required for vision is an impressed quality in the sense. There are no species in the medium required, and, consequently, no sensible or intelligible species are extracted from them. His denial of the existence of species was not solely based on the application of Ockham's razor, but also on the lack of experiential evidence. We are not aware of the existence of species in the medium, and, within Ockham's account of intuitive cognition, we clearly should, if such species would exist in the medium. Ockham was fully aware of the fact that denying the existence of species entails action at a distance.[204] However, he stated that such action at a distance does take place. His examples included magnetic attraction and solar action at a distance. Thus, unlike Bacon, Ockham used burning mirrors as a case in favor of action at a distance, and against neoplatonism and the existence of species in the medium.

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[201] Ibid., pp. 283-341.

[202] 'But our present concern is with that surface which reflects all axes of the sun incident on the whole mirror to a single point [of focus], and this is the concave surface of a paraboloidal body, as the Liber de speculis comburentibus [of Alhazen] demonstrates. … Concerning the exact location of the point of combustion, I have no certain knowledge at present'. Roger Bacon, De speculis comburentibus, proposition 8, translation in Ibid., pp. 337-41.

[203] Tachau, 'The problem of the species in medio', pp. 399-400; Lindberg, *Theories of Vision*, pp. 140-1.

[204] Tachau, 'The problem of the species in medio', pp. 402-3.

Tachau has shown that Ockham's position was certainly not generally accepted.[205] Biagio Pelacani (or Blasius of Parma), who taught at the universities of Bologna, Florence, Pavia and Padua, dealt with species in his 'Quaestiones perspectivae' (1390).[206] In his first question, Pelacani argued against Ockham for the existence of species in the medium. However, he rejected the possibility that species are 'res' or substantial forms.[207] They belong to the Aristotelian category of quality. Pelacani's position seems to be a return to positions of, for example, Henry of Ghent and John Duns Scotus, against which background Ockham had built his denial of the existence of species.[208] Henceforward, Pelacani undermined the metaphysical basis of the notion of species and the notion of ray, which, against neoplatonic metaphysics, became nothing else but mathematical terms. Herewith, Pelacani was at the basis of the 'mathematical turn' of optics, based on Alhazen, which would become highly present in the sixteenth century, as outlined in the first section on the mathematical arts and optics of this chapter.[209]

Finally, another critic of Ockham was Martin Hamerlin, who taught at the University of Vienna, and wrote a commentary on Pecham's 'Perspectiva communis' (1444), which was, in those years, a traditional teaching subject at this university.[210] Against Ockham, Hamerlin defended the Baconian position of the multiplication of species, not understood to be merely qualities, as in the case of Pelacani. When defending this position, Hamerlin had definitely the well-known problem of action at a distance in mind, which Ockham had accepted, because he referred to burning mirrors, raising the rhetorical question, 'how can burning by a crystal ball or in the centre of a concave mirror be saved without species'?[211] Hamerlin's commentary is interesting, because it might have been known to Regiomontanus, who started studying at the University of Vienna around 1450, only six years after Hamerlin's commentary was written.[212] However, in the present state of research, none of the annotations to his version of the 'Speculi almukesi compositio' on the parabolic burning mirror suggests that the problem of species, let alone astrology, might have provided the impetus to Regiomontanus' study of the burning mirror.

The sources provide a better case for arguing in favor of a connection between the study of the parabolic burning mirror and optico-astrological concerns on the Baconian model of the multiplication of species as concerns Oronce Fine. It has already been pointed out above that

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[205] Ibid., 404-43. See also Maier, Anneliese. 'Das Problem der 'Species Sensibiles in Medio' und die Neue Naturphilosophie des 14. Jahrhunderts.' In *Ausgehendes Mittelalter: Gesammelte Aufsätze zur Geistegeschichte des 14. Jahrhunderts*, edited by Anneliese Maier, 419-51. Roma: Edizioni di Storia e Letteratura, 1967.

[206] Vescovini, Graziella Federici. *Studi sulla Prospettiva Medievale*. Torino: G. Giappichelli, 1965, pp. 239-67. See also Vescovini, Graziella Federici. 'Le Questioni di 'Perspectiva' di Biagio Pelacani da Parma.' *Rinascimento* 12 (1961): 163-243, pp. 163-206; Alessio, Franco. 'Questioni Inedite di Ottica di Biagio Pelacani da Parma.' *Revista critica di storia della filosofia* 16 (1961): 79-110, pp. 79-81; Sorge, Valeria. 'Due Questioni di Perspectiva di Biagio Pelacani da Parma.' *Atti dell' Accademia di Scienze Morali e Politiche* 105 (1995): 169-98; pp. 169 -72.

[207] Vescovini, *Studi sulla Prospettiva Medievale*, pp. 245-6; Vescovini, 'Le Questioni di 'Perspectiva'', p. 188.

[208] Tachau, *Vision and Certitude in the Age of Ockham*, pp. 28-39; 55-81.

[209] Vescovini, *Studi sulla Prospettiva Medievale*, pp. 261-2, 269-71.

[210] Rosinska, *Fifteenth Century Optics*, pp. 184-6.

[211] 'Item quomodo salvatur sine speciebus quod sit incensio per cristallum rotundam [sic] et in centro speculi concavi etc.'. Martin Hamerlin, Commentary, in Ibid., p. 153. If a spherical concave mirror is meant, then the notion that the point of combustion is at the center of curvature of the mirror was derived from pseudo-Euclid or a wrong reading of a confusing passage in Bacon's 'De speculis comburentibus', as will be explained below in chapter 4.

[212] Ibid., p. 186.

Fine's 'De speculo ustorio' was remarkable in its adoption of optical theorems as a context to the work on the parabolic burning mirror, although no mention was made in this work of the doctrine of the multiplication of species. However, as already mentioned, Fine edited (or might even have been the actual author) of a work of Claudio Celestino, 'De his quae mundo mirabiliter eveniunt' (1542). It has been shown that this was actually an account and discussion of Oresme's arguments in his 'Quodlibeta (De causis miribilium)'.[213] Oresme (ca. 1320-1382), working in Paris, adopted the Baconian doctrine of the multiplication of species. However, Oresme was a severe critic of the judicial astrology of his own day.[214] Nevertheless, he allowed celestial influences, propagated according to the Baconion optical model, but he limited celestial influence to motion and light propagated as rays that also produce heat, excluding all 'occult' influences.

Celestino and Fine included a chapter 'De influentiis caelorum' to defend astrology against Oresme's attack.[215] They drew highly upon al-Kindi's 'De radiis' for the claim that everything in the terrestrial world depended on the disposition of the stars. Moreover, they argued that there are 'occult' influences from the heavens in addition to light and motion.[216] Notwithstanding that Celestino and Fine made little use of specific optical arguments in their defense of astrology, given the background of the influence of al-Kindi's 'De radiis' and their familiarity with the Baconian doctrine of the multiplication of species, the connection between (astrological) causality, optical radiation and burning mirrors must have been understood. Henceforward, Fine's study of the parabolic burning mirror might have received some impetus from astrological concerns, although, different from Clulee, I think that the concern for instrument-making provided a much stronger impetus in Fine's case.[217] As concerns John Dee, Vanden Broecke has recently argued that his connection with the Louvain mathematicians was much more important than his meeting with Fine in the development of his astrological physics on the basis of a Baconian optical model, eventually published in his 'Propaedeumata aphoristica' (1558).[218]

As already mentioned, Dee visited the Louvain mathematicians, for a few months, in May 1547, and, again, from June 1548 until July 1550. On the basis of the inscriptions on an astrological disc, published by Mercator in May 1551, and a text published by Gerard Mercator's son Bartholomew, 'Breves in Sphaeram Meditatiunculae' (1563), Vanden Broecke has argued that, at that time, there existed in Louvain a programme of astrological reform, which 'gave attention to rays as the physical basis of astrological causation and the relevance of optical analogy in determining the vigour of such rays'.[219] Moreover, as Dee himself retrospectively acknowledged in his 'Mathematicall Praeface', he also met Antonius Gogava (1529-1569) in Louvain.[220]

[213] Caroti, 'Nicole Oresme, Claudio Celestino, Oronce Fine e i 'Mirabilia Naturae'', p. 357.

[214] McCluskey, Stephen C, Jr. 'Nicole Oresme on Light, Color, and the Rainbow: An Edition and Translation, with Introduction and Critical Notes, of Part of Book Three.' Ph. D., The University of Wisconsin, 1974, pp. 8-18; Burton, 'Nicole Oresme's On seeing the stars (De visione stellarum)', pp. 64-5.

[215] Caroti, 'Nicole Oresme, Claudio Celestino, Oronce Fine e i 'Mirabilia Naturae'', pp. 381-91.

[216] Ibid., p. 358.

[217] Compare Clulee, *John Dee's Natural Philosophy*, pp. 57-61.

[218] Vanden Broecke, 'Dee, Mercator, and Louvain Instrument Making', pp. 226-9.

[219] Ibid., p. 228.

[220] 'I was (for *21. yeares ago) by certaine earnest disputations, of the Learned Gerardus Mercator, and Antonius Gogava (and other,) thereto so provoked: and (by my constant and invincible zeale to the veritie) in observations of Heavenly Influences (to the Minute of time,) than, so diligent: And chiefly by the Supernaturall influence, from the

Gogava studied mathematics with Gemma Frisius in Louvain, before leaving for Italy at the end of the 1540s to study medicine at the University of Padua.[221] Around 1550, he established a medical practice in Venice, but he also published some work on music and Aristotelian optics. Due to his connection with the Gonzaga court in Mantua, he moved to Madrid shortly before his death. In Louvain, Govaga published Ptolemy's 'Tetrabiblos' (1548), a standard work on astrology. This edition included Regiomontanus' version of the 'Speculi almukesi compositio' and the first four propositions of Alhazen's 'De speculis comburentibus'.[222] This suggests that Gogava considered the optics of the parabolic burning mirror relevant to an astrological physics.
Thus, Dee's interest in optics and its relevance for astrology originated upon his stay in Louvain between 1548 and 1550. In the next years, he acquired the major optical sources of antiquity and the Middle Ages, and, as already mentioned, developed a specific interest in the work of Roger Bacon in general, and Bacon's optical work in particular.[223] Moreover, Dee wrote a 'De speculis comburentibus libri 5', dated to 1558, the same year of the publication of his 'Propaedeumata aphoristica'.[224] Their was minor treatment in this work, limited to a few drawings, of image formation in a convex mirror, showing in a well-established, but wrong tradition, how an image 'in the air' could be obtained by applying the cathetus rule, and, also, a representation of an instrument to measure angles of incidence and reflection.[225] (Figures 2.19-2.20)

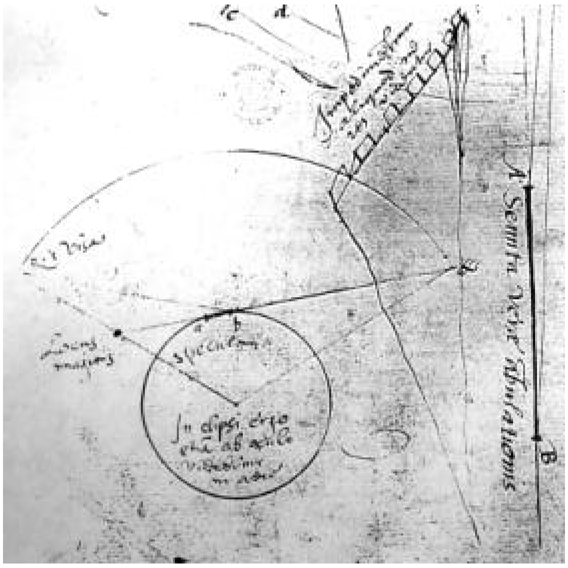

Figure 2.19

Starre of Iacob, so directed: That any Modest and Sober Student, carefully and diligently seking for the Truth, will both finde and confesse, therein, to be the Veritie, of these my wordes'. Dee, *Mathematicall Praeface*, f. B.iiijr.

221 Jacques, Victor. 'Gogava Antoine-Hermann.' In *Biographie Nationale*, 86-88. Bruxelles: Thiry-Van Buggenhoudt, 1866.

222 Clagett, *Archimedes in the Middle Ages*, pp. 4: 319-20; Heiberg, J.L., and E. Wiedemann. 'Eine Arabische Schrift über die Parabel und Parabolische Hohlspiegel.' *Bibliotheca Mathematica* 11 (1910-11): 193-208.

223 Clulee, *John Dee's Natural Philosophy*, pp. 52-7, 68-9.

224 Bodleian Library (Oxford), MS Cotton Vitellius C. VII, ff. 279r-308v.

225 Ibid., ff. 305v, 308v.

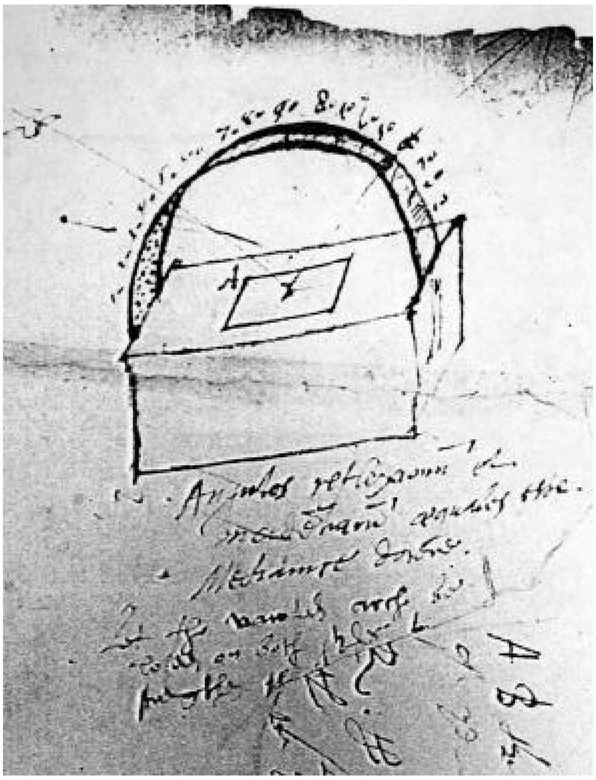

Figure 2.20

However, the best structured part of the work, prepared for publication, adequately dealt with the parabolic burning mirror in the fifteenth and sixteenth centuries tradition outlined above.[226] It discussed the development of a parabolic section from a right-angled cone, as it was usually done in the optical tradition of burning mirrors, following Witelo. Moreover, the context is evidently optical, with references to lines of incidence and reflection, point and distance of combustion, and the development of the parabola from a know distance of combustion or focal distance.

Dee's research on the parabolic burning mirror received its impetus from his interest in an astrological physics in the 1550s, which culminated in the publication of the 'Propaedeumata aphoristica'.[227] Dee's astrological physics was based on two elements, (1) a naturalistic mechanism of causality, based on the multiplication of species, and (2) a mathematical treatment of this causality, based on the geometrical treatment of light rays in optics, intimately connected with the study of burning mirrors. As a model of what is apparently action at a distance, Dee chose magnetism.[228] However, the action at a distance of a magnet is only apparent, because, the magnet influences things at a distance by emitting species or spherically projecting rays in all directions. Likewise, the stars and planets emit species or occult rays of virtue that spread throughout the universe. These species are the cause of what happens on earth. As in Bacon's 'De

[226] Ibid., ff. 279r-305r.

[227] Clulee, *John Dee's Natural Philosophy*, pp. 42-7. See also Bowden, Mary Ellen. 'The Scientific Revolution in Astrology: The English Reformers, 1558-1686.' Ph. D., Yale University, 1974, in particular, pp. 65-72.

[228] Dee, Propaedeumata aphoristica, XXIIII, translation in Shumaker and Heilbron. *John Dee on Astronomy*, p. 133.

multiplicatione specierum', these species are of a more general category than the visible species. However, the propagation of astrological or unseen influences can be considered on the basis of the model of the propagation of the visible species of light, or, as Dee wrote in the 'Propaedeumate aphoristica', 'as it is the prerogative of the first motion that without it all other motions should become quiescent, so it is the faculty of the first and chief sensory form, namely light, that without it all other forms could do nothing'.[229] Thus, this allowed Dee to claim that astrological causality was open to the same mathematical treatment as the propagation of light. Species, also of an occult quality, always propagate themselves in the same way as light.

Consequently, optics became the general science for the study of astrological influences. This was how Dee ultimately regarded optics in his 'Mathematicall Praeface'. No matter what other practical utility it might have, optics was the key discipline for human understanding, 'bycause of the prerogative of Light, beying the first of Gods creatures', which is reminiscent of Grosseteste's neoplatonic cosmogony of light, and, reminiscent of Bacon's multiplication of species, optics 'concerneth all Creatures, all Actions, and passions, by Emanation of beames perfourmed'.[230] Thus, with the mathematical model of optics at hand, Dee claimed that the rays of a star took the form of a right conic, with its base on the star, its vertex on earth and its axis, the strongest ray, passing from the vertex to the center of the star.[231] Following the optical model further, the strength of the astrological influence is dependent upon the strength of the rays, thus, upon the varying size and distance of the stars and planets. Consequently, Dee outlined a programme of measurement of the sizes and distances of stars and planets, which would be necessary for his astrological physics, but which never matured beyond an outline.

Since burning mirrors had been the basic model for the discussion of propagation of light in the optical tradition that Dee inherited, he thought that the study of burning mirrors could provide the instrumental means to study astrological influences. This becomes very clear in the 'Propaedeumate aphoristica', when Dee claimed that by manipulation of burning mirrors 'the industrious investigator of secrets' could simulate and manipulate astrological influences, because 'if you were skilled in 'catoptrics', you would be able, by art, to imprint the rays of any star much more strongly upon any matter subjected to it than nature itself does'.

> By this means obscure, weak, and, as it were, hidden virtues things, when strengthened by the catoptric art, may become quite manifest to our senses. The industrious investigator of secrets has great help offered to him from this source in testing the peculiar powers not merely of stars but also of other things that they work upon through their sensible rays.[232]

[229] 'Sicut primi motus privilegium est, ut sine torpeant omnes reliqui, sic primae & praecipuae Formae sensibilis, (nimirum LUCIS) ea est facultas, ut sine ea caeterae formae omnes agere nihil possint.' Dee, Propaedeumata aphoristica, XXII, translation in Ibid., pp. 130-1.

[230] Dee, *Mathematicall Praeface*, f. b.jr.

[231] Clulee, *John Dee's Natural Philosophy*, pp. 47-50; Shumaker and Heilbron. *John Dee on Astronomy*, pp. 82-99.

[232] 'Katoptricis si fueris peritus, cuiuscunque Stellae radios in quamcunque propositam materiam fortius tu multo per artem imprimere potes, quam ipse per se Natura facit. … Hinc obscurae, debiles, & quasi Latentes rerum Virtutes, arte Catoptrica multiplicatae, sensibus fient nostris manifestissimae. Unde non in stellarum solum, sed aliarum quoque rerum propriis examinandis viribus, quas per Sensibiles exercerent radios, diligens Arcanorum Investigator, maximum sibi oblatum auxilium habet'. Dee, Propaedeumata aphoristica, LII, translated in Shumaker and Heilbron, *John Dee on Astronomy*, pp. 148-9. Discussed in Ibid., pp. 67-73. See also Tait, Hugh. ''The Devil's Looking-Glass': The Magical Speculum of Dr John Dee.' In *Horace Walpole, Writer, Politician, Connoisseur: Essays on the 250th*

For example, since in optical theory the strength of rays depends upon their obliquity, the strongest ray being the perpendicular ray, Dee claimed that the angle of incidence could be manipulated by catoptrical means, or, in his own words, 'the more nearly the radiant axis of any star approaches perpendicularity over any elemental surface, the more strongly the star will impress its forces upon the place exposed to it: directly, to be sure, because of the greater nearness of the agent, but also by reflection, because such reflected rays are joined more closely with the incident ones'.[233] No matter what Dee's procedures actually consisted of, considering their obscurity, his research intention was to use burning mirrors and his knowledge of their optical properties to make comparisons between the radiations of different planets and stars.

Thus, Dee's 'Propaedeumata aphoristica' shows that behind Dee's interest in the study of burning mirrors, there was a concern for the development of an astrological physics. After the 'Propaedeumata aphoritica', Dee's interest shifted more toward alchemy, which would eventually culminate in the writing of the 'Monas hieroglyphica' (1564). Although a study of this work strictly falls outside the scope of this chapter to determine how medieval optics was appropriated by sixteenth century mathematicians, I will touch upon it, without the slightest intention of presenting an exhaustive study, because of the possible link it provides with the protagonist of the next two chapters, Ettore Ausonio. Dee had little interest in alchemy before the 1560s, although already in the 1550s, as Szulakowska has shown, Dee was redirecting his interests toward Paracelsian alchemy.[234] Paracelsus had developed the idea of 'the light of nature', which he distinguished from the light of the human spirit.[235] For Paracelsus, this idea of the light of nature could be examined in mathematical terms. In fact, mathematics could examine how earthly generation and creation were effected by the stars. Herewith, Paracelsus forced a link between alchemy and an astrological mathematics, which was similar in intent to Dee's 'Propeudeumata aphoristica'. Moreover, for Paracelsus, the final aim of alchemy was medical practice.

Another alchemical tradition, which linked alchemy, astrology and medicine, was the pseudo-Lullian corpus. This was a set of alchemical texts, written in the late fourteenth, fifteenth and sixteenth centuries by anomynous alchemists, who attributed their work to Ramon Lull.[236] Lull himself had never positively dealt with alchemy, but the pseudo-Lullian alchemists used Lullian

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Anniversary of Walpole's Birth, edited by Warren Hunting Smith, 195-212. New Haven London: Yale University Press, 1967.

[233] 'Quo magis ad perpendicularitatem super aliquam elementarem superficiem accedit axis radiosus alicuius stellae, eo fortius circa talem suae incidentiae locum suas vires illa stella imprimet: directo quidem modo maiorem agentis vicinitatem: reflexo autem, quia reflexi tales radii, as incidentes, vicinius conduplicantur'. Dee, Propaedeumata aphoristica, LIIII, translated in Ibid., pp. 148-51.

[234] Szulakowska, Urszula. 'Paracelsian Medicine in John Dee's Alchemical Diaries.' *Cauda pavonis* 18 (1999): 26-31. See also Clulee, Nicholas H. 'John Dee and the Paracelsians.' In *Reading the Book of Nature: The Other Side of the Scientific Revolution*, edited by Michael T. Walton and Allen G. Debus, 111-32. Missouri: Sixteenth Century Journal Publishers Inc., 1998.

[235] Pagel, Walter. 'Paracelsus als 'Naturmystiker'.' In *Epochen der Naturmystik: Hermetische Tradition im Wissemschaftlichen Fortschritt*, edited by Antoine Faivre and Rolf Christian Zimmermann, 52-104. Berlin: Erich Schmidt Verlag, 1979, pp. 58-9.

[236] Szulakowska, *The Alchemy of Light*, pp. 18-22; Thorndike, Lynn. *A History of Magic and Experimental Science*. Vol. 4. New York: Columbia University Press, 1934, pp. 3-64; Pereira, Michela. *The Alchemical Corpus Attributed to Raymond Lull*. Edited by Jill Kraye and W.F. Ryan, *Warburg Institute Surveys and Texts*. London: The Warburg Institute, University of London, 1989. See also Szulakowska, Urszula. 'Thirteenth Century Material Pantheism in the Pseudo-Lullian 'S'-Circle of the Powers of the Soul.' *Ambix* 35 (1988): 127- 54.

geometry and diagrams. The corpus is higly understudied, but, in the present state of research, it is safe to conclude that pseudo-Lullian alchemy emphasized the importance of astrology to alchemy. Szulakowska has argued that, in this pseudo-Lullian alchemical tradition, 'the celestial virtue (occasionally related to the sun's rays) is often conceptualized by geometrical paradigms', thus, that 'the geometry of these alchemical treatises signified the use of astrology which is wholly dependent on mathematics and geometry'.[237] Dee collected many pseudo-Lullian alchemical texts.[238] Although Dee did not draw upon any specific alchemical tradition for the writing of his 'Monas', pseudo-Lullian alchemy was part of its general historical context.

Clulee has argued that Dee's 'Monas' should be regarded as developing a new form of writing with, as its principal aim, the restoration of astronomy and alchemy.[239] To write the 'Monas', Dee drew highly upon the Hebraic Caballah, which Dee had been studying since the early 1560s. In the tradition of the Caballah, Hebrew, as the divine language of creation, was considered the vehicle of cosmological truths.[240] The Hebrew alphabet was regarded to be composed out of the divine flame or the light of God. By manipulating the Hebrew alphabet, the cabbalist was said to be able to discover cosmological truths, as such, bridging the gap between the linguistic and the physical world. Dee's 'Monas', a symbol that had already appeared in his 'Propeudaemata aphoristica', was his alternative version of the Hebraic Caballah. For Dee, the 'Monas' mirrored the structure of the cosmos and the mathematical genesis of the cosmos. As in the Hebraic Caballah, by manipulating the 'Monas', Dee thought he revealed the alchemical nature of the universe. This manipulation was a kind of shortcut to the practical alchemical work. Again, a recurrent theme of the 'Monas' was that alchemical processes depended upon the heavens, repeating a general tendency in the alchemical traditions on which Dee was drawing. Consequently, Dee must have considered the manipulation of the 'Monas' as a kind of shortcut to the catoptrical manipulation of celestial virtues in his 'Propeudaemata aphoristica'. Why should the meaning of Dee's 'Monas' be elaborated here, since it definitely was a move away from the Dee's interest in medieval optics in the 'Propaedeumata', and, thus, the scope of this chapter?

As will be seen in the next chapter, Ettore Ausonio was one of the outstanding pseudo-Lullian alchemists of Northern Italy. Sixteenth century Venice, where Ausonio worked during most of his life, saw the introduction of Hebraic or cabbalistic elements into the pseudo-Lullian alchemy. Pereira has shown that it was precisely Ausonio who, quite originally, according to the same scholar, introduced Hebraic or cabbalistic elements into the legend of Lull the alchemist.[241] On his tour of the Continent, Dee visited Venice in June 1563, to return already in November of the same year. From Venice, he travelled to nearby Padua in December, before he went to Antwerp in January 1564 to write and publish his 'Monas'. It was, most likely, also during the summer of 1563 that Dee visited the mathematician Commandino in Urbino. Commandino showed him his work in progress on Apollonius, while Dee left with the Urbino mathematician a manuscript of

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[237] Szulakows ka, *The Alchemy of Light*, p. 27.

[238] Roberts and Watson, *John Dee's Library Catalogue*, pp. 7, 34.

[239] Clulee, *John Dee's Natural Philosophy*, pp. 82-115; Szulakowska, *The alchemy of light*, pp. 55-69.

[240] Ibid., pp. 57-65; Clulee, *John Dee's Natural Philosophy*, pp. 83-96. See also Walton, Michael T., and Phyllis J. Walton. 'The Hebrew Tradition and the Mathematical Study of Nature.' In *Experiencing Nature: Proceedings of a Conference in Honor of Allen G. Debus*, edited by Paul H. Theerman and Karen Hunger Parshall, 43-59. Dordrecht Boston London: Kluwer Academic Publishers, 1997, pp. 44-7.

[241] Pereira, *The Alchemical Corpus Attributed to Raymond Lull*, pp. 48-9.

the 'De superficierum divisionibus' of Machometus Bagdedinus for publication.[242] Although it cannot be argued that Dee's interest in alchemy and the Caballah originated in Venice, it is highly unlikely that Dee would not have met Ausonio in Venice. Both men had very similar interests. Moreover, as will be shown below in chapter 4, in the 1560s, Ausonio corresponded precisely with Commandino on the latter's project on Pappus and its relevance for the study of the parabolic burning mirror. Consequently, it is likely that Ausonio introduced Dee to Commandino, or that Commandino pointed out to Dee that he should visit Ausonio. Although, so far, no trace of Dee has been found in Ausonio's manuscripts, it is very likely that the three men discussed their mutual interests, burning mirrors, and, as concerns, Ausonio and Dee, alchemy and the Caballah. In the following chapters, it will be seen that this connection was also of the utmost importance for the diffusion of optical knowledge essential to the development of the telescope.

3.3. Preliminary Conclusions on Sixteenth Century Mathematical Practitioners and Optics

To conclude, let's return to the original question at the beginning of this chapter. How was the received optical tradition of antiquity and the Middle Ages appropriated by sixteenth century mathematical practitioners? There is no doubt that sixteenth century mathematicians were acquainted with optics, but, as might be expected, how it was appropriated depended upon their professional identity. Their professional identity centered around the development of instruments and astrology. Consequently, there was relatively little interest in the problems of vision, cognition and epistemology, the core of medieval optics, among mathematical practitioners in the sixteenth century. When problems of vision were encountered, it was in the context of measurement problems, and it was thought that such problems could be solved within a Euclidean mathematical framework, to which basic notions of Alhazen's optics were added. However, optics was considered to have consequences for natural philosophy, as in the case of the study of atmospheric refraction, which centered upon the use of a mathematical instrument.

Instead of vision, the study of the propagation of light came more to the fore in the sixteenth century, although this study was certainly not without antecedents in the medieval tradition, in particular, in Bacon's doctrine of the multiplication of species and his study of burning mirrors. Indeed, editions of medieval optical texts in the sixteenth century show that the study of burning mirrors was lifted out of Witelo's 'Perspectiva' to figure prominently. The background of this interest in the study of burning mirrors was provided sometimes, most prominently in the case of John Dee, by astrological concerns of the mathetical practitioners, but, more generally, by their interest in the design of the optical instrument itself, the parabolic burning mirror, and its derivatives, drawing compasses. Henceforward, because of the mathematical practitioners' professional identity, what was of interest to them in the optical tradition was very specific.

In the following two chapters, I will elaborate on the optics of sixteenth century mathematical practitioners, mostly, by focussing on one particular individual of this category, Ettore Ausonio. The two aspects that have stood out so far will be discussed. In chapter 3, the connection between

[242] Watson, Andrew G. 'The Provenance of John Dee's Manuscript of the De Superficierum Divisionibus of Machotemus Bagdedinus.' *Isis* 64 (1973): 382-83; Rose, Paul Lawrence. 'Commandino, John Dee, and the De Superficierum Divisionibus of Machometus Bagdedinus.' *Isis* 63 (1972): 88-93; Rosen, Edward. 'John Dee and Commandino.' *Scripta Mathematica* 28 (1970): 321-26. See also Rambaldi, Enrico I. 'John Dee and Federico Commandino: An English and an Italian Interpretation of Euclid During the Renaissance.' *Rivista di Storia della Filosofia* 44 (1988): 211-47.

optics and instrument design in the sixteenth century will be discussed by taking a closer look at one particular example, the design of the refractive sundial. As it will turn out, its development triggered the intellectual contribution of many sixteenth century mathematical practitioners, from Georg Hartmann to Guidobaldo del Monte. Ausonio seems to be an important link between several of these mathematicians in an area of research that has mostly been neglected in the history of optics. Drawing upon the discussion of the parabolic burning mirror in this chapter, in chapter 4, Ausonio's discussion of burning mirrors, mostly spherical of curvature, will be analyzed. As will become evident, it was of central importance to the development of the telescope in the sixteenth century, in general, and Galileo's optics and telescope, in particular.

III. Optics and Instrument Design in the Sixteenth Century: Breaking Rays and Shadows Running Backwards

1. The Dial of Ahaz and Refractive Scaphe Dials

The category of 'mathematical practitioner', used in the last chapter, is intended to replace the categories of 'scholar' and 'craftsman'.[1] A mathematical practitioner was neither a craftsman with little or no knowledge of the theoretical background to the practical problems he needed to solve, nor was he a scholar, who had little or no connection with craftsmen, whose only contribution consisted of suggesting practical problems which the scholar was able to solve theoretically (and in some cases, after theoretical solutions were proposed, to test them experimentally). Bennett has argued that distinctions were not so clear-cut to the extent that 'scholar' and 'craftsman' are actually categorical fictions of the historian's mind.[2] Instead, when confronted with a practical problem, the mathematical practitioner will take recourse to mathematics, what a 'craftsman' is not supposed to do, and try to advance its solution by practical trials using instruments, what, in its turn, the 'scholar' is not supposed to do. It is important to note that such practical trials involving instruments are not to be equated with any systematic experimentation. In the context of mechanics, as concerns Galileo who likewise belonged to this group of mathematical practitioners, as will be discussed below in chapter 5, Lefèvre has recently remarked that

> 'What makes it difficult to consider men like Galileo engineers [or mathematical practitioners, for consistent terminology] is perhaps not primarily the fact that they produced truly scientific writings; the difficulty might lay elsewhere. It could be that we are not used to recognizing that the theories of these men – and a fortiori it is valid in the case of theories seen as part of the history of modern mechanics – are theories about technical arrangements and procedures. Moreover, we have to recognize them as theories about technical arrangements that were most significant within the sphere of production – including military – whereas arrangements set up for research purposes, i.e. experimental arrangements played a minor role.'[3]

While the 'mathematical practitioner' has some career in the history of mechanics, what if the mathematical practitioner dealt with optics?[4] As outlined in the last chapter, sixteenth century

[1] Hall, Rupert. 'The Scholar and the Craftsman in the Scientific Revolution.' In *Critical Problems in the History of Science*, edited by Marshall Clagett, 3-23. Madison: The University of Wisconsin Press, 1959.

[2] Bennett, J.A. 'The Mechanics' Philosophy and the Mechanical Philosophy.' *History of Science* 24 (1986): 1-28, in particular, pp. 3-9. See also Bennett, J.A. 'The Challenge of Practical Mathematics.' In *Science, Culture and Popular Belief in Renaissance Europe*, edited by Stephen Pumfrey, Paolo L. Rossi and Maurice Slawinski, 176-90. Manchester New York: Manchester University Press, 1991; Settle, Thomas B. 'The Tartaglia Ricci Problem: Towards a Study of the Technical Professional in the 16th Century.' In *Cultura, Scienze e Tecniche nella Venezia del Cinquecento: Atti del Convegno Internazionale di Studio Giovan Battista Benedetti e il Suo Tempo*, edited by Antonio Manno, 217-26. Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1987.

[3] Lefèvre, Wolfgang. 'Galileo Engineer: Art and Modern Science.' *Science in Context* 13 (2000): 281-98, p. 295.

[4] See references in n.2 and n.3, primarily directed toward the history of mechanics; compare also, Laird, W.R. 'The Scope of Renaissance Mechanics.' *Osiris* 2 (1986): 43-68, pp. 51-8; Price, Derek de Solla. 'Philosophical Mechanism and Mechanical Philosophy.' *Annali dell' Istituto e Museo di Storia della Scienza* 5 (1980): 75-85; Machamer, Peter. 'Galileo's Machines, His Mathematics, and His Experiments.' In *The Cambridge Companion to Galileo*, edited by Peter Machamer, 53-79. Cambridge: Cambridge University Press, 1998.

mathematical practitioners appropriated the optics of antiquity and the Middle Ages, and, in this process of appropriation, they often changed the scope and focus of medieval perspectiva. However, as precisely their acquaintance with medieval optics shows, they can hardly be considered craftsmen with little or no knowledge of the theoretical background to their practical problems. On the other hand, as the example of the parabolic burning mirror shows, the problem they were confronted with was, no doubt, the designing of a specific instrument.

In this and the following chapter, I will try to show what it means when sixteenth century mathematical practitioners were involved with optics. What was the kind of problems they were confronted with? How did they solve them? How did they regard and how did they use optical knowledge transmitted from antiquity and the Middle Ages? Did they contribute anything to the optical knowledge transmitted or did they change its meaning? As will become evident, several sixteenth century mathematical practitioners were interested in refraction, and what the optical tradition could offer them on refraction, for the specific practical purpose of designing an instrument. Also, the other way around, working on the design of an instrument, gave them the opportunity to elaborate on the transmitted knowledge of refraction. This instrument was the refractive sundial. In this chapter, I will first outline how these instruments worked. Next, it will be shown who made them and what kind of optical knowledge was involved in making them.

In his 'Letter to the Grand Duchess Christina' (1615), Galileo tried to establish that there was no contradiction between the Copernican system, which he had adopted, and certain passages in the Bible.[5] Galileo argued that the miracle of Joshua, at whose request God made the sun stand still, was much more easily interpreted in agreement with the Copernican system than in agreement with a geocentric system. In this context, Galileo referred to a second miracle in the Bible.

> Dionysius the Areopagite says that it is the primum mobile which stood still, not the sun. St. Augustine is of the same opinion; that is, that all celestial bodies would be stopped; and the Bishop of Avila concurs. What is more, among the Jewish authors endorsed by Josephus, some held that the sun did not really stand still, but that it merely appeared to do so by reason of the shortness of the time during which the Israelites administered defeat to their enemies. *Similarly, with regard to the miracle in the time of Hezekiah, Paul of Burgos was of the opinion that this took place not in the sun but on the sun dial*.[6]

By the time of Galileo, references to these two miracles had become standard fare in the theological as well as the astronomical literature. For example, the appearance of the nova in 1572 was likened to these biblical miracles by Tycho Brahe, to stress the out-of-the-ordinary nature of the astronomical event.[7] Similarly, the standard biblical commentary literature, on

[5] McMullin, Ernan. 'Galileo on Science and Scripture.' In *The Cambridge Companion to Galileo*, edited by Peter Machamer, 271-347. Cambridge: Cambridge University Press, 1998, in particular, pp. 302-14.

[6] 'dicendo Dionisio Areopagita, che non il Sole, ma il primo mobile, si fermò; l' istesso stima S. Agostino, ciò è che si fermassero tutti i corpi celesti; dell' istessa opinione è l' Abulense. Ma più, tra gli autori Ebrei, a i quali applaude Ioseffo, alcuni hanno stimato che veramente il Sole non si fermasse, ma che cosi apparve mediante la brevità del tempo nel quale gl' Israeliti dettero la sconfitta a' nemici. *Cosi del miraclo al tempo d' Ezechia, Paulo Burgense stima non essere fatto nel Sole, ma nell' orivuole*.' Galileo, Lettera a Madama Cristina di Lorena, in *Le Opere di Galileo Galilei*. Vol. 5. Firenze: G. Barbèra Editore, 1968, p. 337. Translation in Drake, Stillman. *Discoveries and Opinions of Galileo*. New York: Double Day Anchor Books, 1957, pp. 204-5.

[7] See the discussion in Lerner, *Le Monde des Sphères II: La Fin du Cosmos Classique*, p. 28.

which Galileo was drawing, had proposed interpretations of these miracles.[8] The Bible passages, to which 'the miracle in the time of Hezekiah', referred, is 2 Kings 20: 8-11 and Isaiah 38.7-8.

> (2 Kings 20: 8-11) … And Hezekiah said unto Isaiah, What shall be the sign that the LORD will heal me, and that I shall go up into the house of the LORD the third day? And Isaiah said, This sign shalt thou have of the LORD, that the LORD will do the thing that he hath spoken: shall the shadow go forward ten degrees, or go back ten degrees? And Hezekiah answered, It is a light thing for the shadow to go down ten degrees: nay, but let the shadow return backward ten degrees. And Isaiah the prophet cried unto the LORD: and he brought the shadow ten degrees backwards, by which it was gone down in the dial of Ahaz. … (Isaiah 38.7-8) And this shall be a sign unto thee from the LORD, that the LORD will do this thing that he hath spoken; behold, I will bring again the shadow of the degrees, which is gone down in the sun dial of Ahaz, ten degrees backward. So the sun returned ten degrees, by which degrees it was gone down.

The translation in 'dial of Ahaz' depended on an interpretation, because the Hebrew word 'ma'alot' was sometimes translated as steps (referring to the steps of the royal palace of Ahaz).[9] At least, one interpretation, namely by Levi ben Gerson, considered refraction of the solar rays by intervening clouds and vapors, as responsible for the retreating of the shadow (on the steps, according to Levi).[10] In the sixteenth century, but maybe earlier, in the fifteenth century, as will become evident, the dial of Ahaz was taken to have been a refractive sundial. A scaphe dial, filled unto the rim with water, shows the shadow of the sun retreating as in the biblical passage.[11]
Scaphe dials are known to have been made since antiquity.[12] They are a subset of altitude sundials. Altitude sundials are sundials on which time is determined from the altitude of the sun above the horizon. Scaphe dials are of a spherical or conical shape. Since most refractive scaphe dials are of the bowl-type, the description is limited here to hemispherical scaphe dials. The bowl shows an image of the celestial vault, so that for every day of the year, the hour of the day can be read from the shadow of tip of the gnomon thrown on the hemisphere. They were different systems of dividing time in hours. Basically, the choice was between an equal and unequal hour system. If unequal hours were used, the duration of a sunlit day, obviously varying throughout the year, was divided into 12 equal parts. An equal hour system considered a full day (from one noon to the next) as divided into 24 equal hours. Here, discussion is limited to the unequal hour system.

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[8] For an overview of this literature and the identification of Galileo's source(s), see Goldstein, Bernard R. 'Galileo's Account of Astronomical Miracles in the Bible: A Confusion of Sources.' *Nuncius* 5 (1990): 3-16.

[9] Ibid., p. 6. See also Turner, A. J. 'A Biblical Miracle in a Renaissance Sundial.' *Bulletin of the Scientific Instrument Society* 61 (1999): 11-14, pp. 13-4.

[10] Goldstein, 'Galileo's Account of Astronomical Miracles in the Bible', pp. 7-8.

[11] For an ahistorical way to establish the retrogradation of the shadow, see Parisot, Jean-Paul. 'La Retrogradation de l' Ombre dans les Cadrans Solaires Analemmatiques.' *Journal for the History of Astronomy* 26 (1985): 43-48.

[12] Turner, A.J. 'Sun-Dials: History and Classification.' *History of Science* 27 (1989): 303-18, p. 305; Rohr, René R.J. *Les Cadrans Solaires: Traité de Gnomonique Théorique et Appliqué*: Gauthier-Villars Editeur, 1965, pp. 22-26.

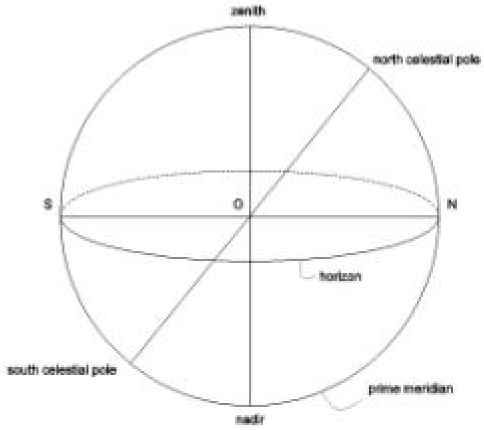

Figure 3.1

To construct a scaphe dial, only some elementary knowledge of the celestial vault is needed.[13] The spot around which all the stars seem to circle is the north celestial pole. (Figure 3.1) Of course, this circular motion is actually the diurnal movement of the earth around its axis. The elevation of the north celestial pole above the observer's horizon equals the observer's latitude. The observer's zenith is the point right above the observer's head (by a line perpendicular to the horizon). Diametrically opposite of this point is the observer's nadir. The prime meridian is the circle that goes through the observer's zenith, nadir and the north celestial pole. It indicates north and south. Thus, all these points and circles depend on the position of the observer. The equinoctial is the plane through the center of the earth perpendicular to the axis of rotation. (Figure 3.2)

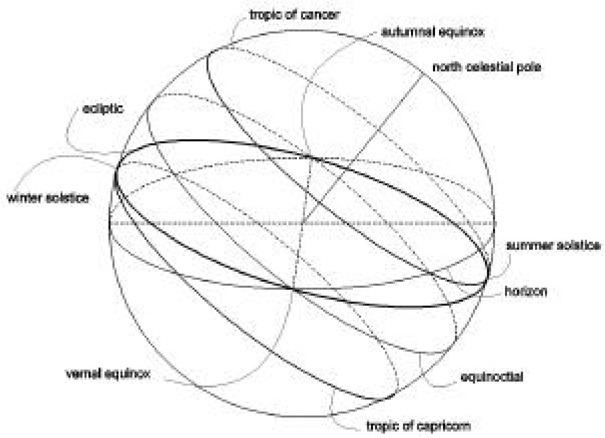

Figure 3.2

[13] None of this knowledge went beyond Sacrobosco. See Heilbron, *The Sun in the Church*, pp. 51-62.

The equinoctial is divided in equal halves by the horizon. Thus, when the sun is on the equinoctial, days and nights are equal. The sun is on the equinoctial on the days of the vernal and autumnal equinox. In summer, it describes a larger arc above the horizon. The arc described by a midsummer sun, parallel to the equinoctial, is the tropic of Cancer. In winter, the sun describes a smaller arc above the horizon. The arc described by a midwinter sun, also parallel to the equinoctial, is the tropic of Capricorn. The tropics are at an angular dstance of the equinoctial of 23°30'. The annual path that the sun seems to follow is the ecliptic. The ecliptic intersects the equinoctial in both equinoxes and the tropics in the summer and winter solstices.

This image of the celestial vault is shown on the bowl of the dial. This can be easily done by a graphic procedure.[14] The nadir is the lowest point of the bowl. The foot of the gnomon is at the lowest point of bowl, while its tip coincides with the center of the sphere. After setting out the prime meridian or noon line, a cardboard semicircular template of the same diameter as the diameter of the bowl is divided into degrees and used to set out the north celestial pole and the equinoctial, dependent on the desired latitude, and transferred to the bowl. Also, the tropics, or the circles through the winter and summer solstice, are set out on the bowl at angular distance of the equinoctial of 23°30'. Next, using an unequal hour system, all three lines are divided into 12 equal parts, and each corresponding three points are connected by a curve indicating the boundary between hours. When the sundial is correctly positioned, the shadow of the gnomon indicates the hour, for example the middle of the ninth hour on the day of the vernal equinox.

However, when the bowl is filled with water, so that the surface of the water coincides with the tip of the gnomon, then the shadow retreats, or the length of the shadow is shortened, with about one hour. The shadow indicates the middle of the eighth hour. Thus, the refractive sundial allowed by optical means to perform the biblical miracle of the Dial of Ahaz. However, refractive sundials were so constructed that they indicate the correct hour on a refracted, thus much contracted, pattern of hour lines. How can these refracted hour lines be constructed? (Figure 3.3)

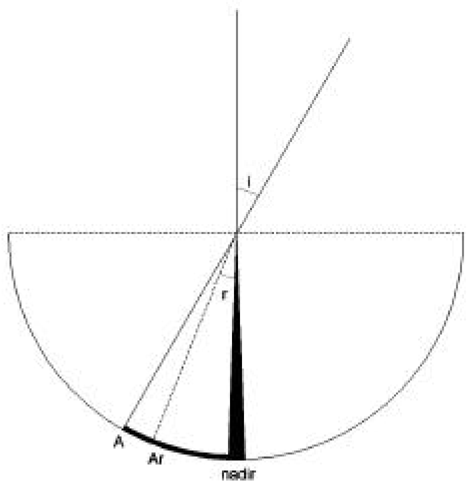

Figure 3.3

[14] This graphic procedure is suggested in Mills, Allan A. 'The 'Dial of Ahaz', and Refractive Sundials in General, Part I: Scaphe Dials.' *Bulletin of the Scientific Instrument Society* 44 (1995): 21-24, pp. 21-2.

When the non-refracted hour lines are already indicated on the scaphe dial, the easiest way to accomplish this is following a practical procedure, which can be applied to each point of the hour lines.[15] First, the angle between the nadir of the bowl and a point A of the non-refracted hour lines is measured. This equals the angle of incidence of the solar ray. Next, the corresponding angle of refraction is calculated. According to this angle of refraction, the corresponding point A_r of the refracted pattern of hour lines is indicated. This procedure is repeated until enough points are set out to draw the curves of the refracted pattern of hour lines of the refractive scaphe dial.

2. The Quantified Study of Refraction in Antiquity and the Middle Ages

In the practical procedure for making a refractive sundial outlined above, it is necessary to calculate the angles of refraction, given the angles of incidence. Nowadays, this is accomplished with the sine law of refraction, sin i/sin r = n, with n the refractive index (1.333 for air-water). However, it was only discovered ca. 1600 by Thomas Harriot, and, again, under a form somewhat different than as it is known today, by Willebrord Snellius ca. 1625.[16] Extensive measurements of angles of refraction were made by Harriot, and, maybe, also by Snellius. Moreover, Harriot's discovery was not known to his contempories, and the knowledge of contempories about Snell's discovery is still discussed. The sine law of refraction was only published, presumably independent of Snellius, by Descartes in 1637.[17] This raises the question how the refracted pattern of hour lines on the refractive sundials were constructed by sixteenth century instrument designers, if they had no knowledge of a law with which to calculate the angles of refraction. However, as is well known, the absence of the sine law of refraction does not entail the absence of any quantitative study of refraction before the seventeenth century.

Ptolemy was the first to undertake a quantitative study of refraction. As discussed in chapter 2, Ptolemy argued that reflection and refraction can be explained along the same physical principles. Moreover, reflection and refraction are not only identical in their physical explanation, but, also, quantitavily.[18] The angles of the visual ray, before and after deviation, with respect to the normal have a quantitative relationship.

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[15] Again suggested in Ibid., pp. 22-3.

[16] On Harriot, see Lohne, Johs. 'Thomas Harriot (1560-1621), the Tycho Brahe of Optics.' *Centaurus* 6 (1959): 113-21; Lohne, Johannes. 'Zur Geschichte des Brechungsgesetzes.' *Sudhoffs Archiv* 47 (1963): 152-72, pp. 159-61. On Snellius, see Hentschel, Klaus. 'Das Brechungsgesetz in der Fassung von Snellius: Rekonstruktion Seines Entdeckungspfades und eine Übersetzung Seines Lateinischen Manuskriptes sowie Ergänzender Dokumente.' *Archive for History of Exact Sciences* 55 (2001): 297-344; De Waard, C. 'Le Manuscrit Perdu de Snellius sur la Réfraction.' *Janus* 39 (1935): 51-73.

[17] There is a general consensus that Descartes' discovery was independent of Snell's. See Schuster, John Andrew. 'Descartes and the Scientific Revolution, 1618-1634: An Interpretation.' Ph. D., Princeton University, 1977, pp. 268-308; Malet, Antoni. 'Gregorie, Descartes, Kepler, and the Law of Refraction.' *Archives Internationales d' Histoire des Sciences* 40 (1990): 278-304; Shea, William R. *The Magic of Number and Motion: The Scientific Career of René Descartes*. U.S.A.: Science History Publications, 1991, pp. 149-64.

[18] Smith, *Ptolemy's Theory of Visual Perception*, pp. 42-3; Smith, A. Mark. 'Ptolemy's Search for a Law of Refraction: A Case-Study in the Classical Methodology of "Saving the Appearances" and Its Limitations.' *Archive for the History of Exact Sciences* 26 (1982): 221-40, pp. 230-2. For Ptolemy's study of refraction, see also Lejeune, Albert. *L' Optique de Claude Ptolémée dans la version Latine d' après l' Arabe de l' Émir Eugène De Sicile*. Louvain: Bibliothèque de l' Université Bureaux de Recueil Publications Universitaires de Louvain, 1956.

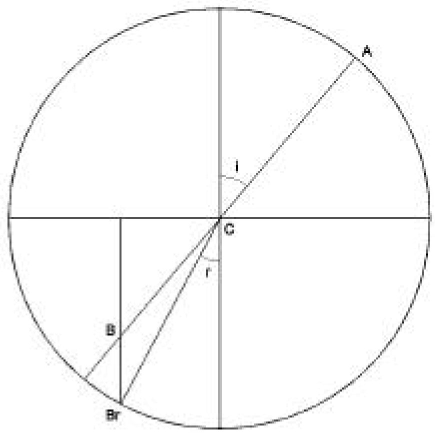

Figure 3.4

In reflection, the quantitative relationship is one of equality. As Ptolemy well knew, the angle of incidence is equal to the angle of reflection. Of course, in refraction, the quantitative relationship is different. To obtain this quantitative relationship, Ptolemy designed an instrument to measure the angles of incidence i and the angles of refraction r, when the ray passes from air to water, from air to glass and from water to glass. This instrument consisted of a graduated disc, equipped with three tiny markers, which allow to establish the line of sight, and, thus, to measure the angle of refraction for angles of incidence at 10 degree intervals.[19] (Figure 3.4) The graduated disc is placed in a water vessel until the level of the water coincides with one of the graduated disc's diameters. The first marker is placed at the center point C of the graduated disc. A second marker is at A along the arc above the water, and a third marker B along the arc under the water. The marker A is placed at, for example, an angle of 40° from the normal, thus, representing the angle of incidence. While sighting along the line between the marker A and the 8center point C, Ptolemy moved the third marker B until it appears to fall in line with the other two. When the marker appears at B, it will be at B_r. Then, the angle of refraction can be read off the disc. By this procedure, Ptolemy claimed to obtain these results.

	i	r		i	r		i	R
air - water	0	0°	air - glass	0	0°	water – glass	0	0°
	10 20	8° 15°30’		10 20	7° 13°30’		10 20	9°30’ 18°30’
	30	22°30’		30	19°30’		30	27°
	40	29°		40	25°		40	35°
	50	35°		50	30°		50	42°30’
	60	40°30’		60	34°30’		60	49°30’
	70	45°30’		70	38°30’		70	56°
	80	50°		80	42°		80	62°

[19] Smith, *Ptolemy's Theory of Visual Perception*, pp. 43-4; Smith, ‘Ptolemy’s search for a law of refraction’, p. 232.

Mark Smith has shown that Ptolemy started measuring with a specific purpose in mind, conditioned by his awareness that reflection and refraction are systematically related phenomena.[20] As for reflection, he expected to find a constant proportionality between the angles of incidence and the angles of refraction. However, when Ptolemy's measurements contradicted his expectations, he retreated to establishing constant second differences of the angles of refraction. For refraction from air to water, the difference between successive angles of refraction is 8°, 7°30', 7°, 6°30', 6°, 5°30', 5°, 4°30'. The second difference, or the difference of the difference between successive angles of refraction is a constant of 0.5°.[21] Moreover, in this process, Ptolemy clearly adapted his measurements, so they would yield constant second differences. However, unable to express such a law mathematically, the only generalization Ptolemy arrived at, was that, as the angle of incidence increases, so does the difference between the angle of incidence and the angle of refraction. Thus, instead of a constant proportionality between the angle of incidence and the angle of refraction, he found a constant disproportionality between the angle of incidence and the angle of refraction.[22]

In medieval perspectiva, a first quantitative law of refraction was formulated by Grosseteste. Grosseteste was not acquainted with the work of Ptolemy and Alhazen. He claimed that the angle of refraction was half the angle of incidence. Eastwood has shown that Grosseteste derived this law from the metaphysical principles of economy and uniformity.[23] Hero of Alexandria showed that, in reflection, the angle of incidence equals the angle of reflection, because this is the shortest path. This is the principle of economy. From the principle of uniformity, it followed, as already seen in the case of Ptolemy, that reflection and refraction are similar phenomena. Thus, since there is equality between the angles in reflection, there must also be equality in refraction. However, in refraction, there is no equality between the angle of incidence and the angle of refraction. Instead, equality is preserved by bisecting the angle between the extension of the direct ray in the refracting medium and the normal. In this way, Grosseteste arrived at his quantitative law of refraction that the angle of refraction was half the angle of incidence.

Tables of refraction made their first appearance in medieval optics in the work of Witelo. These tables were not established by his own measurements, but clearly taken from the work of Ptolemy, which was known to Witelo.[24] With the puzzling exception of the value of the angle of refraction for air-water, which Witelo changed from 8° to 7°55', mistakenly given as 7°45' in Reisner's edition, all the values are the same as Ptolemy's, although, as shown, these values were adapted by Ptolemy to get constant second differences.

[20] Ibid., pp. 230-2.

[21] Ibid., pp. 233-4. See also Govi, Gilberto. *L' Ottica di Claudio Tolomeo*. Translated by Gilberto Govi. Torino: Stamperia Reale della Ditta G. B. Paravia E C., 1885, pp. xxiv-xxxii.

[22] Ibid., pp. 235-7.

[23] Eastwood, Bruce S. 'Grosseteste's 'Quantitative' Law of Refraction.' *Journal of the History of Ideas* 28 (1967): 403-14; Eastwood, Bruce S. 'Metaphysical Derivations of a Law of Refraction: Damianos and Grosseteste.' *Archive for History of Exact Sciences* 6 (1970): 224-36, pp. 232-6. See also Eastwood, Bruce S. 'Mediaeval Empiricism: The Case of Grosseteste's Optics.' *Speculum* 43 (1968): 306-21. These three articles are reprinted in Eastwood, Bruce S. *Astronomy and Optics from Pliny to Descartes*. London: Variorum Reprints, 1989.

[24] Crombie, A.C. *Robert Grosseteste and the Origins of Experimental Science 1100-1700*. Oxford: Clarendon Press, 1953, pp. 219-25; Lindberg, *Opticae Thesaurus*, pp. xx-xxi.

Tabula quãtitatis angulorũ incidentiæ omnibus sequentibus communis.	Anguli refracti ab aere ad aquam.		Anguli refractionis eiusdem.		Anguli refracti ab aere ad uitrum.		Anguli refractionis eiusdem.		Anguli refracti ab aqua ad uitrum.		Anguli refractionis eiusdem.	
	par.	minut.	par.	minut.	par.	minut.	par.	minut.	par.	minut.	par.	minuta
10	7	45	2	5	7	0	3	0	9	30	0	30
20	15	30	4	30	13	30	6	30	18	30	1	30
30	22	30	7	30	19	30	10	30	27	0	3	0
40	29	0	11	0	25	0	15	0	35	0	5	0
50	35	0	15	0	30	0	20	0	42	30	7	30
60	40	30	19	30	34	30	25	30	49	30	10	30
70	45	30	24	30	38	30	31	30	56	0	14	0
80	50	0	30	0	42	0	38	0	62	0	18	0

Tabula quantitatis angulorũ incidentiæ omnibus sequentibus communis.	Anguli refracti ab aqua ad aerem.		Anguli refractionis eiusdem.		Anguli refracti à uitro ad aerem.		Anguli refractionis eiusdem.		Anguli refracti à uitro ad aquam.		Anguli refractionis eiusdem.	
	par.	minut.	par.	minut.	par.	minut.	par.	minut.	par.	minut.	par.	minut.
10	12	5	2	5	13	0	3	0	10	30	0	30
20	24	30	4	30	26	30	6	30	21	30	1	30
30	37	30	7	30	40	30	10	30	33	0	3	0
40	51	0	11	0	55	0	15	0	45	0	5	0
50	65	0	15	0	70	0	20	0	57	30	7	30
60	79	30	19	30	85	30	25	30	70	30	10	30
70	94	30	24	30	101	30	31	30	84	0	14	0
80	110	0	30	0	118	0	38	0	98	0	18	0

Figure 3.5

Moreover, Witelo choose to include also the reciprocal values for water-air, glass-air and glass-water, not included by Ptolemy, although Ptolemy had formulated the principle of reciprocity.[25] (Figure 3.5) However, he also did not measure the reciprocal values, but derived them, incorrectly, from the original values of Ptolemy. The principle of reciprocity states that the relationship between the angle of incidence and the angle of refraction in refraction through a given interface is reversed, when the direction of passage is reversed, thus, the original angle of incidence becomes the new angle of refraction, and vice-versa. Witelo arrived at wrong results for the reciprocal values, because he had wrongly understood the principle of reciprocity.[26] For example, the principle of reciprocity states that, when the angle of incidence is 20° and the angle

[25] On Ptolemy's principle of reciprocity, see Smith, *Ptolemy's Theory of Visual Perception*, p. 46. Witelo's tables are in book 10, proposition 8, see Lindberg, *Opticae Thesaurus*, pp. 412-3.

[26] Ibid., p. xxi; Crombie, *Grosseteste*, pp. 223-5. As a background to Witelo's non-experimentalism, more in general, see also Unguru, Sabetai. 'Mathematics and Experiment in Witelo's Perspectiva.' In *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett*, edited by Edward Grant and John E. Murdoch, 269-97. Cambridge: Cambridge University Press, 1987; Unguru, Sabetai. 'Experiment in Medieval Optics.' In *Physics, Cosmology and Astronomy 1300-1700: Tension and Accomodation*, edited by Sabetai Unguru, 163-84. Dordrecht Boston London: Kluwer Academic Publishers, 1991.

of refraction is 15°30’ for refraction from air to water, than the angle of incidence is 15°30’ and the angle of refraction is 20° for refraction from water to air. Witelo, however, noting that the angle of deviation, that is the ‘angulus refractionis’ in medieval terminology, is 4°30’, stated for refraction in the reverse direction, from water to air, that, if the angle of incidence is 20°, than the angle of refraction, that is the ‘angulus refractus’, is 24°30’. This has the absurd consequence of angles of refraction greater than 90° and no recognition of total internal reflection (for angles of incidence greater than 50° in refraction from water to air). Again, such results cannot be the consequence of any actual measurements.

Thus, in medieval optics there does not seem to have been any attempt at measurement of the angles of incidence and the angles of refraction. Moreover, Witelo did not seem to have realized that Ptolemy’s tabulations were adapted to obtain constant second differences. When making a refractive scaphe dial, the experimental set-up of Ptolemy is present, as such, tied to the instrument itself, not as a deliberate attempt to experiment. As late as 1638, Muzio Oddi, who, as will become evident, had seen refractive scaphe dials, claimed that ‘the making of these sundials, until now, is limited to a mere practice’.[27] Oddi described this practical procedure, which consisted of a scaphe dial, with an erasable non-refracted pattern of hour lines, and a light source, which could be steadily positioned as to control the different positions of the sun.[28] The procedure to trace the refracted pattern of hour lines was based on the systematic comparison of the position of the shadow of the gnomon, manipulated by moving the light source, on any point of interest of the non-refracted pattern of hour lines and, next, observing and indicating its corresponding position when the bowl was successively filled with water. Repeating this procedure for as many positions of the sun as necessary resulted in a refracted pattern of hour lines. (Figure 3.6)

> By means of this light, the shadow of the tip of the gnomon CE, is made to touch the end of whatever hour one wishes P, of the twenty-thirth of Cancer, and having fixed the lantern, and filled the whole bowl with water, so that it covers the complete gnomon, one observes with great care the point of the shadow, which by the breaking of the rays in the water has changed place, and one can note down [this point] with the point of a metal compass, so that it is impressed in the wood [of which the bowl is made]. Then, having thrown out the water, not always all [the water], but as much as reveals the other extremity V, and having made it with the light touched by the shadow of E, and having the bowl again filled with water as first, one indicates in the same way the point S of refraction; and having done the same with T of the Equinoctial in R, then, if they are joined together, SRQ will make the hour line corresponding to the twenty-thirth PTV of the Dial

[27] ‘e però la fabrica di questi Horologi, fino adesso, si riduce ad una mera pratica’. Oddi, Mutio. *De gli Horologi Solari Trattato*. Venetia: Per il Ginammi, 1638, p. 100.

[28] ‘& e, se d’ Ottone, ò d’ altra materia simile si fabricarà con particolare diligenza, vna portione di sfera ABCE, & in essa con alcuni de i modi antecedenti, vn’ Horologio, con linee fatte di materia, che l’ acqua non le dilani; ma però da potersi cancellare, finito che sia quello coi Raggi Rinfranti, mutando tutte sito, dalla Meridiana in poi, che in ambedue è la medesima e si fermarà di poi in modo, che l’ orlo stia perfettamente equidistante all’ Orizonte, il che c’ insegnarà di farlo l’ acqua; e che d’ indi a patto alcuno non possa mouersi, & in luogo lì vicino, si accommodarà vna Lucerna, che si possa alzare, abbassare, e mouersi per molti versi, secondo richiederà il bisogno, & che in qualunque sito farà mestieri lasciarla, quiui se ne resti ferma, col mezzo d’ alcuni bracetti di legno, ò ferro snodati, come se ne vedono per le botteghe de Barbieri’. Ibid., pp. 100-1.

without water; and in the same way, indicating all the others, and the circle of the Horizon, and the Equinoctial, one will have made a Dial, which not without great satisfaction shows the hours under water.[29]

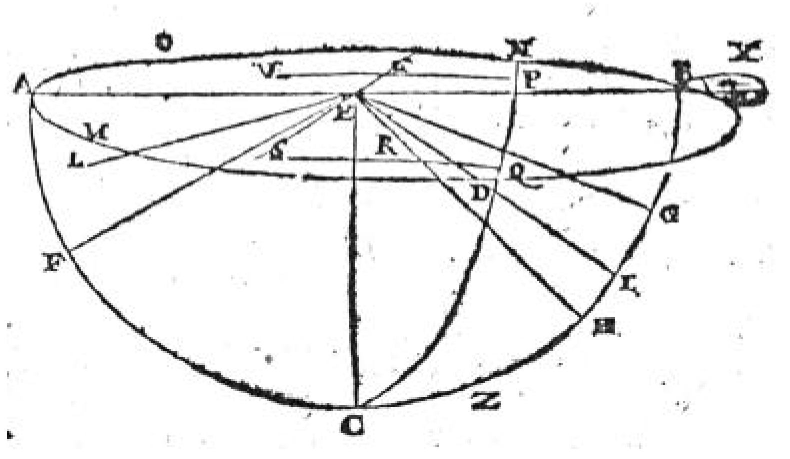

Figure 3.6

If Oddi would have taken the trouble of measuring the angles of incidence and the angles of refraction, or if he would have measured the angle between the point of the shadow of the tip of the gnomon on the non-refracted pattern of hour lines and the corresponding point on the refracted pattern of hour lines, thus, measured the angle of deviation, he would have been able to establish newly tabulated values for the refraction from air to water. Was Oddi's practical procedure followed by sixteenth century designers of refractive dials? Did they measure the angles of refraction? Was there any use for Ptolemy's tables? At first sight, Ptolemy's tables seem to be rather useless, because (1) Ptolemy's values were adapted, and, consequently, if this was realized, they should have been considered as deviating from the real angles of refraction (see table below), and (2) Ptolemy's tables only gave angles of refraction for angles of incidence at 10° intervals, and neither Ptolemy nor medieval optics had established any proportionality between angles of incidence and angles of refraction that would have allowed to calculate the angle of refraction for *any* angle of incidence, as needed in designing a refractive dial.

[29] 'Col mezzo di questo lume dunque si farà che l' ombra del punto E, vertice del Gnomone CE, giunga à toccare il termine d' vna qualche hora verbi gratia P, della ventitressima del Cancro, e fermata la Lucerna, & empito tutto il vaso d' acqua, si che cuopra tutto il Gnomone, si osseruarà con molta diligenza il punto dell' ombra, che per la reprecussione de i Raggi nell' acqua hauerà mutato luogo, e si potrà notare con la punta d' vno Stile d' acciaio, precosso si, che s' imprima nell' ottone. Cauata poi l' acqua, e questa non sempre tutta, ma tanta parte, che si scuopra l' altro estremo V, e col lume fattolo toccare dall' ombra di E, e di nuouo riempito come prima il vaso d' acqua, si notarà con l' istesso modo il punto S della refrattione; e fatto il medesimo con T dell' Equinottiale in R, se si congiungeranno assieme, la SRQ sarà l' horà l' horaria corrispondente a quella della vigesimaterza PTV dell' Horologio senz' acqua, e on l' istesso ordine segnate tutte l' altre, & il cerchio dell' Orizonte, e dell' Equinottiale, si hauerà fatto vn' Horologio, che non senza gran piacere mostra l' hore sott' acqua'. Ibid., pp. 101-2.

Ptolemy's refraction table for air-water	i	R	Calculated angles of refraction with Snell's law and refractive index n = 1.333	r
	0	0°		0°
	10	8°		7°29'
	20	15°30'		14°52'
	30	22°30'		22°01'
	40	29°		28°49'
	50	35°		35°04'
	60	40°30'		40°03'
	70	45°30'		44°49'
	80	50°		47°37'

To answer such questions about the actual procedure involved in making refractive dials, let's take a look at some of the sixteenth century designers of these 'optical' instruments.

3. The refractive sundials of Georg Hartmann

The first who is known, with certainty, to have designed refractive sundials is Georg Hartmann. Hartmann (1489-1564) was born in Eggolsheim in South Germany.[30] In 1506, he began his studies at the Faculty of Arts of the University of Cologne. After travel through Italy sometime between 1510 and 1518, he came to Nuremberg in 1518, where he would stay until the end of his life in 1564. In Nuremberg, he was vicar of the St. Sebald church until 1544. He was also operating a large workshop that produced globes, armillary spheres, astrolabes and sundials.[31] He was part of an extensive social network that included Melanchton, Pirckheimer and the mathematician Johann Schöner.[32] Moreover, he also operated a printing press in his own house.[33]
Hartmann was interested in optics. Notwithstanding he had his own printing press, he published an edition of Pecham's 'Perspectiva communis' with the Nuremberg printer Johannes Petreius. Hartmann's edition was the basis of the subsequent sixteenth century editions of Pecham's 'Perspectiva communis', which were either reprints or translations of Hartmann's.[34] Hartmann's preface, dedicated to the Viennese mathematician and architect, Johann Tscherte, emphasized the utility of optics (and the mathematical arts) for philosophy and painting. As concerns painting, this was presumably due to Hartmann's acquaintance with Dürer, who stressed the importance of the introduction of young painters to perspective, and the Nuremberg context.[35] Also, Hartmann

[30] Klemm, Hans Gunther. *Georg Hartmann aus Eggolsheim (1489-1564): Leben und Werk eines Frankischen Mathematikers und Ingenieurs*. Vol. 8, *Wissenschaftliche und Kunstlerische Beiträge Ehrenbürg-Gymnasium Forchheim*. Forchheim: Gürtler-Druck, 1990, pp. 5-15.

[31] Ibid., pp. 15-29; Lamprey, John P. 'An Examination of Two Groups of Georg Hartmann Sixteenth-Century Astrolabes and the Tables Used in Their Manufacture.' *Annals of Science* 54 (1997): 111-42.

[32] Klemm, *Georg Hartmann aus Eggolsheim*, pp. 29-37, 59-62.

[33] Ibid., pp. 37-41.

[34] Lindberg, *John Pecham and the Science of Optics*, p. 57.

[35] On Dürer and Hartmann, see Klemm, *Georg Hartmann aus Eggolsheim*, pp. 53-4.

announced the publication of a book on shadows, otherwise unknown, for painterly purposes.[36] In several places, Hartmann's edition differed from Pecham's original text. When the changes are substantial, they show Hartmann to have been well acquainted with sixteenth century optics. First, Hartmann was well aware of contemporaneous ocular physiology. For example, Hartmann ascribed functions to the vitreous humour, namely, nourishing the crystalline lens, and the albugineous humour, namely, moistening and protecting the crystalline humour, that have no equivalent in Pecham's original text, as it is known from the manuscript tradition.[37]

Another example of Hartmann's changes of the text is the first proposition of the second book on reflection, which states that 'primary and pure secondary light and unmixed colours rebound from the surfaces of dense bodies', on the cause of reflection.[38] To show the existence of reflected light, Pecham noted that 'because rays are reflected from the surface of the earth, heat is more intense near the earth than in the middle interstice of the air and [is also intense] in valleys to which rays are reflected by the density of the mountains on both sides'.[39] Several times in the text of this proposition Hartmann included an experience that has no counterpart in Pecham's text. Hartmann proposed to visualize this reflected light by setting up mirrors inside a dark room (or camera obscura) and catching the reflected light on a screen.[40] As will become evident in chapter 6, this is a catoptrics-camera obscura set-up that is quite typical for the sixteenth century, although naturally known to medieval optics. Again without any counterpart in Pecham's text, Hartmann also referred to light refracted by a crystal ball and projected unto a screen.[41] Again, this refers to a camera obscura set-up, which stresses the appearance of an image on the screen.

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[36] 'Hic igit' libellus etsi innumeras utilitates habet, sicut vident qui in explica'do Aristotele, & in Physicis versantur, tamen vel methodi causa publicandus erat. Multi qui philosophiae professores videri volunt, à Mathematis tanq' ab artibus ad Philosophia' inutilibus suos dehortantur. Sed hoc faciu't, ut artes iudicio neglexisse videa'tur, vel propter ingenij obtusitate' assequi no' possunt. … Olim Parrhasius & Zeusis pictores inter se ita certabant, ut res non pingi sed fieri videretur. Hoc nostro seculo quidam Itali ingenij bonitate assequuntur. Sed vulgus ita res suas pingit, ut etiam sine diligenti intuitione picta appareant, interim tamen vere perspectivam ad usum conferre volunt videri. Quem defectu' tu nobis precor ut brevi emendes, *Dabo & brevi in lucem Opus nostru' de Vmbris*, quem Herculeum laborem, tua nixus autoritate subij.' Hartmann, Georg, ed. *Perspectiva Communis*. Norimbergae: Apud Iohan. Petreium, 1542, ff. aiij-aiiij, my italics.

[37] 'Hic ut Galenus testatur, Crystallinum fouet & nutrit, & quia est aliquanto subtilior, & vitro liquefacto similis, vitreus humor appellatur. Separantur autem ab inuicem hi duo humores tenui, quadam tunica, ideoq aranea uocata, quae & ambos circundat, & tanquàm in unam sphaeram colligit. Hanc sphaeram ambit alius humor, qui Albugineus dicitur, quem quidam uolunt esse excrementum Crystallini humoris, est ouorum albo similis, est fluidus & aliquanto tenuior. Huius officium est humectare Crystallinum, ne à siccitate telae, eum circundantis corrumpatur, irrigat totu' oculum, defendit & protegit Crystallinum ab accidentibus extrinsecis.' Hartmann, *Perspectiva Communis*, f. diij. For discussion, see Lindberg, *John Pecham and the Science of Optics*, p. 58.

[38] 'Luces primarias & secundarias puras & coloribus immixtas, à de'soru' corporu' superficiebus reverberari'. Hartmann, *Perspectiva Communis*, book II, proposition 1, p. 54. Translation in Ibid., p. 157.

[39] 'Amplius propter reflexionem radiorum a superficie terre est calor intensior prope terram quam in medio aeris interstitio, et in vallibus ad quas utrimque montium densitas radios reflectit.' Ibid., pp. 156-7.

[40] 'Autor perspectivae [Alhazen] hoc in speculis ferreis ostendit, in quibus non est aliqua diaphanitas, incidente enim radio lucis in speculum in aliqua domo, in pariete sensibiliter lux reflexa videbitur … Item si aqua vel alius liquor in domo radijs Solis exponatur, radij sensibiliter videntur in pariete.' Hartmann, *Perspectiva Communis*, p. 54.

[41] 'Occurente vero corpore denso, quia virtus radiantis, & influentia radiosi nondum est terminata, nec per directum transire potest, redit radius per reflexionem in partem unde uenit, *sicut pila cum prijcitur ad parietem*, cum non potest per directu' transire, revertitur more reflexionis, in qua'tum durat virtus impellentis'. Hartmann, *Perspectiva Communis*, p. 54, my italics.

Thus, from Hartmann's editorial changes, it appears that he was well acquainted with optics. Beside editing Pecham's 'Perspectiva communis', Hartmann possessed the manuscript of Ptolemy's 'Optics' that had belonged to Regiomontanus, who had planned to prepare it for publication, but never got around publishing it.[42] Hartmann seems to have had a similar project in mind. However, because of its complexity, about which Hartmann complained, some of which was due to the fragmentary state in which Ptolemy's work was transmitted, the project never materialized beyond a short description of the content in the preface to the edition of Pecham's 'Perspectiva communis'.[43] Recently, the renewed interest in Ptolemy's 'Optics' in the sixteenth century, shown by the relative increase of preserved manuscript copies beyond the extent to be only explainable by the play of chance of transmission, has puzzled historians of optics. There are at least eight known manuscripts of Ptolemy's optics from the sixteenth century, against none of the thirteenth, only three of the early fourteenth, and one of the fifteenth century.[44]

In particular, Smith has attributed the interest in Ptolemy's 'Optics', seemingly incomprehensible, because Ptolemy's extramissionist account of vision was clearly rendered obsolete by medieval optics, to the 'almost slavish classicism of Renaissance scholars inspired by the Golden-Age view of classical Antiquity'.[45] Moreover, he has argued that the increase of manuscript copies of Ptolemy's 'Optics' in the sixteenth century 'indicates not an awaking of scientific interest in the treatise but the enthousiasm of Renaissance scholars eager to revive classical Antiquity and all its works'. Finally, he concluded that, given that Ptolemy's 'Optics' must have been considered surpassed by medieval optics, 'obviously, then, given both the superiority and ready availability of these Perspectivistic sources, no serious scholar of the later Middle Ages or Renaissance was going to waste much, f any time on the Optics'.[46] Then, was Regiomontanus' and Hartmann's interest in Ptolemy's 'Optics' only grounded in humanistic concerns? This explanation makes the mistake of assuming that Regiomontanus and Hartmann would have taken Ptolemy's 'Optics' at face value, confusing its relevance in its own time, and, maybe, to medieval optics with its relevance for fifteenth and sixteenth century editors.

Regiomontanus and Hartmann's involvement with the design of refractive sundials might have provided a much stronger impetus to their interest in Ptolemy's optics than any humanistic concerns. Then, they were less interested in Ptolemy's extramissionist account of vision than in Ptolemy's tabulations of refraction. Of course, the tabulations were also available in Witelo's 'Perspectiva', however, as has been noted, with slight variations. Obviously, the refraction tabulations gained a new importance if they were considered as a manual to understanding the working of a refractive sundial. Regiomontanus was said to have made refractive sundials by Muzio Oddi, a student of Guidobaldo del Monte, who recorded that 'one day, when I was talking

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[42] Hartmann, , *Perspectiva Communis*, f. aij. See also Rose, *The Italian Renaissance of Mathematics*, p. 105.

[43] 'Totam hanc doctrinam Ptolemaeus quinq libris co'plexus est, In primo libro prosecutus est, propietatates lucis & visus: ostendit quomodo & in virtutibus & in motibus co'veniant & discrepe't: assignavitq cuiq suas species cum eoru' differentijs & accidentibus. In secundo docet, quae sint res visibiles, qualis cuiusq sit habitudo, qualiter unaquaeq res visibilis videatur, & quot modis res visibiles vere visu apprehe'di possint. Tertius liber, est de his quae per reflexionem in speculis planis & convexis videntur. Quartus est de his, quae in speculis concavis, co'positis, aut per duo, aut plura specula videntur. Quintus est de his, quae videntur per refractionem.' Hartmann, *Perspectiva Communis*, f. aij.

[44] Lindberg, *A Catalogue of Mediaeval and Renaissance Optical Manuscripts*, pp. 17-9.

[45] Smith, *Ptolemy's Theory of Visual Perception*, p. 9.

[46] Ibid., pp. 60-1.

to Father Christoph Clavius in Rome, he told me that Giovanni da Monteregio [Regiomontanus] also had made one for a ruler in Germany'.[47] However, no preserved example of a refractive sundial of Regiomontanus has come to light so far. If Clavius' testimony is reliable, this is the earliest example of a refractive sundial.

Three refractive scaphe dials designed by Hartmann are still preserved. The three instruments are very similar. A first refractive scaphe dial is in the Museo de Santa Cruz de Toledo.[48] (Figure 3.7)

Figure 3.7

An engraved inscription identifies it as being specifically made as a solution to the miracle of the Dial of Ahaz, because it reads 'a water-operated instrument that wonderfully imitates the Dial of Ahaz on which Isaiah brought back the shadow of the sun by ten degrees in the fourth book of Kings, chapter 20; Isaiah, chapter 38; and Chronicles II chapter 32'.[49] It is an equal hour sundial, set for a latitude of 41°41', and dated 1547.

[47] '& un giorno parlandone io col Padre Christoforo Clauio in Roma, mi disse, che Giouanni da Monteregio n' havea fatto uno ancor lui, per vn Prencipe d' Alemagna'. Oddi, *De gli horologi solari*, p. 100.

[48] Mills, 'The 'Dial of Ahaz': Part I', pp. 22-3.

[49] 'Hydravlicum qvod mirabili artificio horologium Achas in quo Esaias vmbram solis retrorsvm dvxit decem gradibvs qvarto regvm 20. Ca: Esaia 38. Ca: Parali 2. Ca 32/ imitatur.' Translation in Ibid., p. 23.

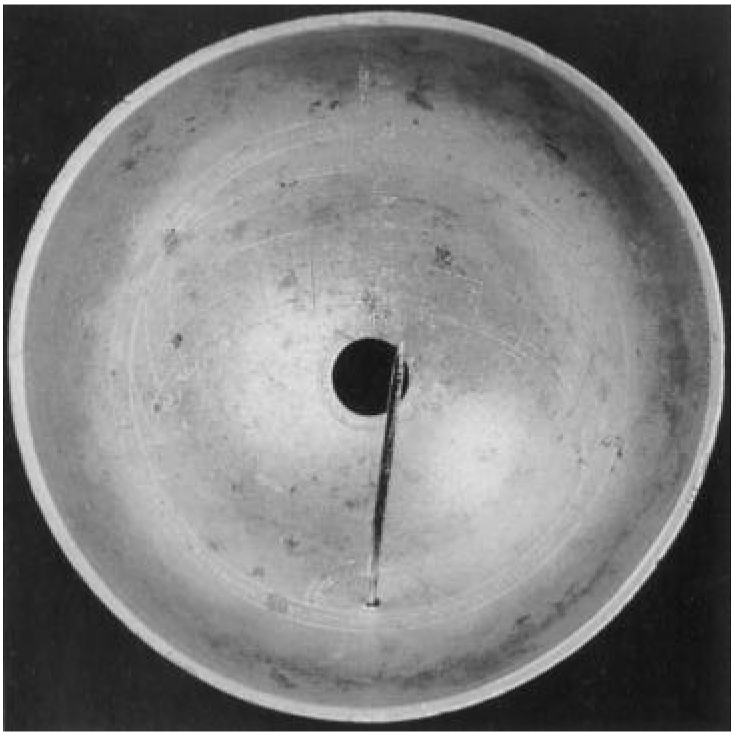

Figure 3.8

Figure 3.9

A second refractive scaphe dial of Hartmann is of the same date, for the same latitude, and bears similar engraved inscriptions.[50] (Figure 3.8) The third instrument is dated 1548 and is in the Collection of Historical Scientific Instruments of Harvard University.[51] (Figure 3.9) The three dials clearly show a contracted pattern of hour lines, typical for a refractive dial. How were these refracted patterns of hour lines traced? Unfortunately, there does not seem to be any hint of Hartmann's procedure in his manuscripts.[52] However, it can be established how another instrument designer, who might have seen the refractive scaphe dials of Hartmann, proceeded. This instrument designer was Ettore Ausonio, whose procedures in making refractive dials have been preserved among his manuscripts. As will be shown, there is a connection between the tabulations of refraction and Ausonio's procedures in making refractive scaphe dials.

4. Ettore Ausonio, a Sixteenth Century Mathematical Practitioner of Venice

Who was Ettore Ausonio? Since little has come to light so far about the man, his intellectual biography and his social network deserve some attention.[53] On several occasions, he referred to himself as Ettore Ausonio Barocio (also: Hettore Ausonio, Hectore Ausonius). He studied medicine at the University of Padua in the early 1540s.[54] Consequently, he was presumably born around 1520. There does not seem to be any evidence that Ausonio was originally from Milan, as Pereira and Eamon claim, beyond the present location of most of his manuscripts.[55] The famous Baroccio family of instrument makers and painters originally came from Milan to Urbino, but, so far, there has not been found any evidence which shows Ausonio to have been a member of this family.[56] In Padua, he lived in the Borgo Santa Croce.[57] He presumably died around 1570, because he disappears from the records after 1569. The latest date in any of his manuscripts is 14 May 1569 on a letter sent from the Urbino mathematician Commandino to Ausonio.[58]

[50] Ibid., p. 23; Turner, 'A biblical miracle', p. 12.

[51] Ibid., pp. 12-3.

[52] I have consulted Hartmann, Georg. *Collectanae Mathematica*. Österreichische Nationalbibliothek (Vienna), Cod. Vindob. 12768; Herzogin Anna Amalia Bibliothek (Weimar), Cod. fol. max. 29.

[53] There is no article on Ettore Ausonio in the *Dizionario biografico degli italiani*. To the best of my knowledge, the only article that has Ausonio as its main focus is Ventrice, Pasquale. 'Ettore Ausonio Matematico dell' Accademia Veneziana della Fama.' In *Ethos e Cultura: Studi in Onore di Ezio Riondato*, 1133-54. Padova: Antenore, 1991.

[54] Ventrice, Pasquale. *La Discussione sulle Maree tra Astronomia, Meccanica e Filosofia nella Cultura Veneto-Padovana del Cinquecento.* Vol. 34, *Memorie Classe di Scienze Fisiche, Matematiche e Naturali.* Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1989, p. 37.

[55] Pereira, *The Alchemical Corpus Attributed to Raymond Lull*, p. 48; Eamon, William. 'Alchemy in Popular Culture: Fioravanti and the Search for the Philosopher's Stone.' *Early Science and Medicine* 5 (2000): 196-213, p. 207.

[56] On the Baroccio family and their workshop in Urbino, see Panicali, Roberto. *Orologi e Orologiai del Rinascimento Italiano: La Scuola Urbinate*. Urbino: QuattroVenti, 1988, pp. 29-64; Gamba, Enrico, and Vico Montebelli. *Galileo Galilei e gli Scienziati del Ducato di Urbino*. Urbino: QuattroVenti, 1989, pp. 29-34.

[57] See, for example, the letter covers in Biblioteca Ambrosiana (Milan), MS D117 Inf., ff. 153v-154v, 203v.

[58] B. A. M., MS G121 Inf., ff. 135r-135v.

Ausonio graduated from the University of Padua in 1543. Four years later, after the death of his teacher, Federico Delfino, he was scheduled to lecture on mathematics and astrology at this same university.[59] However, for unknown reasons, Pietro Catena, and not Ausonio, eventually filled in that position. Instead, Ausonio established himself as a practicing physician in Venice, where he lived in the Canareggio quarter at San Giobbe alle Chiovere.[60] His medical practice received the highest esteem of the Venetian poligrafo Leonardo Fioravanti.[61] Fioravanti dedicated his 'La Cirugia', first published in 1570, to Ausonio, whom he praised for 'his theory and practice'.

> Your Excellency, who through long studies and continuous reading, has become so learned in theory and practical experience, that besides the teaching of many [students], you are so expert in medicating, that you not only heal the sick of their illnesses, but you almost raise the dead from t heir sepulchers, with your divine and precious liquors, which by your genius are invented, and with them, you do so many miracles in the world, very much to the glory of Venice.[62]

Fioravanti had moved to Venice in 1558, but the two men only met between 1564 and 1567. The 1564 edition of Fioravanti's 'Delle specchio universale' did not mention Ausonio, while the 1567 edition of the same work did.[63] The 'theory' mentioned by Fioravanti was Ausonio's pseudo-Lullian alchemy. As seen in chapter 2, Ausonio was one of the important pseudo-Lullian alchemists in Venice, who was, in particular, responsible for introducing Hebraic and cabbalistic elements into the legend of pseudo-Lull the alchemist.[64] It was Ausonio who was responsible for the 'theoretical turn' of Fiorovanti, who, until he met Ausonio, had only had practical interests.[65]

Beside a physician and an alchemist, Ausonio was a mathematical practitioner. He dealt with numerous practical mathematical arts, such as, mechanics, dialling and mathematical instrument design, measuring by sight, geography and cartography, and optics. Notes in more than 30 manuscripts are attributed to Ausonio.[66] However, with the exception of a posthumous

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[59] Ventrice, *La Discussione sulle Maree*, p. 37. On the Paduan professors of mathematics, Delfino and Catena, see Rose, Paul Lawrence. 'Professors of Mathematics at Padua University 1521-1588.' *Physis* 17 (1975): 300-4.

[60] See, for example, the letter cover in B. A. M., MS D178 Inf., ff. 17r-17v. see also Ventrice, *La Discussione sulle Maree*, p. 36. On Ausonio's medical practice, see, for example, B. A. M., MS Suss. A299.

[61] On Fioravanti, see Eamon, *Science and the Secrets of Nature*, pp. 168-93.

[62] 'Vostra Eccelentia, il quale mediante il longo studio & la continua lettura, sete divenuto nella theorica cosi dotto, & nella esperienza cosi pratico, che oltra l' insegnare à molti, sete cosi esperto nel medicare, che non solamente sanate gl' infermi dalle loro infirmità, ma quasi suscitate li morti dalle sepolture, co i vostri divini & preciosi licori, che dal vostro ingegno sono stati inventati, & con essi fate tanti miracoli al mondo, & massime nella inclità Venetia'. Fioravanti, Leonardo. *La Cirugia dell' Eccellente Dottore E. Cavalier M. Leonardo Fioravanti*. Venetia: Apresso gli Heredi di Melchior Seffa, 1582, f. a4.

[63] Fioravanti, Leonardo. *Dello specchio universale … libri tre … Nuovamente ristampato con molto cose agionte*. Venetia, 1567, f. 55v. Compare Fioravanti, Leonardo. *Dello specchio di scientia universale, dell' eccelente medico, & cirurgico M. Leonardo Fioravanti Bolognese*. In Venetia: Appresso Vincenzo Valgrisi, 1564.

[64] Pereira, *The Alchemical Corpus Attributed to Raymond Lull*, pp. 48-9, who also lists Ausonio's manuscripts on pseudo-Lullian alchemy; his 'Trattato sopra l' arte dell' alchimia', B. A. M., MS Q118 Sup., ff. 4r-31r, dated 1551.

[65] Eamon, 'Alchemy in Popular Culture', pp. 208-9.

[66] Kristeller, Paul Oskar. *Iter Italicum: A Finding List of Uncatalogued or Incompletely Catalogued Humanistic Manuscripts of the Renaissance in Italian and Other Libraries*. London: Warburg Institute, 1963; Rivolta, Adolfo. *Catalogo dei Codici Pinelliani dell' Ambrosiana*. Milano: Tipografia Pontificia Arcivescovile S. Giuseppe, 1933; Revelli, Paolo. *I Codici Ambrosiani di Contenuto Geografico*, Milano, Luigi Alfieri Editore, 1929.

publication of his 'Theorica speculi concavi sphaerici' by Magini in 1602, which will be discussed in the next chapter, nothing of his hand seems to have made it to publication. The state of Ausonio's manuscript notes is chaotic. They are, at best, mostly beginnings of books and treatises that were never finished. Among his earliest interests were mathematical instrument design and geography. In 1545, shortly after his graduation, Ausonio wrote four letters to Annibale Bichi in Siena on mathematical instruments. A first letter of 29 September 1545 discussed an elliptical compass, which Ausonio attributed to Michelangelo. (Figure 3.10)

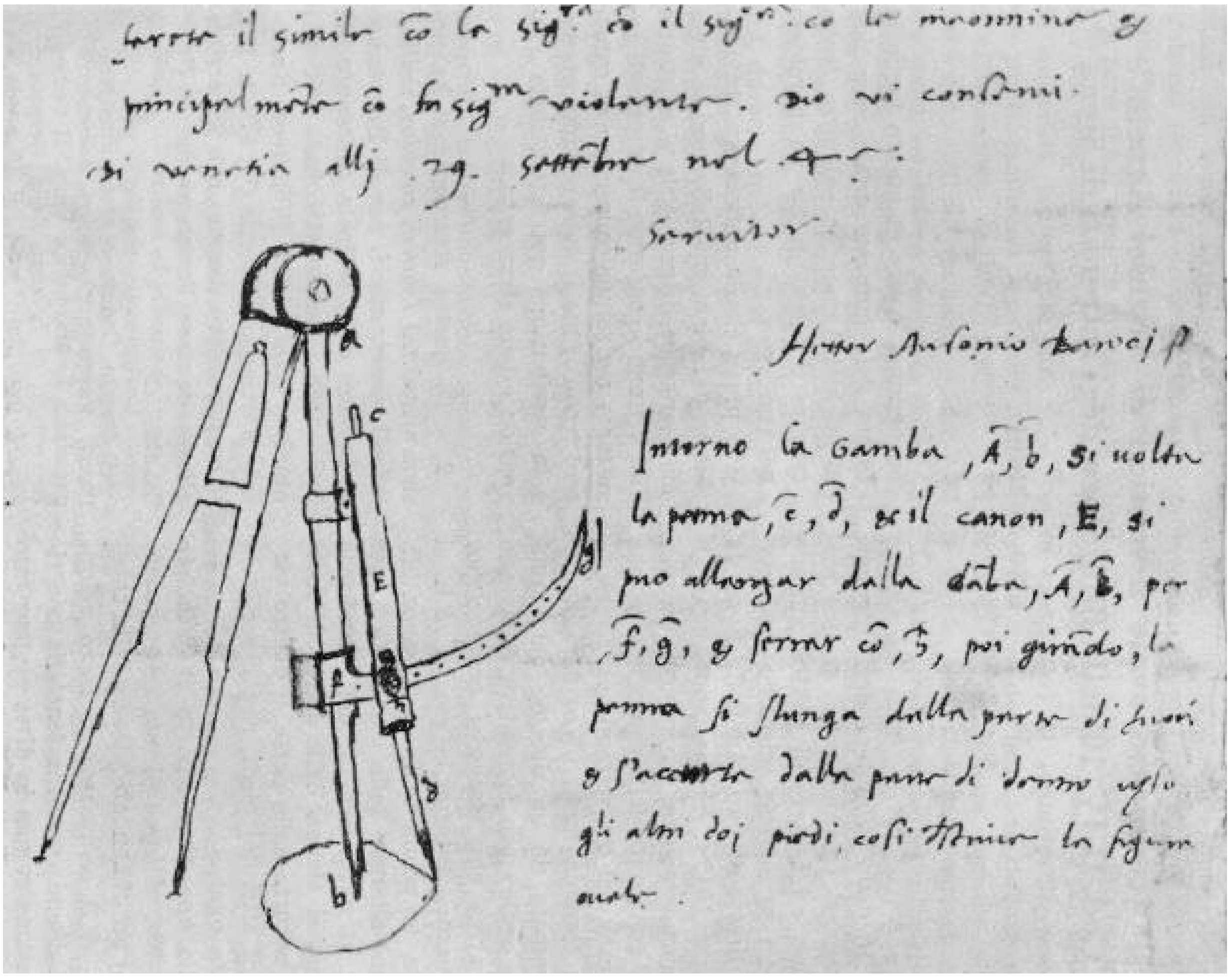


Figure 3.10

The compass is similar to those made in the workshop of Benvenuto Della Volpaia.[67] Two other letters dealt with a nocturnal dial, invented by Alessandro Piccolimini.[68] This dial would return as a focus of Ausonio's interests, later in his life, as would the construction of dials in general.[69] Ausonio also lectured on the astrolabe and discussed instruments for measuring by sight.[70]
Already on 15 December 1546, Ausonio made annotations to Ptolemy's 'Geography'.[71] Cartography and geography was an interest that continued throughout Ausonio's life.[72] As many Venetian poligrafi, prolific writers of popular books in the vernacular, who could live and write independently, away from the courts and universities, Ausonio was connected to the Venetian publishing houses.[73] An undated letter cover links Ausonio to the bottega of the Venetian printer and publisher, Michele Tramezzino, who operated his shop between 1536 and 1574.[74] (Figure 3.11)

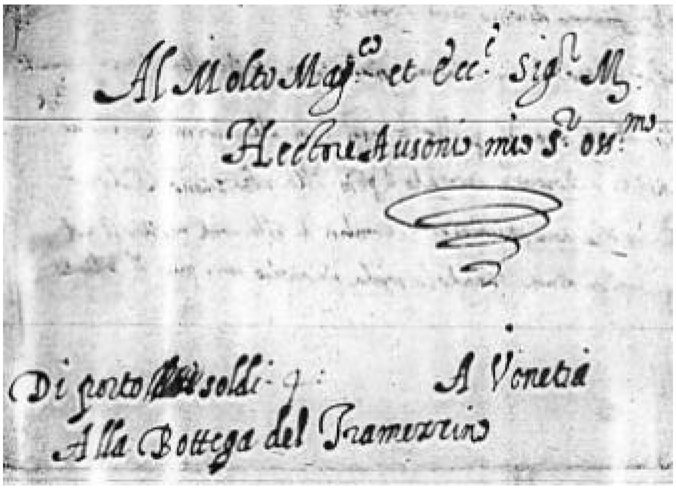


Figure 3.11

[67] Biblioteca Comunale degli Intronati (Siena), L. VI. 10, ff. 92r-93r. See Arrighi, Gino. 'Il "Compasso Ovale Invention Di Michiel Agnelo" del Cod. L. IV. 10 della Biblioteca degl' Intronati di Siena.' *Le Machine* 1 (1968): 103-6; Rose, Paul Lawrence. 'Renaissance Italian Methods of Drawing the Ellipse and Related Curves.' *Physis* 12 (1970): 371-404, p. 384; Frommel, Christoph L., and Nicholas Adams, eds. *The Architectural Drawings of Antonio Da Sangallo the Younger and His Circle*. Vol. 1. Cambridge London: The MIT Press, 1994, p. 196.

[68] Biblioteca Comunale degli Intronati (Siena), L VI 10, ff. 94-98 (27 July 1545; 24 April 1545; undated).

[69] B. A. M., MS D178 Inf., ff. 26r-100v; in particular, on Piccolomini's dial, ff. 54r-55v.

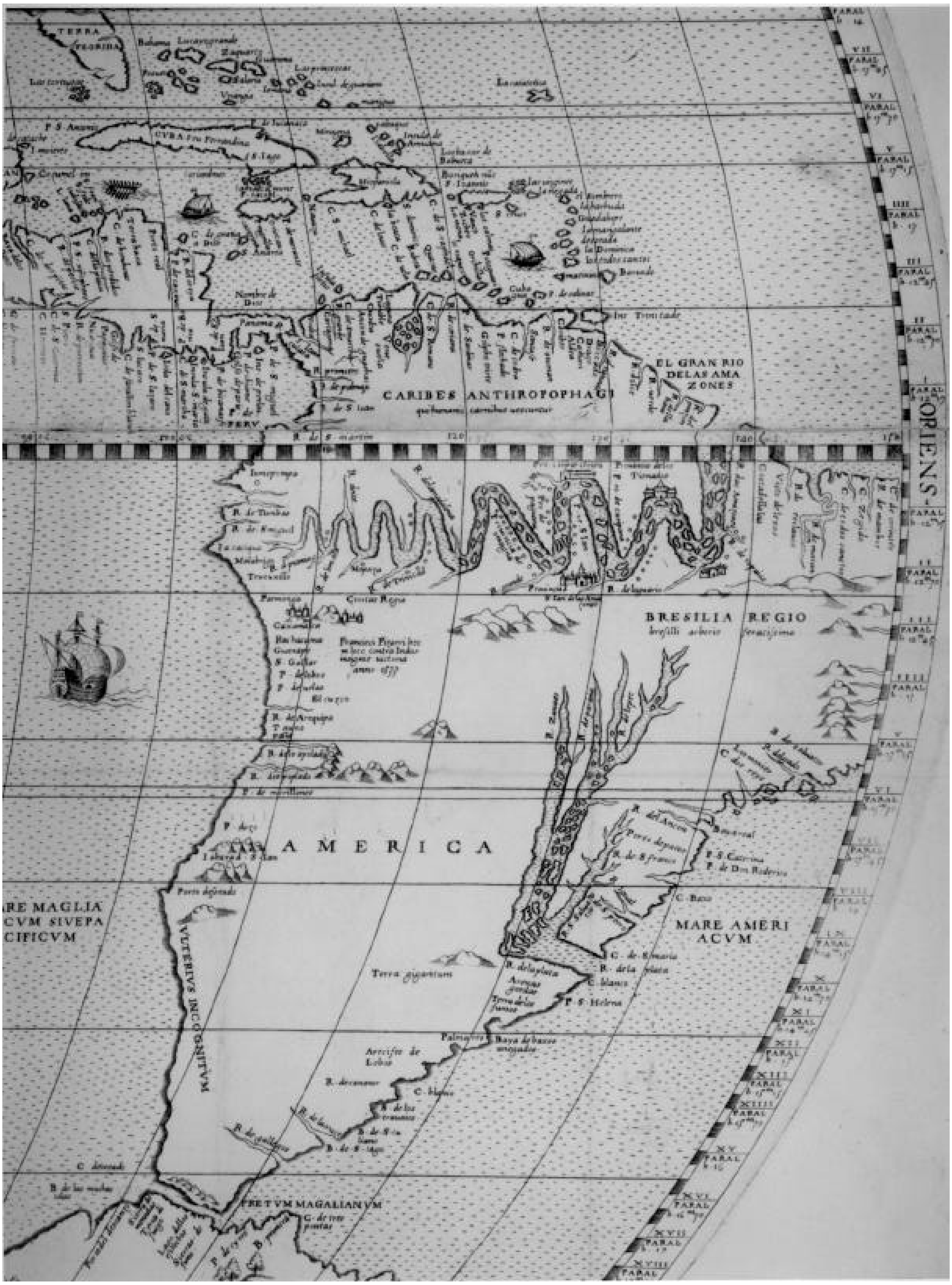
[70] B. A. M., MS D178 Inf., ff. 1r-12v, 103r-105v, contains a summary of Ausonio's course on the astrolabe. Folio's 11r-12v give dates of 20 december 1559, 23 December [1559], 12 January [1560]. There are also course notes on the astrolabe in B. A. M., MS J166 Inf., ff. 37r-56v; B. A. M., MS S97 Sup., ff. 94r-95v. There are notes on measuring by sight in B. A. M., MS D178 Inf., ff. 13r-13v; B. A. M., MS D117 Inf., ff. 139r-142r.

[71] B. A. M., MS D170 Inf., ff. 9r-12v.

[72] B. A. M., MS D170 Inf., contains most of Ausonio's notes on geography, some of them as late as 1568. For Ausonio's cartographical and geographical work in the Ambrosiana manuscripts, see, diffusively, Revelli, Paolo. *I Codici Ambrosiani di Contenuto Geografico*. Milano: Editore Luigi Alfieri, 1929.

[73] On these poligrafi, see Eamon, *Science and the Secrets of Nature*, pp.134-67.

[74] B. A. M., MS G120 Inf., f. 74v. Undated, but, as most of the notes in this manuscript, most likely from the 1550s.


TERRA FLORIDA
CVBA
EL GRAN RIO DELAS AMA ZONES
CARIBES ANTHROPOPHAGI
ORIENS
BRESILIA REGIO
A M E R I C A
MARE AMERI ACVM
VLTERIVS INCOGNITVM
Terra gigantum
FRETVM MAGALIANVM

Figuur 3.12

Tramezzino's shop was responsible for a considerable cartographical production, sometimes reprints of maps, mainly, orginally produced in the Low Countries, sometimes more original cartographical work.[75] To the best of my knowledge, Ausonio is not mentioned in any of the publications coming out of Tramezzino's publishing house. However, it is most likely that Ausonio had a hand in some of Tramezzino's publications, in particular, in his world-map.

Tramezzino's world-map was published in 1554.[76] (Figure 3.12) The engraver is identified on the map as de Musis, but who the cartographer was, has, so far, eluded historians of cartography. Tramezzino's map shows remarkable similarities with the world-map of 1544 by Sebastian Cabot (1476-1557), born in Venice, but most of his life in the service of England and Spain.[77] (Figure 3.13)

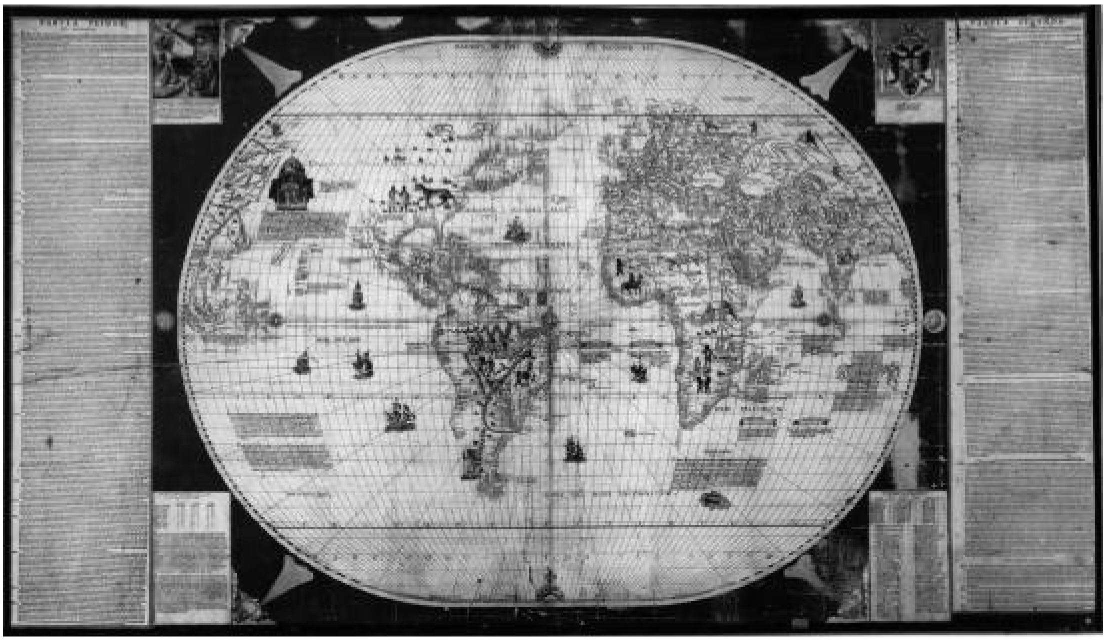


Figure 3.13

[75] On Michele Tramezzino, see Tinto, Alberto. *Annali Tipografici dei Tramezzino*. Venezia Roma: Istituto per la Collaborazione Culturale, 1968; on Tramezzino's cartographic production, of which reprints of maps of the Low Countries were a substantial part, see Schilder, Günther. 'The Cartographical Relationships between Italy and the Low Countries in the Sixteenth Century', in *Imago et Mensura Mundi: Atti del IX Congresso Internazionale di Storia della Cartografia*, edited by Carlo Clivio Marzoli, 265-78. Roma: Istituto della Enciclopedia Italiana, 1985.

[76] Tramezzino's map is discussed in Shirley, Rodney W. *The Mapping of the World: Early Printed World Maps, 1472-1700*. Vol. 9, *Holland Press Cartographica*. London: The Holland Press, 1984, pp. 109-11.

[77] The only extant copy of this map is at the Bibliothèque Nationale de France (Paris), Département des Cartes et Plans, Rés Ge AA. 582. See Kelsey, Harry. 'The Planispheres of Sebastian Cabot and Sancho Gutiérrez.' *Terrae Incognitae* 19 (1987): 41-61; French, Josephine. *Tooley's Dictionary of Mapmakers Revised Edition*. Tring: Map Collector Publications, 1999, pp. 103-12; Shirley, *The mapping of the world*, pp. 92-3; on Cabot, in general, see Tucci, U. 'Caboto', in *Dizionario biografico degli italiani*, Vol. 15, pp. 703-23.

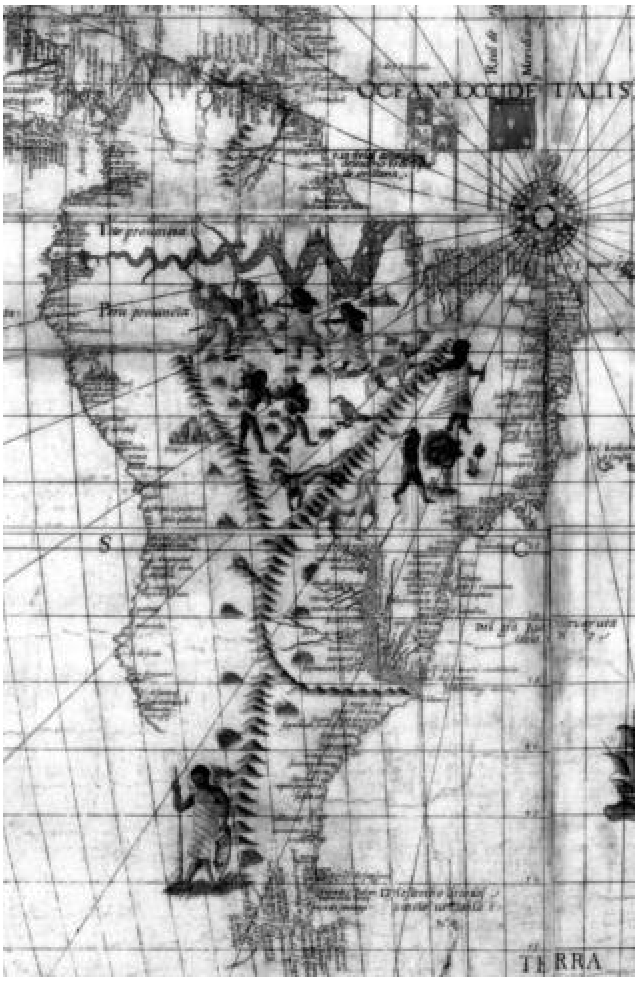


Figure 3.14

Prominent features on the South-American continent, for example, the serpentine east-west course of the Amazon and the shape of the estuary of the Rio de la Plata, the destination of Cabot's voyage in 1526, on Tramezzino's world-map seem to have been taken from Cabot's map. Ausonio was well acquainted with Cabot's world-map. (Figure 3.14) Among his notes, there is a folio with states that it is 'extractum ex charta Sebastiani Caboti'.[78] This is a copy of one of the notes in Latin that Cabot had added to his map in both Latin and Spanish. This note dealt with a method for finding longitude from magnetic variation that Cabot claimed to have invented.

According to Cabot, the differences between north as indicated by the magnetic needle and true north was not the same in all places and observing its variation, with a 'variation compass' of his own invention, would allow determining longitude. The Venetian cartographer Livio Sanuto (ca. 1520 – ca. 1576) described Cabot's method in the first book of his 'Geografia dell' Africa',

[78] B. A. M., MS R105 Sup., ff. 226r-226v: 'Extractum ex charta Sebastiani Caboti'. This is a copy of note 17 attached to Cabot's map, which begins as 'Sebastianus Cabotus Dux & archigubernius S. C. C. M. domini Caroli Imperatoris, huius nominis quinti, & Regis Hispaniae domini nostri Iesu Christi 1544.'

posthumously published in 1588.[79] It is presumably no coincidence that Sanuto and Ausonio were well acquainted. A remarkable collection of 'angelic visions', which appears not to have been atypical within the pseudo-Lullian alchemical tradition, which bears the title 'Le erudite narrationi et i saggi discorsi dello eccellentissimo Ettor Ausonio sopra alcune visioni apparse in diversi tempi ad uno amico suo', was written down and organized by Livio Sanuto.[80] Given the attention of Ausonio and his circle of cartographers to Cabot's method of finding longitude and Ausonio's acquaintance with Cabot's world-map, it is not unlikely that Ausonio was, to some extent, involved with the making of the map of Tramezzino and its Cabot-like features.

Sanuto was one of the cartographers who were elected a member of the Venetian Accademia della Fama. This academy was founded by the Venetian patrician Federigo Badoer in 1557.[81] Already in 1561, the Academy stopped to exist, as a consequence of its disastrous financial policy. The Accademia della Fama was characterized by two main trends, as concerns the sciences, (1) the desire for establishing the unity of knowledge or the encyclopedia of the sciences, strongly inspired by the Ramist reform, and (2) an interest in instruments and technology, much inspired by the Vitruvian commentary tradition in Venice.[82] This was the background to the ambitious publication programme of the Academy, with, eventually, only limited realization, which, as concerns the sciences, included the publication of classical as well as 'modern' works. For example, as concerns optics, the Academy intended to publish Euclid and Ptolemy as well as Alhazen and Roger Bacon.[83] One of the departments of the Accademia della Fama was the Consiglie delle Scientie, which reflected the encyclopedia of knowledge. It was divided into four stanze, theology, philosophy, mathematics and umanità. Ausonio was appointed Regent of the 'Stanza delle Matematiche', and, as such, supervised the publications on mathematics (geometry, arithmetic, astrology, music and cosmography) that were part of the publication programme of the Academy.[84] For example, as Ventrice has shown, Ausonio was responsible for seeing the 'De fluxu maris' of his teacher Delfino through publication.[85]

There were strong patronage and commercial connections between the Accademia della Fama and the Duke of Savoy, Emanuele Filiberto. Filiberto was one of the dukes sought by Badoer to support his Academy, and the Accademia della Fama was also granted permission to trade grains

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[79] On Sanuto's account of Cabot's mehod to find longitude, see the first book of Sanuto, Livio. *Geografia Dell' Africa*. Edited by R. A. Skelton. Amsterdam: Theatrvm Orbis Terrarum Ltd., 1965, also, pp. vii-viii.

[80] Bodleian Library (Oxford), MS Bywater 37, ff. 119-175. On 'visions' and pseudo-Lullian alchemy, see Thorndike, *A History of Magic and Experimental Science*, Vol. 4, pp. 63-4. It is not known when Sanuto and Ausonio met. There is a letter from Sanuto to Ausonio of 19 April 1562, B. A. M., MS R118 Sup., f. 100r.

[81] Rose, Paul Lawrence. 'The Accademia Venetia: Science and Culture in Renaissance Venice.' *Studi Veneziani* 11 (1969): 191-242; Bolzoni, Lina. ''Rendere Visibile il Sapere': L' Accademia Veneziana fra Modernità e Utopia.' In *Italian Academies of the Sixteenth Century*, edited by D.S. Chambers and F. Quiviger, 61-78. London: The Warburg Institute, University of London, 1995; Bolzoni, Lina. 'L' Accademia Veneziana: Splendore e Decadenza di una Utopia Enciclopedica.' In *Università, Accademie e Società Scientifiche in Italia e in Germania dal Cinquecento al Settecento*, edited by Laetitia Boehm and Ezio Raimondi, 117-68. Bologna: Società editrice il Mulino, 1981.

[82] Rose, 'The Accademia Venetia', pp. 195-9.

[83] Ibid., p. 205.

[84] Ventrice, 'Ettore Ausonio Matematico dell' Accademia Veneziana della Fama', pp. 1137.

[85] Ibid., pp. 1140-3; Ventrice, *La Discussione sulle Maree*, in particular, pp. 16-40.

in the Savoy to finance its workings.[86] It was presumably through the Accademia della Fama that Ausonio came into contact with the Duke of Savoy.

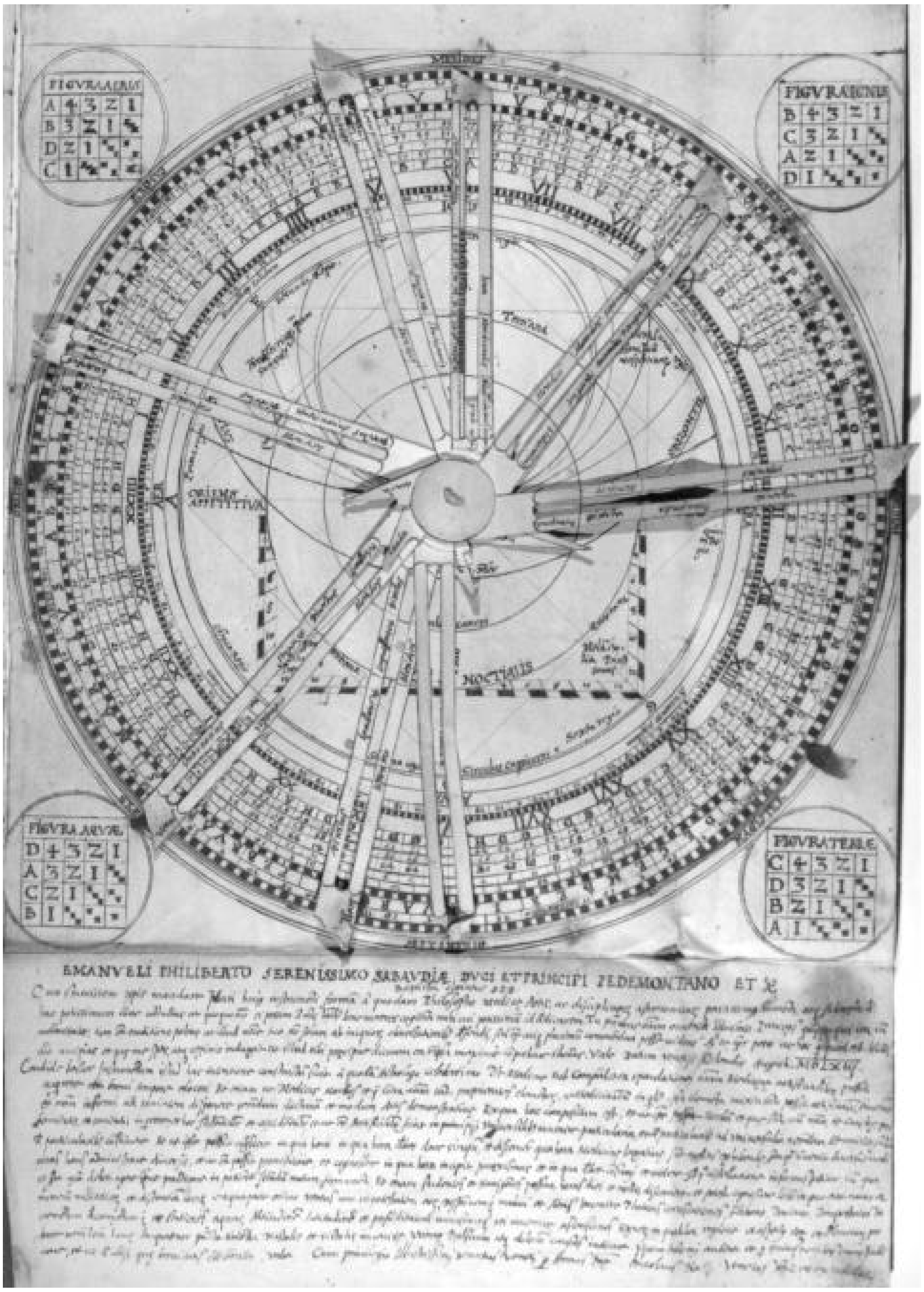

Figure 3.15

[86] Rose, ‘The Academia Venetia’, pp. 207, 214, 234-5. The letter from the Duke of Savoy is dated 7 January 1560. See also Ventrice, ‘Ettore Ausonio Matematico dell' Accademia Veneziana della Fama’, p. 1140.

Around 1560, as appears, for example, from the date of 1563 on a volvelle intended for the duke, Emanuele Filiberto asked Ausonio to provide his library with an instrument collection.[87] (Figure 3.15) An undated list of the instruments that Ausonio intended to make for Filiberto, 'Gli instromenti che si devenano apparecchiar a sua Altezza', is still preserved.[88] It included instruments of perspective, such as, instruments to measure refraction and reflection, a concave spherical mirror, a cylindrical mirror, a parabolic mirror, lenses or 'christalli', one or more burning mirrors; instruments of cosmography and geography, such as, an armillary sphere, a jacob's staff or 'baculo di Levi', a terrestrial and a celestial globe, and a portale sundial; musical instruments; instruments of geometry, in particular measuring by sight, a compass to divide a line and a circle in equal parts, and a compass to draw a parabola; and, finally, some hydraulic instruments.

What is of particular interest here, are the optical instruments that stand out in Ausonio's proposal to the Duke of Savoy. First, they included a wide range of mirrors. Ausonio had a high reputation as a designer of mirrors. Fioravanti not only admired Ausonio for his medical practice and his pseudo-Lullian alchemy, but also as a mirror designer. In his 'Specchio della scientia universale' (1567), Fioravanti applauded him as 'Etor Eusonio from Venice, inventor of the most marvelous mathematical things ever seen or heard of; he made certain concave mirrors of reasonable size that show marvelous things'.[89] The other optical instruments, no doubt, at first sight, less appropriate for a collection of a duke, were instruments to measure reflection and refraction. As will become evident, this referred to the instruments described in Witelo's 'Perspectiva'. These instruments to measure refraction went together with refractive sundials, which Ausonio also wanted to sell to Emanuele Filiberto. This is shown in a letter that Ausonio sent to an unknown correspondent, but presumably the Duke of Savoy, on 6 July 1562. In this letter, Ausonio apologized for the delay of delivery of 'cristallo' glass to make lenses and mirrors for Filiberto, which he blamed on the glassmakers of Murano. He promised however to send him immediately, among other mathematical instruments, some refractive sundials.

> Immediately after I received the letter from Mr. Rocche, I quickly ordered some other instruments, but I still haven't anything finished. When something is finished, I will give notice to Your Highness. I apologize for this delay, but I have to wait until September to make the forms of the cristalli of Murano, because they have already put out the fire in the ovens. This man will take some time in between, so that I doubt that I will do it in time. I do not lack a piece of a concave mirror of cristallo glass, but if I find it difficult to succeed in making it, we will make some pieces of another material and the refractive sundials.[90]

[87] For the volvelle, see B. A. M., MS D178 Inf., f. 25r.

[88] B. A. M., MS R105 Sup., ff. 292r-292v.

[89] 'Etor Eusonio da Venetia inventore delle piu belle materie matematiche che mai si sieno viste ne udite al mondo: percioche ha fatto certi specchi concavi di estimabile grandezza, ne i quali se veggono cose maravigliose'. Fioravanti, *Dello Specchio Universale*, f. 55v.

[90] 'subito che hebbi la lettera di monsignor di Rocche forte diedi ordine per alcuni altri instromenti, ma anchora non ho di finito cosa alcuna, quando sara finita alcuna cosa darò aviso all' Altezza Vostra. … mi rincresse che per queste tardità io habbi da aspettar fino al settembre per fare le figure delli christalli da murano, perche di già sono per cavar fuogo di fornace et questo gentilhuomo metterà qualche tempo di mezzo siche dubito che non farò a tempo. Per un pezzo d' un specchio di christallo concavo non ho mancato, ma si trova fatica nel farlo riuscire faremo alcuni pezzi d' altra materia et li horologij delle refrattioni'. B. A. M., MS D178 Inf., f. 106r.

Ausonio's meddling with concave spherical mirrors will be discussed in the next chapter. First, Ausonio's refractive dials and his study of refraction will be dealt with in this chapter.

5. Ausonio's Optics, the Study of Refraction and Refractive Sundials

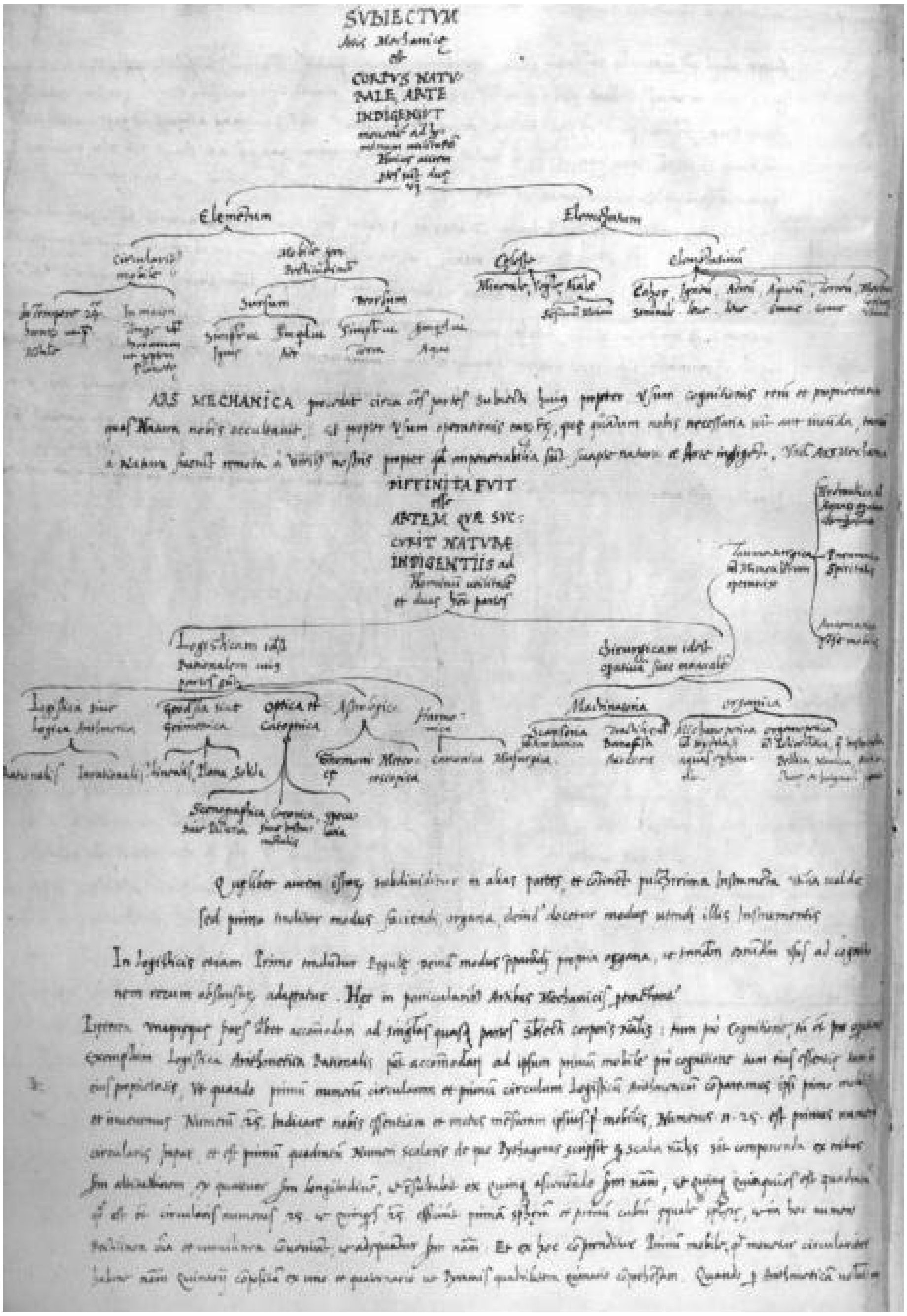

Figure 3.16

Ausonio taught, most likely privately, on several mathematical disciplines, including mechanics, Ptolemy's geography and optics. As concerns optics, in December 1559 and January 1560, Ausonio taught a course on the astrolabe, which, as will become evident, he would have considered optics. In March 1560, he started an introductory course to the 'Perspectiva' of Witelo.[91] As will become evident, in general, Witelo was Ausonio's most important source of medieval optics. Ausonio's lecture notes are particularly revealing of his concept of optics. Among his notes on mechanics, he showed the classification of the sciences in a tree structure. (Figure 3.16) Optics and catoptrics are divided into three parts, (1) scenography or painting *(scenographia sive pictoria)*, (2) regular vision and instruments *(canonica sive instrumentalis)*, and (3) mirrors *(specularia)*.[92] This is not to be considered a standard division of the science of optics. As a mathematical practitioner, for Ausonio, optics and catoptrics was about painting and the design of mathematical and optical instruments, for example, the astrolabe and the mirror.

The division of the science of optics that was presented in his lectures seems to follow more closely the way Witelo's 'Perspectiva' was divided into chapters. In his lecture of 31 March 1560, he divided optics into four parts, (1) the geometry and the arithmetic necessary for the science of optics (book 1 of Witelo), (2) direct vision (books 2-4), (3) reflection and refraction (books 5-10), and (4) the optics of interest to the study of astronomy (book 10). Also, Ausonio emphasized the utility of optics to a whole range of practical mathematical disciplines, including navigation, painting, geography, geodesy, and the arts of *disegno*, such as architecture and fortification.[93] In one of the most structured texts on optics, which seems to have been prepared for publication, of 17 November 1564, Ausonio defined the utility of optics once more. Again, he referred to the utility of optics for painting, architecture, measuring by sight and astronomy.

> Our aim is the utility which this introduction can give to natural contemplation, that is, to Astrology, and the consolation and satisfaction that men might have from this knowledge in judging those paintings of perspective which are admired by men, and in organizing the parts of buildings in Architecture, and in measuring distances, heights and depths with the eye. Because with this science, we can much better judge the differences of the natural things and the heavenly appearences than with any other science.[94]

[91] Ausonio's lecture notes on mechanics are in B. A. M., MS G121 Inf., ff. 1r- 9v, 14v-16r (2 April 1569), 20r-32v, 59r-91v, 105r-106v. For the notes on Ptolemy's 'Geography', see B. A. M., MS D170 Inf., ff. 6r-8v, undated. For Ausonio's course notes on the astrolabe, see the references cited in n. 69. For the lecture notes on Witelo's optics see, B. A. M., MS R105 Sup., ff. 258r-261v; B. A. M., MS G120 Inf., ff. 43ff, dated 31 March 1560, 6 April [1560].

[92] B. A. M., MS G121 Inf., f. 16v.

[93] 'La prima [parte] contenera quelle propositioni et ragioni Geometrice et Arithmetice che sono più necessarie in questa scienza et la scierà fuori quella che si potrano havere appresso altri authori di Geometria. La seconda parte dichiarirà tutte quelle cose lequali sono necessarie nella illuminatione et visione semplice che si chiama retta visione. La terza fara cognoscer quelle cose ch' apartengono alla visione obliqua chiamata reflessa, et refracta visione. Et no dirà le cose che no sono molto necessarie et utili alla cognitione dell' Astrologia, perche no si puo io brevità dicchiarire il tutto, che noi havemo per fine. La brevità della dicchiarazione delle scienza di Perspettiva, che ordinamo questa scienza alla Astrologia alla quale la congiugneremo con la quarte parte di questio compendio dove trattaremo di quelli visioni, le quali no comprendono tutto quello che è necessario nella visione retta et obliqua, et delli modi della visione delle cose del cielo et delle impressioni che appareno nell' Aria.' B. A. M., MS G120 Inf., ff. 43v-44r.

[94] 'La causa di questa nostro fino estata l' utilità che vi puo dar' questa introduttione alla contemplatione naturale, cioè alla Astrologia, e la consolatione et il piacere che l' huomo puo havere da questa cognitione nel giudicar quelle pitture di perspettiva che sono ammirate degli huomini, et nell' ordinare le parti delli edificij nella Architettura, et nel misurar' con l' occhio le distanze, le altezze e le profondità. Perche con questa scienza potremo giudicar' molto

Of all these mathematical disciplines, Ausonio considered the utility of optics to astronomy to be most important. Ausonio's definition of optics as a mathematical discipline with applications in a wide range of practical mathematical arts, intimately connected to the design and use of instruments, was in agreement with the definition of the scope and focus of optics by other sixteenth century mathematical practitioners. As such, it is not different from the utility of optics for practical mathematical arts emphasized by Jean Pena and John Dee.

As it turns out, Ausonio's treatise on optics is nothing else but a paraphrase of Witelo's 'Perspectiva'. Moreover, Ausonio has little original to contribute with respect to Witelo. However, it is interesting to follow the structure of Ausonio's text to see what he took from Witelo and, as such, how he reorganized Witelo's 'Perspectiva'. Consequently, such analysis can show how a sixteenth century mathematical practitioner regarded Witelo and the medieval optical tradition. Ausonio argued that the primary object of study for optics is illumination. Illumination is known from sensible and intellectual principles. Ausonio considered intellectual principles to be, for example, the definitions of the visual cone, the line of incidence and the line of refraction, but what can be defined as an intellectual principle is only discussed at the end of the treatise. First, he discussed the sensible principles that are the instruments and their operations.[95]

What are these instruments and their operations? Ausonio discussed Witelo's instrument, similar to Ptolemy's instrument described above, as discussed in the first proposition of the second book. This instrument is first used by Witelo, and Ausonio, to show that 'all luminous rays, as well as the multiplication of forms, stretch forth in straight lines', thus that light travels in straight lines.[96] Next, with reference to Witelo's third proposition of the second book, Ausonio claimed that the physical rays can be dealt with mathematically, as if they were mere unidimensional.

> Every straight line by which the light of a luminous body comes to an opposite body is understood to be a natural and sensible line, which has a little width and depth, and in this minimal illumination one can understand the mathematical line without width.[97]

On the basis of this claim, it can be argued that a geometrical description is pertinent for optics. Thus, Ausonio claimed that optics is the study of the mathematical angles formed by the rays.

meglio le differenze delle cose naturali, e le apparenze delli cieli che con ciascun altra.' B. A. M., MS R105 Sup., f. 183r.

[95] 'Li principij della cognitione della Illuminazione sono di due sorti. Alcuni sono sensati et altri intellettivj. Li sensati sono gli instromenti, et alcune operationi che si faremo per venire in cognitione del modo della Illuminazione, delli Instromenti adunque si dirà prima et poi delle altre operationi'. B. A. M., R105 Sup., f. 184r.

[96] Witelo, Perspectiva, book II, proposition 1, translation in Unguru, Sabetai. *Witelonis Perspectivae Liber Secundus et Liber Tertius: Books II and III of Witelo's Perspectiva, a Critical Latin Edition and English Translation with Introduction, Notes and Commentaries*. Vol. 28, *Studia Copernicana*. Wroclaw Warszawa Krakow Ossolineum: The Polish Academy of Sciences Press, 1991, p. 40.

[97] 'Ogni linea dritta per la quale viene la luce dal corpo luminoso al corpo opposto si comprende essere linea naturale et sensibile che tiene un poco di larghezza et di profondità e che in questa minima illuminatione la linea mathematica senza larghezza si puo intendere'. B. A. M., MS R105 Sup., f. 187v. The corresponding proposition 3 of the second book of Witelo stated that 'every line along which light from a luminous body reaches to an opposite body is a natural sensible line, having a certain width, within which a mathematical line is to be assumed imaginarily.' Translation in Unguru, *Witelonis Perspectivae Liber Secundus*, p. 46.

> Now that we have found out by means of the sense that illumination in its minimum has in it the three dimensions of length, width and depth, we still have to demonstrate by means of the sense how the luminous ray obtains angles and mathematical figures, and, when talking about the most simple angles, we will speak about those angles which originate without the impediment of penetration, and, next, we will talk about those angles which originate from the impediment of penetration.[98]

As will become evident, both kinds of angles referred to the study of refraction. Ausonio discussed the five propositions, propositions 42 to 46, of the second book of Witelo. It were precisely these propositions which Witelo claimed to establish by means of his instrument, for example, immersed in a recipient of water, to measure air-water refraction, or by positioning a glass hemisphere inside the graduated disc, to measure air-glass refraction. Ausonio's 'angles which originate without the impediment of penetration' referred to Witelo's proposition 42 which stated that 'a perpendicular ray drawn from the center of the luminous body to the surface of the opposite body always penetrates through the midst of the second transparent [medium] without being refracted'.[99] Witelo claimed to show from experience with his instrument that a solar ray entering the water from the rising sun penetrated the denser medium without being refracted. A perpendicular ray in this case actually meant a ray parallel to the water surface, and only in this mathematical limit case, the proposition is valid. As Ausonio also noted, grazing angles to the water surface, as at sunset or sunrise, need to make larger angles of refraction than smaller angles of incidence, as Witelo himself had stated in proposition 5 of book X of the 'Perspectiva'.[100]

Ausonio's fourth principle was taken from proposition 43 of the second book of Witelo. Again, Ausonio claimed to show by means of experience with the instrument that 'from the translucent body, denser than the first [medium] in which the luminous body is, always by an oblique ray, the refraction is toward the part where the line perpendicular to the said surface falls in the point of refraction'.[101] Thus, refraction from a rarer to a denser medium always takes place toward the normal. Proposition 44, equivalent to Ausonio's fifth principle, showed that 'a [luminous] ray [arriving] perpendicularly incident to the surface of the opposite body from the centre of the luminous body penetrates the second transparent medium [which is] rarer than the first without being refracted'.[102] Ausonio's sixth principle corresponded with proposition 45 of the second book of Witelo's 'Perspectiva'. By means of the instrument, it is shown that refraction from a denser to a rarer medium, such as from glass to air or from glass to water, is always away from

[98] 'Hora che havemo ritrovato per via del senso che la illuminazione per minimina ch'e sia hà in se le tre dimensioni di lunghezza di larghezza, et dj profondità, dovemo anchora dimostrare per via del senso come il riaggio luminoso se' habbia con li angoli, et con le figure mathematici, et dicendo parlare delli angoli piu semplici diremo di quelli angoli che nascono senza l' impedimento della penetratione et poi diremo delli angoli che nascono per l' impedimento della penetratione.' B. A. M., MS R105 Sup., f. 188r.

[99] Witelo, Perspectiva, book II, proposition 42, translation in Ibid., p. 80.

[100] B. A. M., MS R105 Sup., f. 190r. Compare Lindberg, *Opticae thesaurus*, pp. 408-10.

[101] 'Dalla superficie del corpo trallucido più denso dal primo nel quale stia il corpo luminoso sempre per raggio obliquo la rifrattione si fa' verso la parte dove cade la linea perpendicolare sopra la detta superficie nel punto della refrattione.' B. A. M., MS R105 Sup., f. 190r. Witelo's proposition 43 of the second book stated that 'the refraction of oblique rays in a second transparent medium which is denser than the first transparant [medium] is performed from the anterior surface of the second transparant [medium] toward the perpendicular (issuing from the point of refraction) to the surface of the second body.' Translation in Unguru, *Witelonis Perspectivae Liber Secundus*, p. 83.

[102] Witelo, Perspectiva, book II, proposition 44, translation in Ibid., p. 85; B. A. M., MS R105 Sup., f. 191v.

the normal.[103] Proposition 46, equivalent to Ausonio's seventh principle, claimed to show by means of the instrument, that 'every incident and refracted ray be situated in the same plane surface'.[104] In his eighth principle, Ausonio referred to propositions five to seven of the tenth book of Witelo that show how, with the instrument, the angles of incidence and the corresponding angles of refraction are measured for the different media, which is claimed to result in the tabulations of refraction of proposition 8 of Witelo's tenth book.[105]

Only after the discussion of refraction, Ausonio dealt with reflection. This part is much less elaborated as it only refers to the corresponding propositions in Witelo's 'Perspectiva'. First, Ausonio referred to Witelo's instrument 'with which all the types of reflections from the various regular mirrors can be empirically demonstrated' in proposition 9 of the fifth book of Witelo.[106] Second, Ausonio claimed that it was shown with this instrument that the angle of incidence is equal to the angle of reflection, following proposition 10 of the same book.[107] Third, Ausonio referred to proposition 11 which stated that 'it will be demonstrated with the aid of the apparatus that any ray falling perpendicularly upon plane mirrors is reflected back on itself'.[108] Fourth, the next principle of Ausonio claims that, with the instrument, it can be shown for all kinds of regular mirrors that the angle of incidence and the angle of reflection are equal, and that a perpendicular ray is reflected back on itself, following propositions 12 to 17 of the fifth book of Witelo.[109]

Finally, Ausonio discussed a 'third kind of angles which can be found in illumination', namely, 'angles of diffusion'.[110] This concerned the cone of illumination formed by rays spreading out from any point of a luminous body along straight lines. Moreover, again, Witelo argued in proposition 20 of his second book that this could be shown with his instrument, namely he claimed that the circle of light on the second marker of his instrument would be larger than on the first marker where the light entered.[111] To this discussion of the angles of refraction, the angles of reflection and the angles of diffusion, Ausonio added a reference to the 'congregation of rays', that is to a burning mirror, not of a paraboloid shape, but a concave spherical burning mirror, as taken from proposition 68 of book VIII of Witelo's 'Perspectiva', to show that rays issuing from the sun are sensibly equidistant.[112] This proposition will be of central importance to the discussion of Ausonio's concave spherical burning mirrors, so we will postpone its discussion until the next chapter. Here, it should be noted that its basis was, again, an 'instrumental proof'.

[103] Witelo, Perspectiva, book II, proposition 45, translation in Ibid., p. 90; B. A. M., MS R105 Sup., f. 191v.

[104] Witelo, Perspectiva, book II, proposition 46, translation in Ibid., p. 93; B. A. M., MS R105 Sup., f. 191v-192r.

[105] 'Bisogna anchora con l' aiuto del senso saper il modo di poter misurar li predetti angoli et per far questo vi servirete della Vta, Vi.a et Vij.a del Decimo di Vitellione'. B. A. M., MS R105 Sup., f. 192r.

[106] Witelo, Perspectiva, book V, proposition 9, translation in Smith, A. Mark. *Witelonis Perspectivae Liber Quintus - Book V of Witelo's Perspectiva*. Vol. 23, *Studia Copernicana*. Wroclaw Warszawa Krakow Gdansk Lodz Ossolineum: The Polish Academy of Sciences Press, 1983, p. 92; B. A. M., MS R105 Sup., f. 192v. For a description of Witelo's instrument to measure angles of reflection, see Ibid., p. 160.

[107] Witelo, Perspectiva, book V, proposition 10, translation in Ibid., p. 95; B. A. M., MS R105 Sup., f. 192v.

[108] Witelo, Perspectiva, book V, proposition 11, translation in Ibid., p. 97; B. A. M., MS R105 Sup., f. 192v.

[109] Witelo, Perspectiva, book V, propositions 12-17, translation in Ibid., pp. 98-100; B. A. M., MS R105 Sup., f. 192v.

[110] 'della terza sorte delli angoli che nella illuminatione si possi ritrovare'. B. A. M., MS R105 Sup., f. 193r.

[111] Witelo, Perspectiva, book II, proposition 20, translation in Unguru, *Witelonis Perspectivae Liber Secundus*, p. 59.

[112] B. A. M., MS R105 Sup., f. 193r.

Thus, Ausonio's treatise on optics contributed nothing original with respect to Witelo, because it is nothing else but a summary of Witelo's 'Perspectiva'. However, Ausonio's summary was highly selective. He singled out all propositions that discussed Witelo's instruments and only those propositions that Witelo claimed to have been able to demonstrate with his instruments. Consequently, Ausonio reorganized Witelo's 'Perspectiva', so that what came forward, more strongly than in Witelo's 'Perspectiva', was the science of optics as the geometrical study of the propagation of light. Moreover, the basic principles of this mathematical discipline were claimed to be established by instrumental experiences and measurements with instruments. Also, Ausonio did not show any interest for Witelo's discussion of vision and cognition. Henceforward, he changed the focus of the optical tradition, although he does not seem to contribute anything, nor did he claim to do so. In Ausonio's reorganization of Witelo's 'Perspectiva', once the rectilinear propagation of light experientally established, the central focus of optics became the study of the refraction and reflection of light. This reorganization paralleled Ausonio's involvement with the design of refractive sundials and burning mirrors. These two instruments were the embodiment of the optics of Ausonio as a mathematical practitioner. Indeed, a refractive scaphe dial is not too different from the Ptolemy-Witelo 'experimental' set-up for the quantitative study of refraction.

It is not known how Ausonio came about designing refractive scaphe dials, but he might have seen some of the Hartmann instruments. At the Adler Planetarium, there is an ornated box compendium, which consists of a sundial, a magnetic compass to orient the sundial, and an astrolabe.[113] According to the engravings on the instrument, the compendium was made for the Duke of Savoy in 1558 by Georg Hartmann. Thus, it might be that Hartmann also made refractive scaphe dials for Emanuele Filiberto, and that Ausonio saw such an instrument and was asked by the Duke to make similar ones for him. However, the instrument is now often considered as being presumably a 19th century forgery, so the connection between Hartmann and the Duke of Savoy can no longer be established.[114] A more likely hypothesis is that a refractive scaphe dial of Hartmann came directly to Venice from Nuremberg. In the sixteenth century, there was a strong commercial connection between Nuremberg and Venice, also, as concerns, the instrument trade. For example, German merchants bought Venetian glass, 'cristallo', in Venice, and, instruments made in Nuremberg travelled the other way. Most Nuremberg merchants in Venice lived in a quarter, known as the 'Fondaco dei Tedeschi'.[115] Ausonio had good contacts among the merchants of the Fondaco dei Tedeschi. Some of the covers of letters addressed to Ausonio show him to have stayed in the workshop of a certain Giacomo Nassimbene at the Fondaco dei Tedeschi.[116]

[113] The Adler Planetarium (Chicago), A-7; See Falke, Otto v. 'Eine Instrumentenkassette von Georg Hartmann.' *Pantheon* 5 (1932): 356-57; Coole, P. G., and E. Neumann. *The Orpheus Clocks*. London: Hutchinson Educational Ltd., 1972, pp. 145-6; Klemm, *Georg Hartmann aus Eggolsheim*, p. 77.

[114] Ibid., p. 77. I would like to thank Bruce Stephenson and Marvin Bolt of The Adler Planetarium, Chicago, for information on this instrument and pointing out that it is now considered a forgery.

[115] Keil, *Augustanus Opticus*, p. 223.

[116] For example, see the letter covers in B. A. M., MS R116 Sup., f. 173v; MS G119 Inf., f. 58r, dated 29 April 1564.

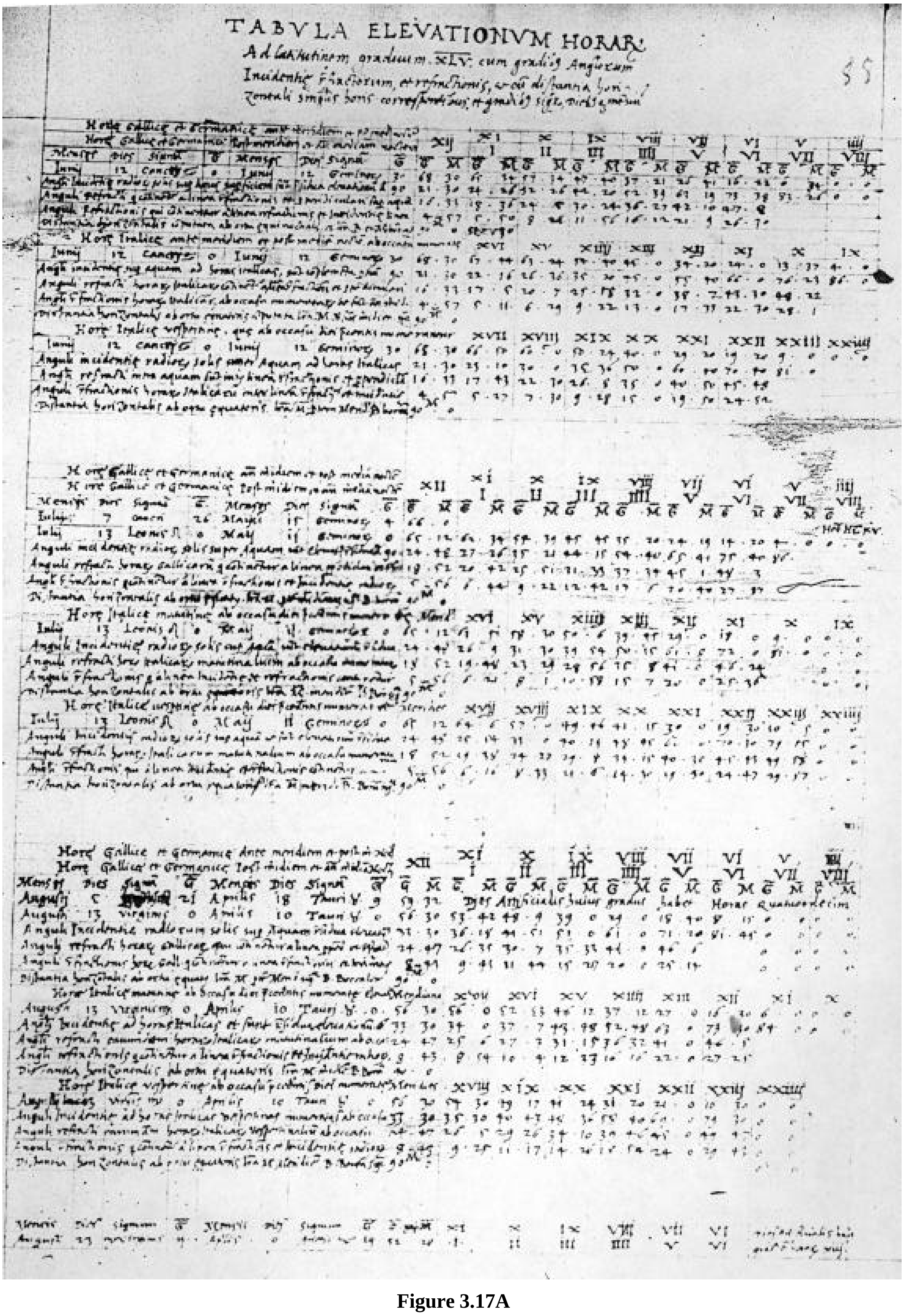
TABVLA ELEVATIONVM HORAR.

Figure 3.17A

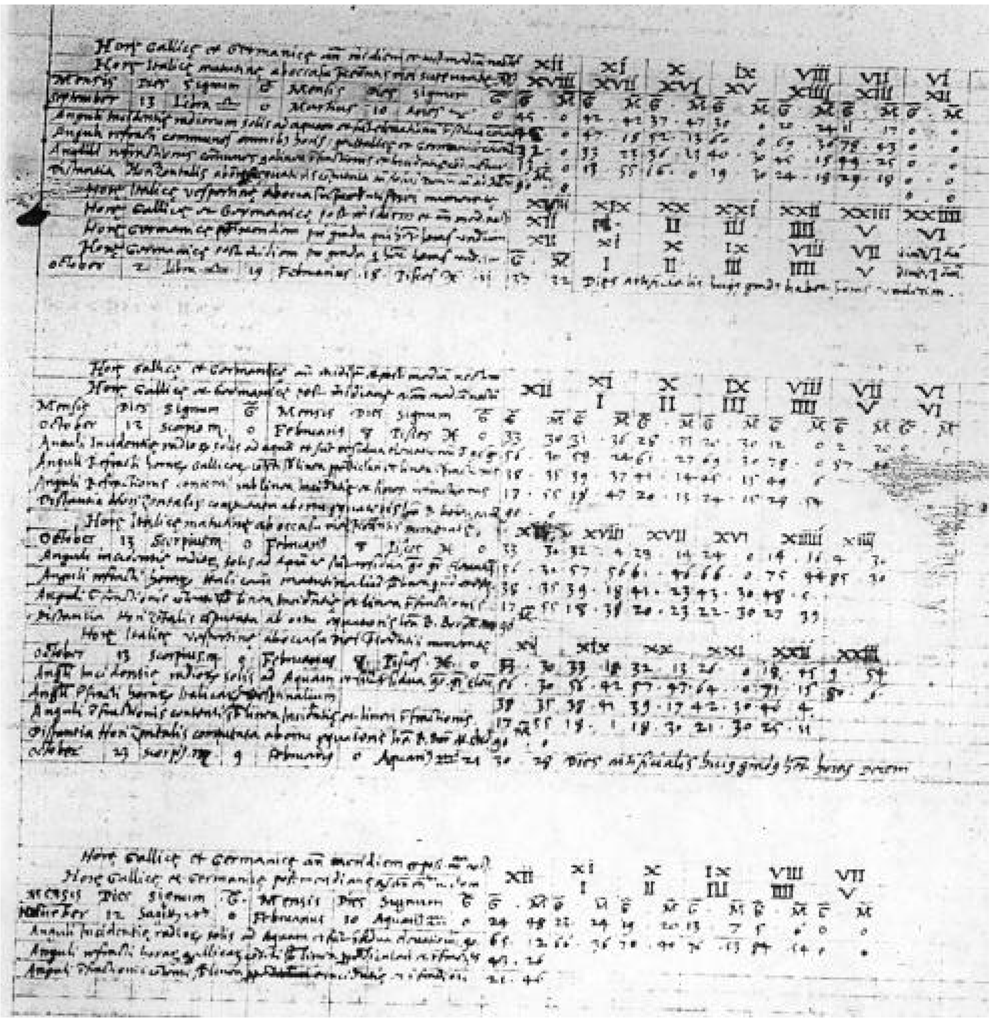

Figure 3.17B

There, Ausonio presumably saw the famous Nuremberg sundials, because he mentioned a ‘horologio solare di Germania’.[117] Moreover, many of the refractive scaphe dials he described were designed for 48°, the latitude of Nuremberg.[118]

How did Ausonio make his refractive scaphe dials? Ausonio’s approach to optics, as shown in his treatise on optics discussed above, with its strong emphasis on the ‘instruments of optics’, establishing the fundamental concepts of optics, seems to suggest that he measured the angles of refraction. Among Ausonio’s notes, there are two refraction tables that were intended for the design of refractive dials. The first table is incomplete and meant to be used on a dial for a latitude of 48°.[119] (Figure 3.17A-B)

[117] B. A. M., MS D178 Inf., f. 81r; for the Nuremberg sundials, see Lloyd, Steven A., Penelope Gouk, and A. J. Turner. *Ivory Dyptich Sundials 1570-1750*. Cambridge: The Collection of Historical Scientific Instruments, Harvard University, 1992, pp. 33-98.

[118] B. A. M., MS D178 Inf., ff. 73r, 82v.

[119] B. A. M., MS D178 Inf., f. 82v.

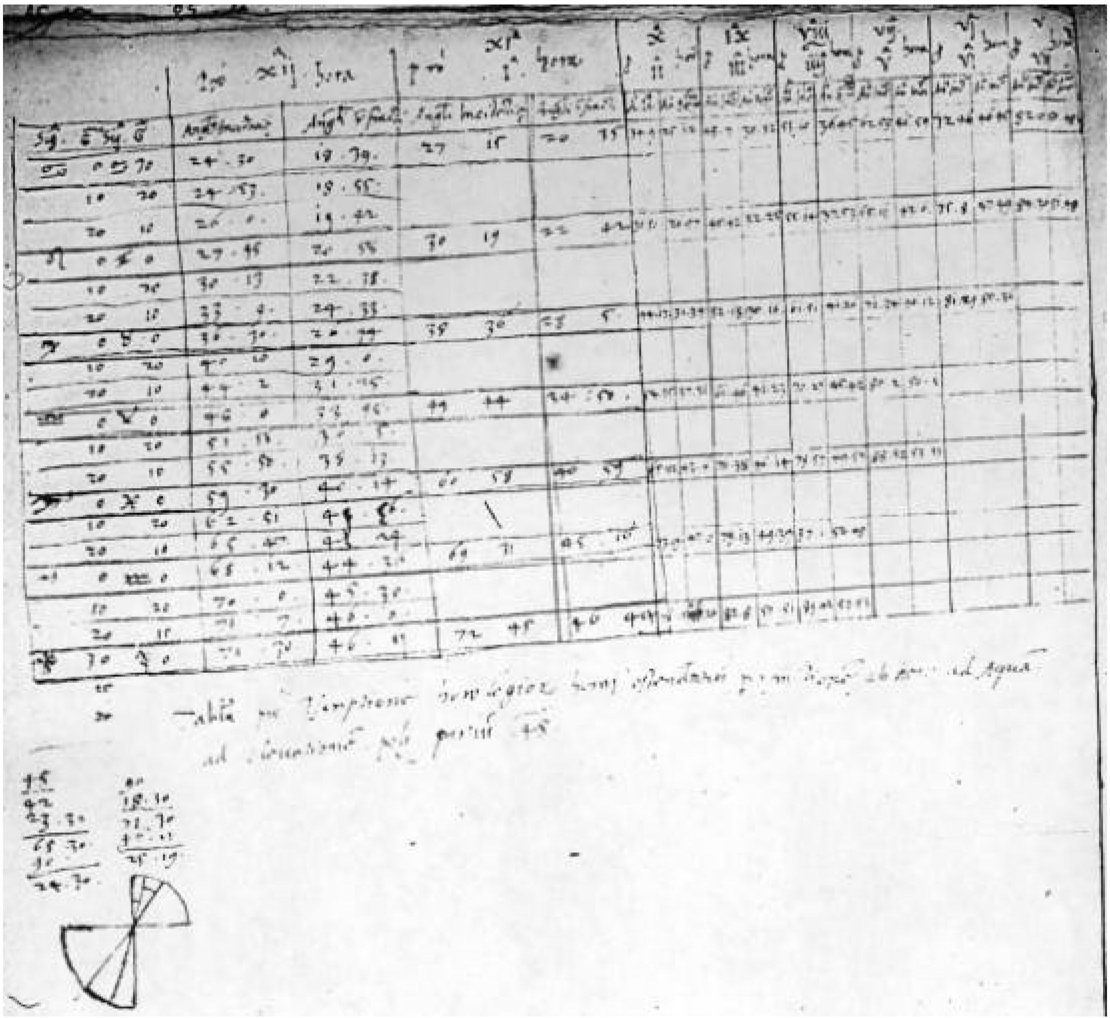

Figure 3.18

The second table is completed for use on a dial for 45°, the latitude of Venice.[120] (Figure 3.18) These tables systematically give the angle of refraction for an angle of incidence of the solar rays on a given date, established by the position of the sun, in any of the signs of the zodiac, along the ecliptic, and a given hour. An equal hour system of 'German' hours is used.[121] XII indicates noon, when the sun is on the prime meridian. Thus, the columns to the far right of the folio give times around sunrise and sunset. Consequently, here, 0° values are often indicated. On the table of the refractive dial for the latitude of 48°, the first row gives the altitude of the sun for a certain date and hour. The second row indicates the corresponding angle of incidence of the solar rays (90°- Altitude of the Sun). The third row shows the angle of refraction (angulus refractus) and the fourth row the angle of deviation (angulus refractionis). The values for the angles of incidence

[120] B. A. M., MS D178 Inf., ff. 88r-88v.

[121] On the different hour systems used on sundials, see Ibid., pp. 13-5.

and the angles of refraction, extracted from the top section of Ausonio's table for the design of the refractive dial for the latitude of 45°, are tabulated below.

i	r (Ausonio)	r (calculated)
21°30'	16°33'	15°57'
24°26'	18°36'	18°04'
32°26'	24°05'	23°43'
42°20'	30°24'	30°20'
52°39'	36°27'	36°36'
63°19'	42°10'	42°05'
73°38'	47°08'	46°02'
83°26'		48°11'
0°0' [90°]		48°36'

Since these values are in between the 10° intervals for the angles of incidence of Ptolemy's table, Ausonio could not simply have taken them from these tables, which Ausonio knew and copied from Witelo, as shown by the value of 7°45' (against Ptolemy's 8°) for an angle of incidence of 10°.[122] (Figure 3.19)

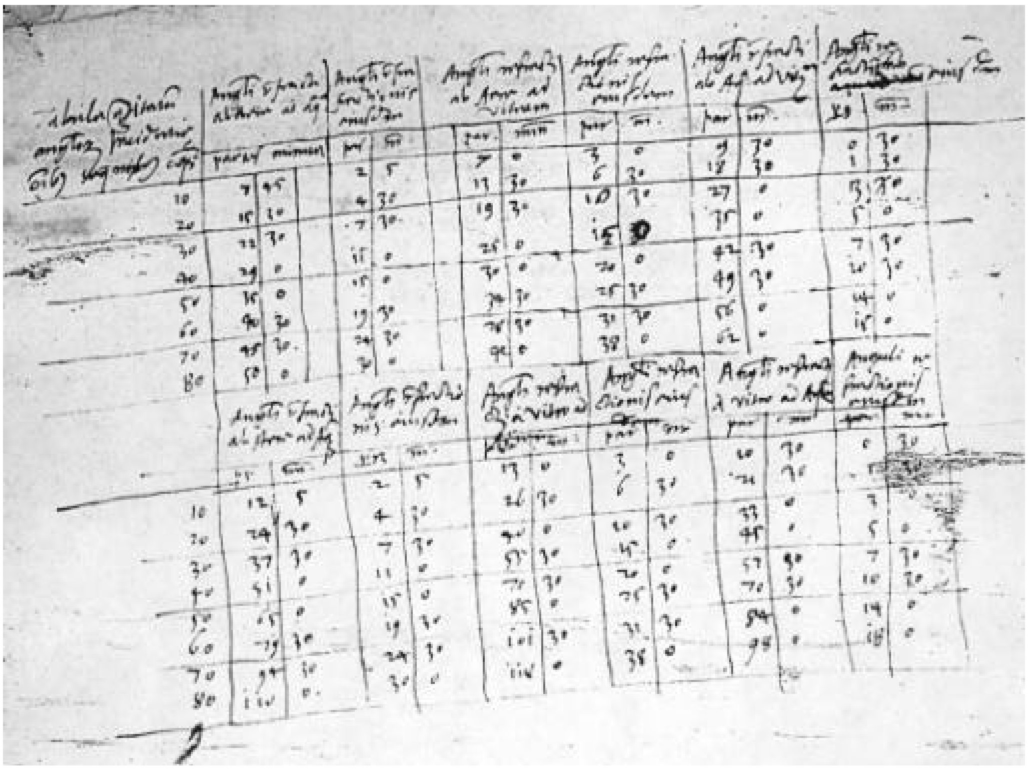

Figure 3.19

[122] B. A. M., MS D178 Inf., f. 73v.

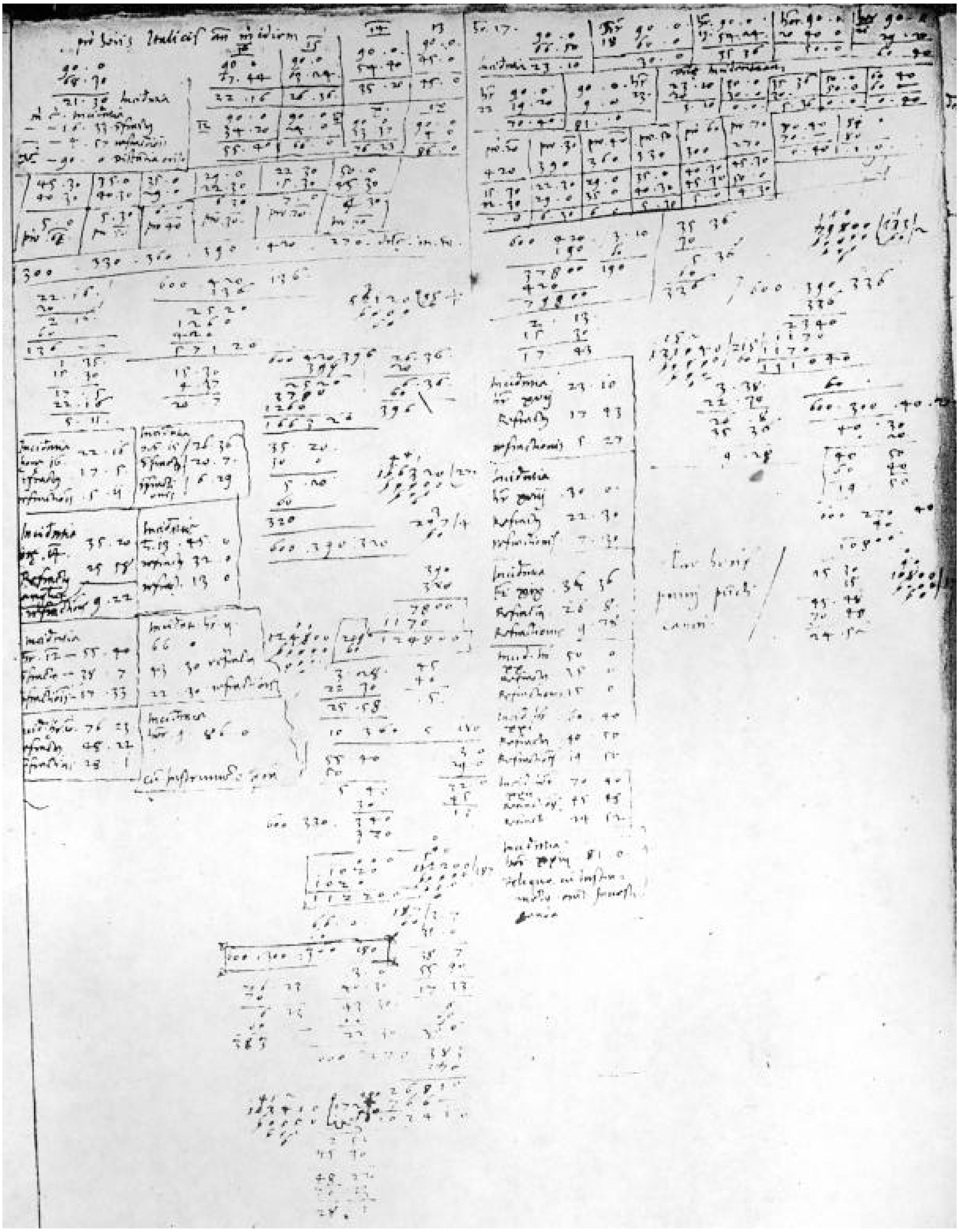

Figure 3.20

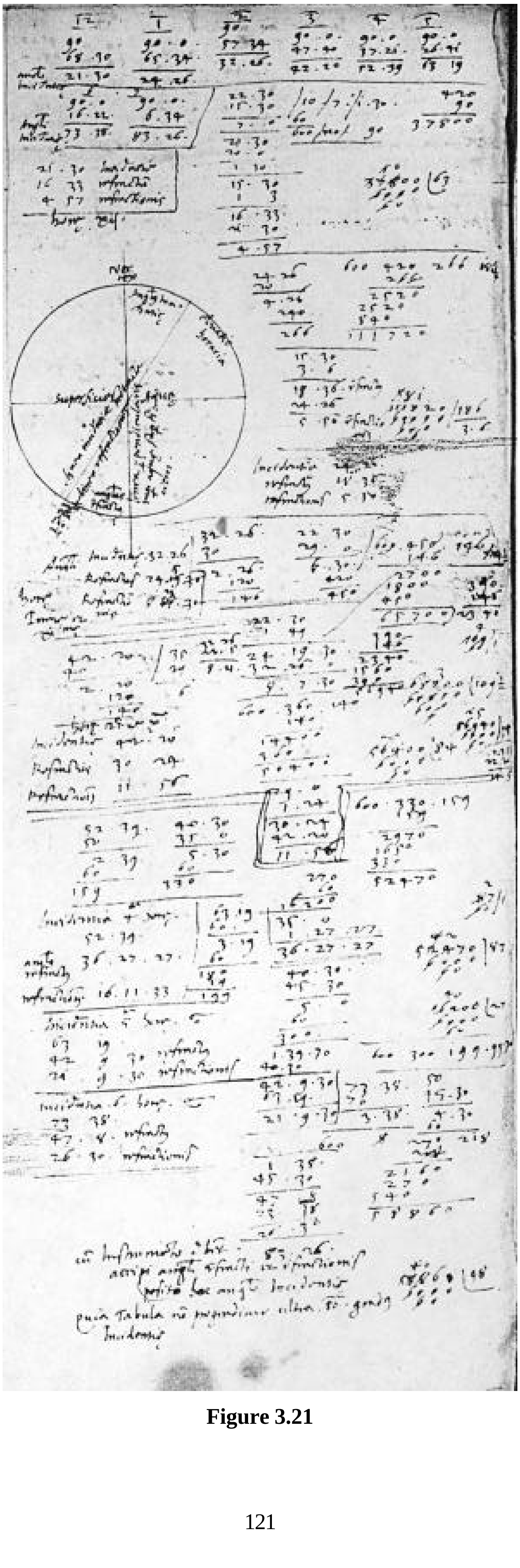

Figure 3.21

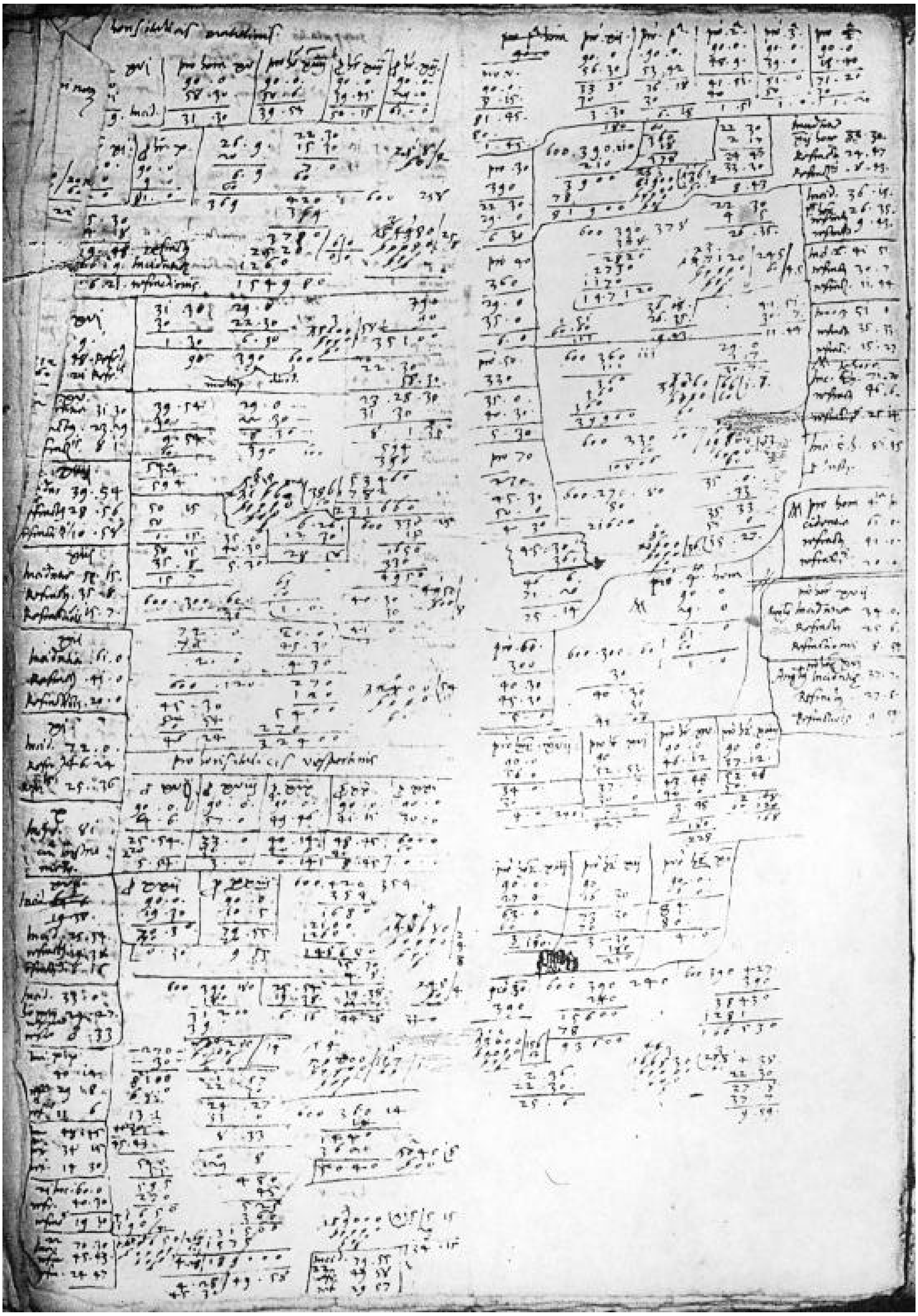

Figure 3.22

Did Ausonio measure these angles? Several folio's of calculations show that he did not.[123] (Figures 3.20-3.21-3.22) Instead, he did calculate them from the values of Witelo's table. First, Ausonio noted the difference between successive angles of refraction in Witelo's table, as they were adapted to yield constant second differences, and converted them to the value as written in minutes (see figure 3.20, top left corner).

45.30	35.0	35.0	29.0	22.30	50.0
40.30	40.30	29.0	22.30	15.30	45.30
5.0	5.30	6.0	6.30	7.0	4.30
pro 60	pro 50	pro 40	pro 30	pro 20	pro 70
300	330	360	390	420	270

Then, below this table, the folio shows, for example, how he calculated an angle of refraction of 17°05' for an angle of incidence of 22°16'. First, he calculated the difference between this angle of incidence and the smaller angle of incidence in Witelo's table. Thus, 22°16' minus 20° is 2°16', converted to minutes, is 136'. Next, he multiplied this by the value of the difference between successive angles of incidence for the relevant interval. 420' is the value for the 20-30° interval. 136 was multiplied by 420, which equals 57120. This value was divided by the difference between the two successive angles of incidence, in between which the given angle of incidence is situated, converted to minutes. 57120 divided by 600 (or 10°) equals 95 1/5. This value is then added to the value of the angle of refraction corresponding to the smaller angle of incidence, as tabulated by Witelo. 95' is 1°35', added to 15°30', the value of the angle of refraction for an angle of incidence of 20° in Witelo's table, equals 17°05'. Ausonio took 17°05' to be the angle of refraction, if the angle of incidence is 22°16'. In this way, Ausonio calculated every angle of refraction for any given angle of incidence. Thus, the general formula he used to calculate r, is

$$r = r_1 + (i - i_1).(r_2 - r_1)/(i_2 - i_1)$$

with i_1 and i_2 successive angles of incidence, and r_1 and r_2 successive angles of refraction of Witelo's table, for any given angle of incidence i. In this way, all values in Ausonio's tables for his refractive sundials are calculated. For example, in the above table, for the given angle of incidence i of 63°19', the calculated angle of refraction r is 42°10', by r = 40°30' + (63°19' – 60°).(45°30' – 40°30')/(70° - 60°), or r = 40°30' + 99.5' (~ 1°40'). Consequently, that no angles of refraction for angles of incidence larger than 80° are given in Ausonio's table is not related to practical difficulties encountered when measuring the angles of refraction for grazing angles of incidence. When the sun is at the horizon at dawn or dusk, it might become impossible to measure the angle of refraction, when the water surface is not level with the rim of the bowl. However, values of angles of refraction for angles of incidence exceeding 80° were not present in the tables, because Ausonio's calculation asked for the angle of refraction for an angle of incidence of 90° not given in the Ptolemy-Witelo tables of refraction from air to water. Moreover, the absence of such angles of refraction was in agreement with the already discussed

[123] B. A. M., MS D178 Inf., ff. 74r-74v, 76r-76v, 79r-79v.

proposition 42 of the second book of Witelo, which entailed the conclusion that light at grazing angles of incidence, as at sunrise and sunset, passed unrefracted into the second medium.[124]

Ausonio's refractive scaphe dials might be the missing link between Hartmann's refractive scaphe dials and the design of refractive dials by other Italian mathematicians in the 1570s. Giovanni Battista Benedetti (1530-1590) became the court mathematician of the Duke of Savoy, Emanuele Filiberto, in 1567.[125] The patronage was continued, also, by Emanuele Filiberto's son, until Benedetti's death. In his 'De gnomonum umbrarumque solarium usu liber' (1574), dedicated to Emanuele Filiberto, Benedetti seems to have been the first to take up in print a discussion of refractive dials.[126] It might be that Benedetti was familiar with the refractive dials that Ausonio had made for Filiberto in the 1560s. Benedetti described, again, Witelo's 'experimental' set-up for the measurement of the angles of refraction, and he claimed that, once the angles of deviation are known, it was possible to construct the refractive dial from the non-refractive dial.[127] The analysis of Ausonio's procedures might serve as the caveat that this should not necessarily be taken at face value as evidence that Benedetti actually made measurements.

Another path of transmission might lead from Ausonio's refractive dials to Guidobaldo del Monte (1545-1607).[128] In a letter of 16 June 1610, Guidobaldo's son Orazio asked Galileo's help to publish some of the works of his father.[129] One of these works was a treatise entitled 'De radiis in aqua refractis' of which the present location is not known. However, the content of this work

[124] See references cited in n. 99 and n. 100.

[125] Drake, Stillman. 'Benedetti, Giovanni Battista.' In *The Dictionary of Scientific Biography*, edited by Charles Coulston Gillispie, Vol. 1, 604-9. New York: Scribner's sons, 1970, in particular, p. 605; Maccagni, Carlo. 'Contributi alla Biobibliografia di Giovanni Battista Benedetti.' *Physis* 9 (1967): 337-64.

[126] Turner, Anthony J. 'Dialling in the Time of Giovan Battista Benedetti.' In *Cultura, Scienze e Tecniche nella Venezia del Cinquecento: Atti del Convegno Internazionale di Studio Giovan Battista Benedetti e il Suo Tempo*, edited by Antonio Manno, 311-22. Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1987, p. 315.

[127] 'collocata deinde rota in vase aqua pleno ad solem, ita tamen vt dimidium eius exactè in aqua mergatur, vnaque quarta divisa penitus emineat aquae, altera penitus in aquam mergatur, postmodum dicta vas, ita ad solem situm sit, vt quarta indiuisa ad solem vergat, divisa autem in oppositam partem soli, atque intra solem, & rotam aliquid collecetur, quod obstet ne sol radijs suis rotam aquamue feriat, quo perfecto, sumatur termissimum speculum, medianteq; solis radio ab eo reflexo in singulos gradus per rimas dictus radius pertranseat, recta ad axim, centrú versus, obserueturq; quo loco dicti axis vmbra in quarta diuisa in aquam mersa cadat: quo loco, exactissimè cernetur quantaum radius refractus in aqua declinet ab incidente per aerem quolibet gradu altitudinis ab orizonte, ope vt predictum est speculi.' Benedetti, Giovanni Battista. *De Gnomonum Umbrarumq Solarium Usu Liber.* Avgvstae Tavrinorum: Apud haeredes Nicolai Beuilaquae, 1574, ff. 105r-105v.

[128] On Guidobaldo del Monte, see Rose, *The Italian Renaissance of Mathematics*, pp. 222-42. See also Gamba and Montebelli, *Galileo Galilei e gli Scienziati del Ducato di Urbino*, pp. 23-6, 36; Arrighi, Gino. 'Un Grande Scienziato Italiano: Guidobaldo del Monte in Alcune Carte Inedite della Biblioteca Oliveriana di Pesaro.' *Atti dell' Accademia Lucchese di Scienze, Lettere ed Arti* 59 (1965): 183-99, pp. 183 -90.

[129] 'Io mi ritrovo in essere alcune opere di mio padre b. m., che le vorrei dar fuori; ma li stampatori di Venetia mi hanno tradito troppo con le scorrettioni ne' Problemi Astronomici. Se fosse possibile che in Padova io fossi servito di buon correttore, io le darei fuori volentieri, perchè son consigliato et importunato farlo, et le opere son curiose: La Coclea che inalza l' aqua, divisa in 4 libri; Opuscoli: *In Quintum; De motu terrae; De horologiis, De radiis in aqua refractis; In nono (?) opere (?) Scoti; De proportione composita,* et la fabrica di alcuni istromenti ritrovati da lui, delle quali tutte cose vi sono le figure intagliate. Io prego V. S. Ecc.ma avisarmi come potrei fare'. Orazio del Monte to Galileo, 16 June 1610, in Galileo, *Opere*, Vol. 10, p. 372.

on ‘rays refracted in water’ presumably was related to Guidobaldo’s design of refractive dials. According to his student, Muzio Oddi, Guidobaldo made several refractive dials in Urbino.[130]

> The most famous Mr. Guidobaldo de Marchesi del Monte had one made by Simone Baroccio, excellent artificer, in a hemisphere of wood, which I have had in my hands many times, and this one served next as a model of one, which the Duke Francesco Maria ordered. A second one was made in the basin of the fountain that is in the hanging garden of his most magnificent palace of Urbino, as one can see until today.[131]

Thus, the first refractive dial, made by Guidobaldo and the famous instrument maker Simone Baroccio in 1572, according to Oddi, was a refractive scaphe dial. The second refractive dial, made after the first one, was made in the basin of the fountain of the palace garden in Urbino. This second dial was not a refractive scaphe dial, but the refractive variant of a horizontal planar dial, with the hour lines set out on the bottom of the basin of a fountain, of the type also made by the Augsburg instrument maker, Christoph Schissler the Elder in 1578.[132] Panicali has dated the construction of this dial, still visible today, between 1587 and 1631.[133] Consequently, it did not necessarily involve the direct contribution of either Guidobaldo del Monte, who died in 1607, or Simone Baraccio, who died in 1608, so it will be of lesser concern for our purposes.

How did Guidobaldo learn about refractive scaphe dials? Guidobaldo was a student of Federico Commandino (1509-1575). Commandino and Ausonio might have met early in their careers. Commandino is said to have read medicine and philosophy at the University of Padua between ca. 1535 and ca. 1545.[134] The latter part of this period coincides with Ausonio’s medical studies at the same university. Also, later in his career, Commandino visited Venice and his publications around 1560 appeared under the auspices of Paolo Manuzio, the official publishing house of the Accademia della Fama.[135] However, it can be established with certainty that Commandino and Ausonio corresponded in the 1560s, because a few traces of this correspondence are preserved.[136]

Since both men were corresponding, also on optics, as will become evident in the next chapter, it is likely that Commandino learned from Ausonio about the construction of refractive sundials, and that Commandino had a hand in Guidobaldo’s design of a refractive scaphe dial in 1572.

[130] On Muzio Oddi, see Promis, Carlo. ‘Vita di Muzio Oddi.’ *Antologia Italiana* 4 (1848): 377-400; Gamba and Montebelli, *Galileo Galilei e gli Scienziati del Ducato di Urbino*, pp. 16-8.

[131] ‘L’ illustrissimo Signor Guidobaldo de Marchesi del Monte ne fece fare vno da Simone Baroccio, eccellente artefice, in vna mezza sfera d’ Ottone, & hollo hauuto nelle mani molto tempo, il quale seruì poi come per modello d’ vno, che d’ ordine del Duca Francesco Maria. Secondo, ne fù fabricato entro la tazza della fonte, che è nel Giardino pensile del suo Magnificentissimo Palazzo d’ Vrbino; come si vede fino al giorno d’ hoggi’. Oddi, *De gli Horologi Solari*, pp. 99-100.

[132] Mills, Allan A. ‘The 'Dial of Ahaz', and Refractive Sundials in General, Part II: Horizontal Planar Dials.’ *Bulletin of the Scientific Instrument Society* 45 (1995): 25-7, p. 27.

[133] Panicali, *Orologi e orologiai del Rinascimento italiano*, pp. 122-9.

[134] Rose, *The Italian Renaissance of Mathematics*, p. 187.

[135] Ibid., pp. 190-7.

[136] Two letters of Commandino to Ausonio are still extant; B. A. M., MS D117 Inf., ff. 122r-122v, dated 22 February 1568; B. A. M., MS G121 Inf., ff. 135r-135v, dated 14 May 1569. On this correspondence and a reproduction of the first letter, see Ventrice, ‘Ettore Ausonio Matematico dell' Accademia Veneziana della Fama’, pp. 1151-4.

According to Oddi, different from Benedetti and Guidobaldo, Commandino seems to have been looking for new tables of refraction that might facilitate the construction of refractive dials.

> I still have in my possession some folio's made up by Commandino, which, as far as I can conjecture, look for the cause of the variety of the angles of refraction. Since the shadows made by the gnomon are not retreating uniformily, when the sun is close to the horizon with respect to when it is high above the earth, although it traversed equal intervals, [these folio's] maybe were meant to compose the tables of this effect. [These tables] are not the same as those of Alhazen and Witelo.[137]

In the absence of these folio's of Commandino, it is of course impossible to determine whether the tables of Commandino were really different from Witelo's, based on measurements of the angles of incidence and the angles of refraction, or only, like Ausonio's, apparently different from Witelo's, but actually derived from calculations based on Witelo's tables of refraction. What can be concluded is that, even if measurements were taken, their aim was the designing of an instrument that would work. Apparently, there did not seem to have been any search for a law of refraction. Oddi wrote his 'De gli horologi solari' in the same year that Descartes (1637) published the sine law of refraction, and, as will be discussed in the next chapter, more than thirty years after Kepler's discovery of a partial law of refraction in his 'Paralipomena' (1604).

The involvement with refractive sundials of mathematical practitioners shows how Witelo's 'Perspectiva' was appropriated in the sixteenth century. Ausonio reorganized Witelo's 'Perspectiva' so that it highlighted Witelo's 'instrumental' approach to optics as the geometrical study of the propagation of light. After Ausonio's reorganization, Witelo's 'Perspectiva' is to be read as a treatise about refractive sundials (or, as will be our concern in the next chapter, about burning mirrors). Ausonio's refractive dials were the material embodiment of Witelo's tables of refraction. For Ausonio, the refractive sundial was the central cognitive problem that needed a solution. Only in such a practical context, there was a need solve the problem of finding the angle of refraction for any angle of incidence. As has been shown, Ausonio did not make measurements, but he solved the problem by taking the constant second differences as the basis of the formulation of his rule to calculate the angle of refraction for any angle of incidence. This might have worked appropriately for the construction of refractive dials, since Ptolemy's values for the angles of refraction, and Ausonio's calculated values, are not too far off the actual angles of refraction (see tables above). Ausonio's refractive dials would have worked well without attaining such precision. However, the refractive sundial is not to be confused with a deliberate experimental set-up aimed at finding a law of refraction. Even if Benedetti, Commandino or Guidobaldo actually made measurements, they were only aimed at the design of a well-working refractive sundial. This instrument-centered view of optics is not limited to the involvement with refractive dials. We will encounter a similar attitude when discussing the telescope.

[137] 'Si conservano ancora presso di me alcuni fogli disegnati dal Commandino, che, per quanto hò potuto conietturare, giua cercando la ragione della varietà de gl' angoli delle refrattioni, non retirandosi vniformemente l' ombre fatte dal Gnomone, quando il Sole è vicino all' Orizonte, da quando è alto da terra, benche habbia trascorso intervalli vguali, forse per comporne le tauole à questo effetto, non essendo le medesime, che quelle d' Alazeno, e di Vitellione'. Oddi, *De gli horologi solari*, p. 100.

IV. The Point of Inversion in Sixteenth Century Optics: The 'Theory of the Concave Spherical Mirror' of Ettore Ausonio

1. From Vision to Light: Kepler, Refraction and Images

Ausonio designed, beside refractive sundials, burning mirrors. As shown in the last chapter, around 1560, he formulated a proposal to deliver a whole range of mirrors, of which several were burning mirrors, to the Duke of Savoy, Emmanuele Filiberto. Apparently, Ausonio was highly regarded as a mirror designer. Ausonio's interest in burning mirrors is hardly surprising, if we consider the general rise of interest in burning mirrors among mathematical practitioners of the fifteenth and sixteenth centuries. Consequently, in chapter 2, I have characterized the mathematical practitioners' appropriation of optics as resulting in a shift from vision to light. While vision and cognition were the focus of medieval optics, Renaissance optics, in the appropriation of mathematical practitioners, changed the scope of optics as it became primarily concerned with the study of light. Burning mirrors were an important object for the study of the propagation of light. It should be emphasized that such distinctions of scope and focus are relative distinctions. As discussed in chapter 2, the optics of antiquity and the Middle Ages saw several treatises on burning mirrors, from Diocles' 'On burning mirrors' and Alhazen's 'De speculis comburentibus' to the propositions on burning mirrors in Witelo's 'Perspectiva'.

Simon has argued that the main difference between ancient and medieval optics and modern geometrical optics, developed in the seventeenth century, beginning with Kepler's 'Paralipomena ad Vitellionem' (1604), is that the first was a science of vision, while the second was a science of light.[1] In the optics of Euclid and Ptolemy, the object of analysis is the visual ray, issuing from the eye or the center of sight toward a given point on the visible object. This visual ray is not to be confused with a luminous ray or a light ray. Consequently, since the visual ray was meant to represent a line of sight, its function was to explain vision, not the physics of light radiation or the propagation of light. Euclid's ray-concept missed any dynamic content, as it described a physical path of visual radiation, only understood as a process in space, not in time.[2] As discussed in chapter 2, Ptolemy and, in particular, Alhazen, introduced mechanical analogies which gave some dynamical content to the ray-concept. However, as Smith has emphasized, for Alhazen, these mechanical analogies were mere analogies.[3] Not before the seventeenth century did the mechanical analogies lose their analogical status to develop into a particle-model of light that considered a light ray as the representation of the passage of discrete matter through space and time.[4] Although Alhazen, and his medieval followers of the perspectiva, considered light to have an ontological status independent of vision, they also regarded it a 'form'.[5] Consequently,

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[1] Simon, Gérard. 'Derrière le Miroir.' *Le Temps de la Réflexion* 2 (1981): 298-332; Simon, Gérard. *Le Regard, l' Être et l' Apparence dans l' Optique de l' Antiquité*. Paris: Editions du Seuil, 1988, pp. 11-20; Simon, Gérard. 'La Notion de Rayon Visuel et ses Conséquences sur l' Optique Géométrique Grecque.' *Physis* 31 (1994): 77-112, pp. 79-105.

[2] Smith, A. Mark. 'Extremal Principles in Ancient and Medieval Optics.' *Physis* 31 (1994): 113-40, pp. 122-3.

[3] Ibid., pp. 135-9. See also Smith, *Descartes' Theory of Light and Refraction*, pp. 49-51.

[4] Smith, 'Extremal Principles in Ancient and Medieval Optics', p. 140.

[5] Rashed, Roshdi. 'Le 'Discours de la Lumière' d' Ibn Al-Haytham (Alhazen): Traduction Française Critique.' *Revue d' Histoire des Sciences* 21(1968): 197-224, pp. 199-201; Rashed, Roshdi. 'Lumière et Vision: L' Application des Mathématiques dans l' Optique d' Ibn Al-Haytham.' In *Roemer et la Vitesse de la Lumière*, edited by R. Taton, 19-

notwithstanding the introduction of mechanical analogies, they thought of the actual propagation of light in terms of the continuous qualitative transformation through a material medium.

That Ptolemy's optics was based on a visual ray model, and not on a light ray model, is evident from the absence of the discussion of burning mirrors in his discussion of image formation in concave mirrors in the fourth book of his 'Optics'.[6] Until the seventeenth century, the discussion of image formation was based on the cathetus rule of image location.[7] According to this rule, the image is located at the intersection of the reflected ray and the cathetus (of incidence), which is the perpendicular drawn from the object to the reflecting surface. It was applied to plane as well as to curved mirrors. In a plane mirror EF, the ray AN from the object A is reflected to the eye at O. The image Z of the object A is at the intersection of the produced reflected ray NO and the cathetus AB, drawn perpendicular to the mirror EF. (Figure 4.1)

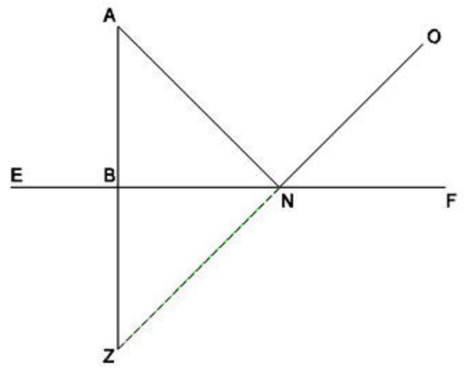

Figure 4.1

44. Paris: Librairie J. Vrin, 1978, in particular, pp. 26-37. Reprinted in Rashed, Roshdi. *Optique et Mathématique: Recherches sur l' Histoire de la Pensée Scientifique en Arabe*. Aldershot, Hampshire; Brookfield, Vermont: Variorum, 1992; Sabra, A. I. 'Form in Ibn Al-Haytham's Theory of Vision.' *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 5 (1989): 115-40. Reprinted in Sabra, A.I. *Optics, Astronomy and Logic: Studies in Arabic Science and Philosophy*. Aldershot Broofkield: Variorum, 1994; Simon, Gérard. 'L' Optique d' Ibn Al-Haytham et la Tradition Ptoléméenne.' *Arabic Sciences and Philosophy* 2 (1992): 203-35, pp. 212-8.

[6] Smith, 'Ptolemy's Theory of Visual Perception', p. 42. For Ptolemy's discussion of image formation in concave mirrors, see Lejeune, Albert. *Recherches sur la Catoptrique Grecque d' après les Sources Antiques et Médiévales*. Brussel: Koninklijke Academie van België, 1957, in particular, pp. 77-8, 104-11.

[7] The rule was formulated by pseudo-Euclid. See Turbayne, Colin M. 'Grosseteste and an Ancient Optical Principle.' *Isis* 50 (1959): 467-72. On its interpretation in the sixteenth century, see Lorch, Richard. 'Pseudo-Euclid on the Position of the Image in Reflection: Interpretations by an Anonymous Commentator, by Pena, and by Kepler.' In *The Light of Nature: Essays in the History and Philosophy of Science Presented to A. C. Crombie*, edited by J.D. North and J. J. Roche, 135-44. Dordrecht Boston Lancaster: Martinus Nijhoff Publishers, 1985.

If the mirror is curved, the cathetus is the line drawn from the object to the center of curvature of the mirror. Thus, in the concave mirror EF, the locus of the image Z of the object A seen by the eye O is at the intersection of the cathetus AB and the reflected ray NO. (Figure 4.2)

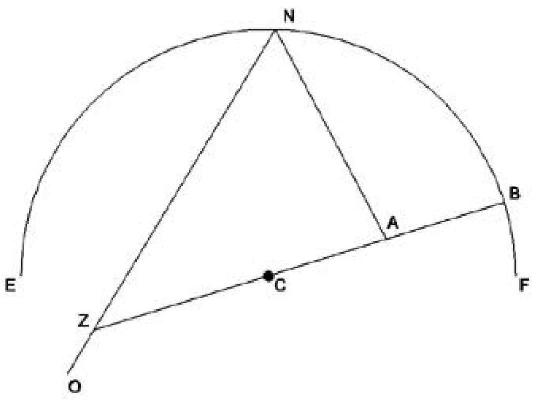

Figure 4.2

Finally, the cathetus rule was also applied to find the locus of an image produced by a refracted surface. If the ray ON from the eye O is refracted at the surface of a denser medium to A, then the image Z is at the intersection of the ray produced ONZ and the cathetus AB perpendicular to the refracting surface EF. (Figure 4.3) Consequently, in these accounts of image formation, there are no focal points.

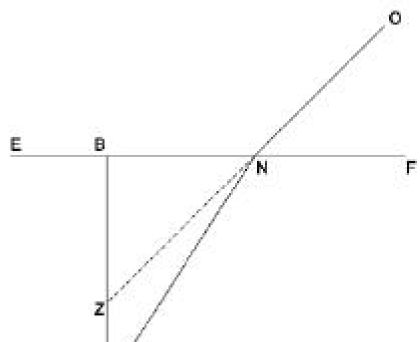

Figure 4.3

On the other hand, as already shown, burning mirrors were widely discussed in antiquity, and, to a lesser extent, in the Middle Ages, and they became an important object of research in the fifteenth and sixteenth centuries when mathematical practitioners appropriated the heritage of medieval optics. However, burning mirrors were discussed in separate treatises or, if discussed in more general optical treatises, for example in Witelo's 'Perspectiva', it will become evident that they played no role in the discussion of image formation. To the best of my knowledge, there is only one exception to this general tendency. In the 9th century, Ahmad ibn Isa, a compiler of several works of al-Kindi, and Qusta ibn Luqa discussed in their treatises on burning mirrors the catoptrical properties of mirrors.[8] They attempted to show that 'in burning mirrors bodies appear larger than their size', however, without determining the locus of the image. Again, the focal properties of mirrors, discussed in the section on burning mirrors of their treatises, played no role in this part of their analysis. Thus, as Simon has pointed out, the link, so obvious to us, between burning mirrors and vision, that is image location, was inconceivable in antiquity and the Middle Ages.[9] In this chapter, I will argue that it was precisely this connection between vision and burning mirrors that was established by Ausonio. Moreover, in chapter 6, it will be argued that the establishment of this connection was at the basis of the attempts to develop telescopes in the second half of the sixteenth century. However, to facilitate the understanding of Ausonio's contribution, it might be helpful to discuss first Kepler, who in his discussion of refraction and image formation in the 'Paralipomena', established the connection on theoretical grounds.

Kepler discussed refraction in the fourth chapter of his 'Paralipomena'. The point of entrance of his discussion of refraction was atmospheric refraction, in particular, the debate on atmospheric refraction between Rothmann and Tycho Brahe at the end of the sixteenth century. The debate between Rothmann and Tycho was the continuation of the discussion of atmospheric refraction by Pena, as discussed in chapter 2. Indeed, more than any other element of his 'De usu optices', Pena's discussion of (the absence of) atmospheric refraction, as an argument against the existence of the Aristotelian spheres, and his discussion of the nature and locus of comets, in the same context, had turned out influential. Christoph Rothmann (d. after 1597) was mathematician at the court of the German prince Wilhelm IV of Hesse-Kassel from 1577 to 1590.[10] At this court, the Landgrave Wilhelm IV had established a permanent observatory, the first of its kind in Europe. The observatory was equipped with precision instruments, made by Jost Bürgi, of which the design, involving the use of transversal points to subdivide the degrees of arc scale and the use of a pair of fine sighting slits, was, at least partially, due to Tycho Brahe, who visited the observatory in 1575.[11] In his book on the comet of 1585, Rothmann adopted the optical theory of comets, as developed by Apian, Gemma Frisius and Pena.[12] Moreover, from the absence of parallax, he argued for the supralunar locus of the comet, in fact, he placed them in the sphere of Saturn. However, since Rothmann still considered comets to be made up of terrestrial vapors, it

[8] Rashed, Roshdi. *Oeuvres Philosophiques et Scientifiques d' Al-Kindi*, pp. 139-43.

[9] Simon, 'Derrière le Miroir', p. 313.

[10] Moran, Bruce T. 'Christoph Rothmann, the Copernican Theory, and Institutional and Technical Influences on the Criticism of Aristotelian Cosmology.' *Sixteenth Century Journal* 13 (1982): 85-108, p. 85.

[11] Ibid., pp. 89-97.

[12] Barker, Peter, and Bernard R. Goldstein. 'The Role of Comets in the Copernican Revolution.' *Studies in History and Philosophy of Science* 19 (1988): 299-319, p. 314.

was impossible that the planetary spheres were solid and impenetrable, if the comets were supposed to have risen to the sphere of Saturn all the way up from the earth.[13]

In this same treatise, and in his 'Observationum stellarum fixarum', Rothmann accepted Pena's idea that there is nothing else between the earth and the sphere of the fixed stars than ordinary air. However, this does not mean that no atmospheric refraction could be detected. Rothmann claimed that there was no detectable atmospheric refraction of the sun and stars above an altitude of 30°.[14] Consequently, Rothmann limited the argument of Gemma Frisius. If there was a difference of density between the celestial ether and the air just below the moon, as conventional Aristotelian cosmology wanted it, then there should be a detectable atmospheric refraction all the way to the zenith. Since no such atmospheric refraction can be found with his precision instruments above an altitude of 30°, Rothmann concluded that atmospheric refraction must take place in clouds and vapors near the earth, denser than the air above, thus causing effects of atmospheric refraction on celestial bodies near the horizon. Because of this absence of refraction above an altitude of 30°, Rothmann rejected the solid spheres in favor of fluid heavens.[15]

Tycho Brahe also rejected the solid planetary spheres in favor of fluid heavens, but he disagreed with Rothmann as concerns the substance of the heavens.[16] In fact, he strongly rejected the idea, which Rothmann had borrowed from Pena, that the substance of the heavens was nothing else but ordinary air. Tycho's judgement of Pena's 'De usu optices' was highly negative. However, it is unlikely that he knew Pena's preface directly, since he complained that he had difficulties obtaining a copy. He probably was acquainted with Pena's opinions through his correspondence with Rothmann from 1586 onwards, and through the reading of the manuscript of Rothmann's work on comets, which was sent to him in the same year.[17] From 1587 onwards, Tycho started using the argument of atmospheric refraction against the existence of solid and impenetrable planetary spheres.[18] Atmospheric refraction was certainly not Tycho's primary argument against the existence of solid spheres. Prior to 1587, the trajectory he had calculated for the comets,

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[13] Lerner, *Le Monde des Sphères II*, p. 62.

[14] Rothmann's tables contain separate entries for the sun and the stars. The two tables are identical up to 7 degrees. For altitudes higher than 7 degrees, there is less correction for the stars and the sun, until any atmospheric refraction disappears for the stars at an altitude of 29°. See Barker, Peter. 'Brahe and Rothmann on Atmospheric Refraction.' In *Optics and Astronomy: Proceedings of the Xth International Congress of History of Science (Liège, 20-26 July 1997)*, edited by Gérard Simon and Suzanne Débarbat, 43-50. Turnhout: Brepols, 2001, p. 45.

[15] Goldstein, Bernard R., and Peter Barker. 'The Role of Rothmann in the Dissolution of the Celestial Spheres.' *British Journal for the History of Science* 28 (1995): 385-403, pp. 391-5. See also Granada, Miguel A. 'Eliminazione delle Sfere Celesti e Ipotesi Astronomiche in un Inedito di Christoph Rothmann: L' Influenza di Jean Pena e la Polemica con Pietro Ramo.' *Rivista di Storia della Filosofia* 52 (1997): 785-821, pp. 787-96.

[16] On Tycho Brahe, see Christianson, John Robert. *On Tycho's Island: Tycho Brahe and His Assistants, 1570-1601*. Cambridge: Cambridge University Press, 2000; Thoren, Victor E. *The Lord of Uraniborg: A Biography of Tycho Brahe*. Cambridge: Cambridge University Press, 1990; Dreyer, J. L. E. *Tycho Brahe: A Picture of Scientific Life and Work in the Sixteenth Century*. New York: Dover Publications Inc., 1963; Chapman, Allan. 'Tycho Brahe - Instrument Designer, Observer and Mechanician.' In *Astronomical Instruments and Their Users: Tycho Brahe to William Lassell*, edited by Allan Chapman, 1-15. Aldershot Brookfield: Variorum, 1996.

[17] On Rothmann and Brahe, see Barker, 'Brahe and Rothmann on Atmospheric Refraction', pp. 44-5; Goldstein and Barker, 'The Role of Rothmann in the Dissolution of the Celestial Spheres', pp. 395-7; Gingerich, Owen, and Robert S. Westman. *The Wittich Connection: Conflict and Priority in Late Sixteenth-Century Cosmology*. Vol. 78, *Transactions of the American Philosophical Society*. Philadelphia: American Philosophical Society, 1988, pp. 72-6.

[18] Barker, 'Brahe and Rothmann on Atmospheric Refraction', pp. 46-7.

together with the mutability of the heavens, the appearance of comets had suggested to him, against Aristotelian cosmology, and his hypothesis of a geoheliocentric system, in which the orbits of Venus and Mars intersected the orbit of the sun, had already suggested to him that the existence of solid and impenetrable planetary orbs was something highly impossible.[19]

Tycho presumably became first interested in atmospheric refraction, when visiting the Landgrave of Hesse in 1575, who told him about the apparent retrotardation of the sun near the horizon, which was attributed to refraction.[20] Tycho's first measurement of solar refraction date from the early 1580s, but it was only, after he received the letters and work of Rothmann, that he developed a more extensive and systematic program of observation of the stars and the sun in order to measure atmospheric refraction. However, Tycho's measurements yielded different results from Rothmann's. For Tycho, a measurable effect of atmospheric refraction did not disappear until above an altitude of 45°.[21] This result was enough to reject the existence of solid spheres, which would have spoiled by refraction the observations all the way up to zenith. Tycho insisted, with Rothmann, that the refraction was caused near the earth.[22] However, since atmospheric refraction could be detected, against Pena's absence of atmospheric refraction, there was no need to accept Pena's and Rothmann's arguments for pneuma reaching to the stars. Tycho kept insisting on a difference between the air of the earth's atmosphere and celestial matter.

Where did Kepler stand in this debate? As shown by the preface to his 'Dioptrice' (1611) in which he discussed the arguments of Pena's 'De usu optices', Kepler was well acquainted with Pena's preface to Euclid's 'Optica et Catoptrica'. Kepler agreed with Pena that the substance of the heavens is 'just that of the air in which we humans draw breath in the way that fish draw water'.[23] However, he disagreed with Pena on the existence of atmospheric refraction, which Kepler considered established by the observations of Tycho. Consequently, Kepler assumed a difference of density between the fluid aether, which he brought down to within half a German mile, about 7.4 km, of the earth, and the air of the earth's atmosphere.[24] As concerns atmospheric refraction, Kepler criticized both Rothmann and Tycho. In Kepler's opinion, 'if they had applied the genuine measure of refractions, it would not have been necessary for Tycho to introduce a

[19] Lerner, *Le Monde des Sphères II*, pp. 59-65.

[20] Dreyer, *Tycho Brahe*, p. 80.

[21] Similar to Rothmann, Tycho's atmospheric refraction tables had separate entries for the sun and the stars. The quantity of refraction for stars is always 4'30" less than for the sun. The measurable effect of atmospheric refraction, according to Tycho's table, disappears for the stars at an altitude of 20°. For the sun, the quantities are larger, because Tycho assumed an effect of solar parallax of 3', going against the effect of atmospheric refraction. See Barker, 'Brahe and Rothmann on Atmospheric Refraction', pp. 46-7; Goldstein and Barker, 'The Role of Rothmann in the Dissolution of the Celestial Spheres', p. 395; Thoren, *The Lord of Uraniborg*, pp. 220-35; Dreyer, *Tycho Brahe: A Picture of Scientific Life and Work in the Sixteenth Century*, pp. 334-7.

[22] Goldstein and Barker, 'The Role of Rothmann in the Dissolution of the Celestial Spheres', pp. 395-6.

[23] Kepler, Johannes. *Ad Vitellionem Paralipomena Quibus Astronomiae Pars Optica Traditur.* Edited by Max Caspar. Vol. 2, *Gesammelte Werke*. München: C.H. Beck'sche Verlagsbuchhandlung, 1937, p. 79. References are to the page numbers in the original 1604 edition. Translation in Kepler, Johannes. *Optics: Paralipomena to Witelo & Optical Part of Astronomy*. Translated by William H. Donahue. Santa Fe: Green Lion Press, 2000, p. 96. There is also a French translation, see Kepler, Johannes. *Les Fondements de l' Optique Moderne: Paralipomènes à Vitellion (1604)*. Translated by Catherine Chevalley. Paris: Librairie Philosophique J. Vrin, 1980. For similar quotes in other works of Kepler, see Barker, 'Jean Pena and Stoic Physics in the Sixteenth Century', p. 102.

[24] For discussion, see Randles, W. G. L. *The Unmaking of the Medieval Christian Cosmos, 1500-1760: From Solid Heavens to Boundless Aether*. Aldershot: Ashgate, 1999, pp. 77-9.

twofold cause of refractions, by which I mean the two bodies, the one of air and the other vapor; nor would Rothmann have denied that light is refracted by some imperceptible amount, even near the zenith'.[25] Moreover, Kepler considered the causes of atmospheric refraction on which Tycho as well as Rothmann drew, to be a violation of the physical principles of refraction. As has been discussed in chapter 2, in the tradition of medieval optics, with the exception of Oresme, refraction was considered a surface phenomenon.

> Thus Tycho would have determined that there is no successive attenuation of air into aether and effacing of the density pertaining to air, but *an obvious and evident division*; … Rothmann, on the contrary, would not have collided with optical principles, that the surface of a denser medium is struck by light without suffering anything in return or being refracted; and that something is in the doubled parts that is not in the single ones; and that *the rays are refracted by the depth of the media, not by the surfaces; all of which are absurd*.[26]

Henceforward, Kepler attempted to find a law of refraction, based on the tabulations of refraction for different media of Witelo and on Tycho's tables for atmospheric refraction. As is well known, Kepler's search was not completely successful.[27] Eventually, and through a very different path than blind data fitting, he only found an approximate law for angles of incidence smaller than 30°, which is equivalent to $r = 2/3i$. However, for our purposes, it is most important to see why Kepler did not succeed. Kepler was very close to discovering the sine law of refraction.

> Nor did I leave this remaining case untried: whether, once the horizontal parallax is established from the density of the medium, the rest might correspond to the sines of the distances from the zenith. But computation did not give approval to this, not was there any need to make the enquiry. For refractions would increase in the same pattern in all media, which does not agree with experience.[28]

However, Kepler did not take in consideration the angle of refraction, but the angle of deviation, to be a function of the sine of the angle of incidence. He did not take the angle of refraction in consideration, because this angle had no physical meaning within the framework of Kepler's physics of light.[29] As discussed in chapter 2, Alhazen had elaborated the mechanical analogy for light introduced by Ptolemy, but this mechanical analogy was ultimately dependent upon a teleological or animistic element, because when a ray was refracted toward the normal, it was thought of as either choosing a direction of easier passage or as preserving its ease of passage uniformly when passing from one medium to another. Since within Alhazen's physics of light, taken over by medieval optics, the normal is privileged, it made sense to take in consideration the angle of refraction. However, it was precisely this animistic element that Kepler rejected.

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[25] Kepler, *Ad Vitellionem Paralipomena*, p. 79. Translation in Kepler, *Optics*, p. 95.

[26] Ibid., p. 79. Translation in Ibid., p. 95, my italics.

[27] My discussion of Kepler's search for a law of refraction is much indebted to three authors arguing along similar lines, Simon, Gérard. 'Structures de Pensée et Objets du Savoir chez Kepler.', Université de Paris IV, 1976, pp. 478-514; Buchdahl, Gerd. 'Methodological Aspects of Kepler's Theory of Refraction.' *Studies in the History and Philosophy of Science* 3 (1972): 265-99; Hallyn, 'Kepler, Snellius en de Lichtbrekingswet', pp. 127-31. See also Hallyn, Fernand. 'Tout Peut-il Marcher?' *Sartoniana* 5 (1992): 41-69.

[28] Kepler, *Ad Vitellionem Paralipomena*, p. 84. Translation in Kepler, *Optics*, p. 100.

[29] For discussion, see Simon, 'Structures de Pensée et Objets du Savoir chez Kepler', in particular, pp. 484-9; Hallyn, 'Kepler, Snellius en de Lichtbrekingswet', pp. 128-31.

> They [Alhazen and Witelo] did not explain the matter much better than Macrobius, *Saturnalia* book 7, who attributed to the sense of vision a hesitation, and a retreat into itself from the encounter. It is exactly as if the form of light were endowed with mind, by which it might reckon both the density of the medium and its own injury, and, using its own judgement and not an extrinsic force, acting and not being acted upon, might of itself perform its own refraction.[30]

Thus, since Kepler stripped the normal of its privileged status, the angle of refraction had no physical meaning, and, thus, could not have any function in a law governing refraction.
Kepler tried another path that brought him very close to discovering the sine law of refraction. Instead of starting from the angles of incidence and deviation, he started from the localisation of the images.[31]

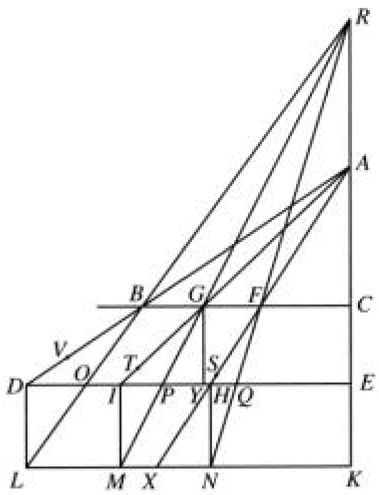

Figure 4.4

In Kepler's diagram, BC is the refracting surface and A is the eye. (Figure 4.4) By application of the cathetus rule, the image of M is seen at I, at the intersection of AG produced and the cathetus, the perpendicular from M to the refracting surface BC. As Lohne has argued, Harriot precisely discovered the sine law of refraction by applying trigonometric functions to the triangle GMI.[32] However, this path of discovery was not open to Kepler, because he rejected the cathetus rule as a way of finding the image. Eventually, Kepler considered the search for a law of refraction from the determination of the location of images by the cathetus rule to be in vain.

[30] Kepler, *Ad Vitellionem Paralipomena*, p. 84. Translation in Kepler, *Optics*, p. 100.

[31] For discussion, see Buchdahl, 'Methodological Aspects of Kepler's Theory of Refraction', pp. 283-4; Simon, 'Structures de Pensée et Objets du Savoir chez Kepler', pp. 495-6.

[32] Lohne, 'Thomas Harriot', p. 117. See also Lohne, J.A. 'Kepler and Harriot - Their Search for a Law of Refraction.' In *Four Hundred Years: Proceedings of Conferences in Honour of Johannes Kepler*, edited by Arthur Beer and Peter Beer, 857-58. Oxford New York Toronto Sydney Braunschweig: Pergamon Press, 1975.

> Our consideration of the image, or of the place of the image, was in vain, for the very reason that it is an image. For what it is that happens to the sense of vision, from whose error the image results, has nothing to do with the density of the medium, nothing to do with the real affect itself, the bending back of light.[33]

In the third chapter of his 'Paralipomena', Kepler had replaced the cathetus rule by a more general rule for image location in catoptrics, because, as shown, in the framework of Kepler's physics of light, the cathetus had no meaning.[34] First, Kepler showed that the cathetus rule had only a limited application, that is, beside its validity for nearly perpendicular incident rays or paraxial rays, it is valid for rays laying out of the initial plane of incidence.[35] However, Kepler deduced the limited validity of the cathetus rule not from some inherent, animistic characteristic of the cathetus, as in medieval optics, but only from the geometry of the imaginary mathematical lines which represent the propagation of light. Kepler's point of departure is his acceptance of the psychological definition of an image as an error of sight in not seeing an object in its true place.

> The Optical writers say it is an image, when the object itself is indeed perceived along with its colors and the parts of its figure, but in a position not its own, and occasionally endowed with quantities not its own, and with an inappropriate ratio of parts of its figure. Briefly, an image is the vision of some object conjoined with an error of the faculties contributing the sense of vision. Thus, the image is practically nothing in itself, and should rather be called imagination.[36]

Next, Kepler formulated a more general rule for image location, based on the 'distance-measuring triangle' to explain the judgment of distances, to replace the cathetus rule. In proposition 8, Kepler argued that distances are determined by a triangle that uses the distance between our two eyes, the base of the triangle, and the angle of convergence of the axes of the eyes, converging toward the object, that is, the vertex of the triangle.[37] Kepler argued that the distances can also be judged by one eye, if the diameter of the pupil is taken as the base of the triangle. Since the eye is unaware of any change of direction of rays before they enter the eye, it judges an object to be in the place where the reflected or refracted rays come from. Consequently, as Kepler claimed in proposition 17, 'the genuine place of the image is that point in which the visual rays from the two eyes meet, extended through their respective points of refraction or reflection, by 8 of this third chapter'.[38] From this principle of image location, Kepler deduced the limited validity of the cathetus rule.[39] (Figure 4.5)

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[33] Kepler, *Ad Vitellionem Paralipomena*, p. 89. Translation in Kepler, *Optics*, p. 104.

[34] For discussion of this third chapter, see Chen-Morris, Raz Dov, and Sabetai Unguru. 'Kepler's Critique of the Medieval Perspectivist Tradition.' In *Optics and Astronomy: Proceedings of the XXth International Congress of History of Science (Liège, 20-26 July 1997)*, edited by Gérard Simon and Suzanne Débarbat, 83-92. Turnhout: Brepols, 2001; Shapiro, Alan E. 'The Optical Lectures and the Foundations of the Theory of Optical Imagery.' In *Before Newton: The Life and Times of Isaac Barrow*, edited by Mordechai Feingold, 105-78. Cambridge: Cambridge University Press, 1990, pp. 122-4; Simon, 'Structures de Pensée et Objets du Savoir chez Kepler', pp. 464-77.

[35] For discussion, see Shapiro, 'The Optical Lectures', pp. 120-1.

[36] Kepler, *Ad Vitellionem Paralipomena*, p. 60. Translation in Kepler, *Optics*, p. 77.

[37] Ibid., p. 62. Translation in Ibid., p. 79.

[38] Ibid., p. 69. Translation in Ibid., p. 85.

[39] Ibid., p. 69. Translation in Ibid., p. 85. For discussion, see Shapiro, 'The Optical Lectures', pp. 123-4; Simon, 'Structures de Pensée et Objets du Savoir chez Kepler', pp. 468-9.

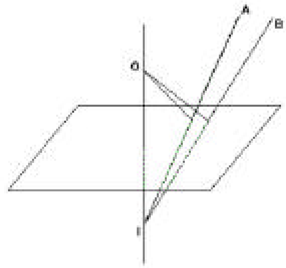

Figure 4.5

If both eyes A and B lie in different planes of incidence, then the image I of the object O is at the intersection of those two planes. By necessity, this image will also be in a plane perpendicular to the reflecting surface, namely on the perpendicular from the object O to the reflecting surface, known as the cathetus. However, when the eyes are in the same plane of incidence, the image will not be on the cathetus. As Kepler showed in proposition 18, the image of the object O, seen in a convex mirror by two eyes A and B, in the same plane of incidence, is at the intersection I of the reflected rays AE and BF, while the cathetus rule predict it to be at either T or S, dependent on the choice of eye.[40] (Figure 4.6)

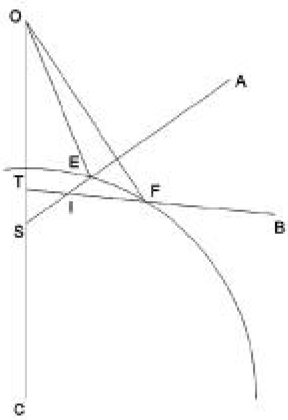

Figure 4.6

Consequently, the cathetus rule is not universally applicable for image location, unless, Kepler concluded, 'the sense of vision be so located with respect to the mirror as nature shows'.[41] Kepler seems to suggest that it is natural to look at objects with the head held vertical.

[40] Kepler, *Ad Vitellionem Paralipomena*, pp. 70-2. Translation in Kepler, *Optics*, pp. 86-8.

[41] Ibid., p. 72. Translation in Ibid., p. 88.

In the fifth chapter of his ‘Paralipomena’, he applied his rule of image location to image formation in a sphere filled with water. It is important to note that Kepler’s rule of image location is meant for the perceived location of images. As has been shown, Kepler used a fundamentally psychological definition of an image, in agreement with the medieval optical tradition. Only the first seven propositions of the fifth chapter of the ‘Paralipomena’ are devoted to the perceived location of images. He consistently used the principle that the image must lie at the vertex of the two-eyes-based optical triangle. In the first proposition, he located the image of point A, seen through the sphere filled with water EFG with two eyes B and C at D, the intersection of the rays AEFC and AGHB.[42] (Figure 4.7)

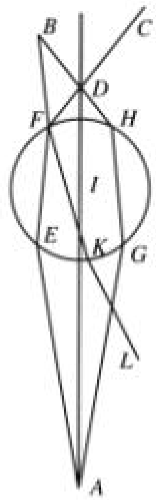

Figure 4.7

However, in the following propositions, Kepler adduced reasons why the image is seldom seen at D. In proposition 2, he claimed that ‘the sense of vision looks upon things that are very close with greater difficulty than upon things that are more distant’.[43] Consequently, it is easier for the eyes to look at the sphere filled with water than at D, the position of the image. In proposition 3, he argued that vision is attracted by the sphere’s stronger light.[44] This claim was elaborated in proposition 4 that ‘darkness accommodates an image, but when a stronger light arises from the direction of the image, the image passes away’.[45] Consequently, Kepler argued that the light refracted through the sphere filled with water will illuminate the locus of the image at D to the extent that the image disappears.[46]

[42] Ibid., pp. 178-9. Translation in Ibid., p. 192.

[43] Ibid., p. 179. Translation in Ibid., p. 192.

[44] Ibid., p. 179. Translation in Ibid., pp. 192-3.

[45] Ibid., p. 180. Translation in Ibid., p. 193.

[46] Ibid., p. 180. Translation in Ibid., p. 193.

Finally, in proposition 5, Kepler concluded, detracting from his first proposition, that 'in front of an aqueous ball or globe there is no place for the image of an object hiding behind the ball'.[47] Kepler brought together the conditions, explained in the former propositions, to criticize image formation as discussed in Della Porta's 'Magiae naturalis' in a way that made very clear that the perceived location of images was for Kepler not only a matter of mathematics. His point of departure were the 'images hanging in air', to which references were often encountered in sixteenth century optics, as shown in chapter 1, and which will also be discussed below.

> Pertinent to this is what Porta had taught in chapter 10 preceding, 'with a convex crystalline lens, to see an image hanging in air'. … For this reason, he adds, 'If you will place a piece of paper in the way, you will see clearly that a lighted candle appears to be burning upon the paper.' That is, the image will be seen weakly and hardly at all in the bare air itself, by Porta's admission. But if you put a piece of paper in the way – if, I say, you interpose a piece of paper between the lens and the sense of vision (for, with me, Porta here is still speaking about the image, not yet about the picture, of which this is true, as will be clear below), the image will now appear, not hanging in air, but fixed on the paper. For the paper, striking the eyes more obviously, steadies them on the place of the image, so that they may be turned towards each other in that direction. And nonetheless, because the paper is then brighter than the image, the paper will be seen primarily, the image secondarily. For it is not mathematical dimensions alone that create the image, but also, and much more, the colors and lights and physical causes, with which prop. 2, 3, and 4 of this chapter are concerned. If you should focus the eyesight upon one place, namely, upon the place of the image previously investigated, as it has been described in prop. 1 of this chapter, when a clearly visible object is placed nearby, then the eyes coming together upon this object, will also see the required image secondarily.[48]

While criticizing Della Porta, Kepler made a distinction between an image, seen by the eye, and a picture, caught on a piece of paper. The first is a psychological image, the second is not.

> 'Since hitherto an Image [Imago] has been a Being of the reason, now let the figures of objects that really exist on paper or upon another surface be called pictures [Pictura].'[49]

Consequently, from proposition 8, Kepler described image formation in a sphere filled with water in a different way based on his concept of pencils of rays, which he used instead of the isolated rays of the optics of antiquity and the Middle Ages. Kepler located the picture, without regard to any eye, at the intersection of pencils of rays along the axis of the sphere filled with water. Kepler's location of pictures was based on his concept of a refracting focus. Proposition 8 stated that 'rays from some point at a distance beyond comparison that flow to any points of an aqueous globe, meet the axis after a double refraction, that is they meet the line that is drawn from the radiating point through the center of the globe'.[50] In proposition 9, Kepler proved that, from those incoming parallel rays, those farther away from the axis intersect the axis closer to the globe after being twice refracted.[51] (Figure 4.8)

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[47] Ibid., p. 180. Translation in Ibid., p. 193.

[48] Ibid., pp. 180-1. Translation in Ibid., pp. 193-4.

[49] Ibid., p. 193. Translation in Ibid., p. 210.

[50] Ibid., p. 183. Translation in Ibid., p. 196.

[51] Ibid., pp. 185-6. Translation in Ibid., pp. 198-9.

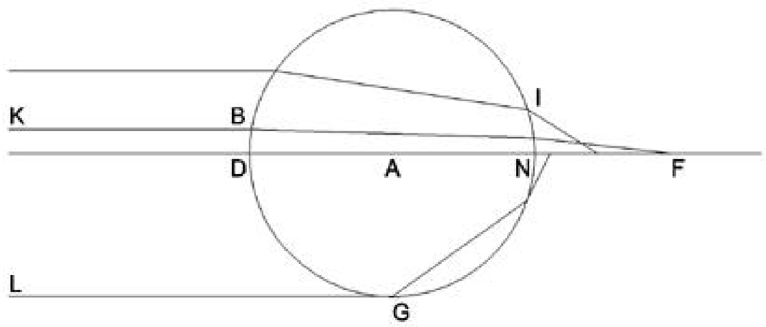

Figure 4.8

The incoming ray LG farthest from the axis will meet the axis after being refracted at I, the closest to the sphere among all the intersections. The closer the incoming rays are to the axis, the farther away from the sphere they intersect the axis. The upper boundary of these intersections is F. To determine F, Kepler assumed the ray KB, so that the arc BD equals 10°. Next, by applying his approximate law of refraction, he calculated that the intersection with the axis must lie approximately at a distance equal to the radius AN. In proposition 15, he proved that F was not far removed from this intersection point at a distance AN. Moreover, all paraxial rays intersect the axis more or less in the same point F.[52]

In a modern terminology, F is the focus. The image in a refracting sphere filled with water will be formed on a piece of paper located at the focus. Thus, from this determination of the focus, Kepler moved toward image formation. In proposition 20, he stated that 'through a globe of a denser medium, any point more remote than the intersections of parallels strongly depicts itself upon paper, located at the last boundary of the intersection of its radiations, not before and not after this point; and the picture comprising all the points is seen inverted'.[53] Kepler's picture is at the basis of the modern theory of optical imagery, first formulated in the 1660s by Isaac Barrow.[54] Barrow gave up the distinction between a perceived image, Kepler's 'imago', and a geometrical image, Kepler's 'pictura'. Thus, Barrow's principle is that an image is always perceived at the locus of the geometrical image. The geometrical image is located at the point of convergence (for real images) or the point of divergence (for virtual images) of the refracted or reflected rays. Only with Barrow's theory of optical imagery, the connection between light and image location, inconceivable to antiquity and the Middle Ages, was completely established.

However, Malet has argued that Kepler's pictures are not be equated with geometrical images.[55] First, Kepler's pictures are of a more limited applicability, namely, they only serve for real, and not for virtual images, and only for images in lenses, not in mirrors. Second, Kepler did not use

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[52] Ibid., pp. 190-1. Translation in Ibid., pp. 205-6.

[53] Ibid., p. 194. Translation in Ibid., p. 211.

[54] Shapiro, 'The Optical Lectures', pp. 130-44.

[55] Malet, Antoni. 'Keplerian Illusions: Geometrical Pictures Versus Optical Images in Kepler's Visual Theory.' *Studies in the History and Philosophy of Science* 21 (1990): 1-40, in particular, pp. 6-21.

them as geometrical images. Proposition 17 showed how Kepler's images, not his pictures, were related to the refracting focus.[56] (Figure 4.9)

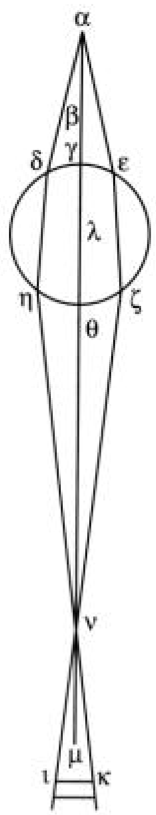


Figure 4.9

βγ is the region of 'boundaries of the intersections of parallels', or, in a modern terminology, the focal region, and β the focus, for parallel rays refracted through a sphere filled with water ξη. θμ is the focal region of point α. It is assumed that the eye is at point α, while KI is the object seen through the sphere filled with water. Kepler claimed to prove that 'if the eye be removed from the globe beyond the boundary of the intersections of Proposition 15, whatever is placed after the globe beyond the last boundary of the intersections, which these nonparallel radiations of the eye make, its image appears in an inverted situation on the surface of the globe'.[57] Thus, as usual in his accounts of image formation in the propositions before the concept of a picture was introduced, Kepler located the image at the surface of the sphere filled with water. If Kepler had used the theory of optical imagery of modern geometrical optics, he would have used the focal region of the object KI to locate its image, without any reference to an eye. However, to account for what the eye can see, he used the focal region θμ of point α. In his proof, he fell back on

[56] Kepler, *Ad Vitellionem Paralipomena*, pp. 191-2. Translation in Kepler, *Optics*, pp. 208-9. For discussion, see Malet, 'Keplerian Illusions: Geometrical Pictures Versus Optical Images in Kepler's Visual Theory', pp. 20-1.

[57] Kepler, *Ad Vitellionem Paralipomena*, p. 191. Translation in Kepler, *Optics*, p. 208.

proposition 6, which stated that 'images of objects seen through an aqueous globe with one eye adhere to the nearer surface of the aqueous globe', thus, to a proposition from before he had introduced the concept of a picture.[58]

Thus, on the one hand, Kepler's 'Paralipomena' was not the complete turn-around, immediately establishing modern geometrical optics. On the other hand, it has not been realized that Kepler's account of image formation in a refractive sphere filled with water was highly dependent upon sixteenth century optics. At the beginning of chapter 5 of his 'Paralipomena', Kepler gave an overview of image formation in a refractive sphere under camera obscura conditions.

> For if one were to stand with a crystalline or aqueous globe of this kind in some room next to a glazed window, and provide a white piece of paper behind the globe, distant from the edge of the globe by a semidiameter of the globe, the glazed window with the channels overlead with wood and lead, enclosing the edges of the windows, are depicted with perfect clarity upon the paper, but in an inverted position. The rest of the objects do the same thing, if the place be darkened a little more, to such an extent that if the globe be brought into the chamber … and set opposite the little window, whatever things are able to reach through the breadth of the little window or opening to the globe are all depicted with perfect clarity and most pleasingly through the crystalline upon the paper opposite. And while the picture appears at this distance uniquely (that is, a semidiameter from the globe to the paper), and nearer and farther there is confusion, nevertheless, exactly the opposite happens when the eye is applied. For if the eye be set at a semidiameter of the globe behind the glass, where formely the picture was most distinct, there now appears the greatest confusion of the objects represented through the glass. … If the eye comes to be nearer to the globe, it perceives the objects opposite erect and large, where they are poured together on the paper; if it on the other hand recedes farther from the globe than the semidiameter of the globe, it grasps the objects with distinct images, inverted in situation, and small, and clinging right to the nearest surface of the globe. Before, however, when the paper was placed there, the picture had entirely vanished.[59]

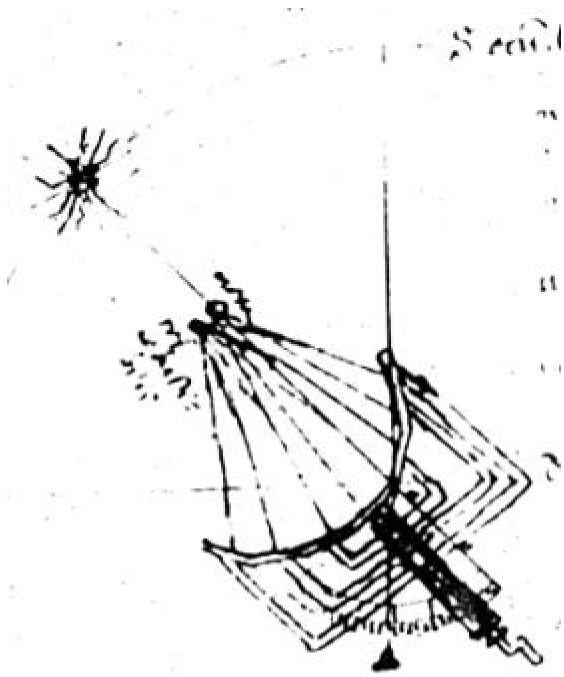
Kepler's concept of a picture, depicted on a piece of paper at the point of combustion or the focus, as distinct of an image, was entirely original with him.

However, as is also evident when he criticized Della Porta, quoted above, his observations that, when they eye is located at the focus, the perceived image appears confused, that, when the eye is closer to the sphere than the focus, the image is perceived erect and large, and, that, when the eye is farther from the sphere than the focus, the image is small and inverted, are not. This account was dependent upon the introduction of the point of combustion in image formation, and its identification with the point of inversion, in sixteenth century optics. As will become evident, this account became only available in the sixteenth century, as it was not present in medieval optics. It was introduced by Ettore Ausonio, however, not for refractive spheres, but for concave spherical mirrors.

[58] Ibid., p. 181. Translation in Ibid., p. 194.

[59] Ibid., p. 178. Translation in Ibid., p. 191.

2. The Attribution of the 'Theory of the Concave Spherical Mirror'



Figuur 4.10

As the many other mathematical practitioners of the fifteenth and sixteenth centuries, discussed in chapter 1, Ausonio was interested in parabolic burning mirrors from the point of view of optical design, and, consequently, as his notes show, his design of a parabolic shape took as its point of departure the focal distance of the mirror, or 'the point where the mirror would burn'.[60] However, the extremely chaotic state of his scattered notes has not allowed to evaluate to what extent Ausonio mastered the mathematics of the construction of a parabolic mirror. The traces of his already mentioned correspondence with Commandino indicate that he might have been a better mathematician than the chaos of his notes might suggest. At the end of the 1560s, Commandino corresponded with Ausonio on his edition and Latin translation of Pappus, which was published in 1589, and its possible relevance for optics, in particular, Witelo's discussion of burning mirrors. In his letter of 22 February 1568, Commandino referred to proposition 37 of book IX of Witelo's Perspectiva', on the making of a burning mirror out of several concave mirrors.[61] Ausonio was interested in the construction of a burning mirror out of several mirrors, but notes and a drawing suggest that he intended several plane mirrors to be used.[62] (Figure 4.10)

[60] B. A. M., G 120 Inf., ff. 29r-30r.

[61] 'Io vederò meglio la propositione di Vitellione et trascriverò un' altra volta; perché ci intendiamo bene, io parlo della 37.a del nono libro'. B. A. M., D 117 Inf., f. 122r. For a transcription of this letter, see Ventrice, 'Ettore Ausonio Matematico dell' Accademia Veneziana della Fama', pp. 1153-4. Proposition 37 of book IX of Witelo's 'Perspectiva' argued that 'ex plurium speculorum sphaericorum concavorum intersectione speculum comburens constitui est possibile'. See Lindberg, *Opticae Thesaurus*, pp. 392-4.

[62] B. A. M., G 120 Inf., f. 4r; B. A. M., R 105 Sup., f. 293r.

Apparently, Commandino considered the proposition not well proved by Witelo. Anyway, that Commandino considered Ausonio an appropriate correspondent to send him his theorems of Pappus for approval, suggests that Ausonio might have mastered the mathematics involved.

However, Ausonio's original work was not on parabolic burning mirrors, but on spherical burning mirrors. His most fully developed study of these mirrors was his 'Theorica speculi concavi sphaerici'. As will become evident, Ausonio's original appears to be lost, but the 'Theorica' is preserved in three different copies. A first copy is on a large folio, in Galileo's handwriting, in the Galileiana 83 at the Biblioteca Nazionale of Florence. (Figure 4.11) A facsimile was reproduced by Favaro in the third volume of Galileo's collected works.[63] Sosio has identified a second incomplete manuscript copy among the optical notes and drawings, known as the 'Manoscritto dell' Iride e del Calore' of Paolo Sarpi.[64] (Figure 4.12) A third copy, first identified by Savelli, and, later, apparently unaware of Savelli's article, by Ventrice, was published by Giovanni Antonio Magini in 1602.[65] (Figure 4.13) By a comparative study of these different copies, it is possible to show that (1) the work was originally by Ausonio, that (2) Galileo's copy must have been identical to Ausonio's orginal and that (3) Galileo copied it between 1592 and 1601, at a time when Ausonio's original was part of the library of Gian Vincenzo Pinelli in Padua.[66]

[63] Biblioteca Nazionale (Firenze), Gal. 83, f. 4r, reproduced by Favaro in Galileo, *Opere*, Vol. 3, pp. 865-70.

[64] Sarpi, Paolo. *Pensieri Naturali, Metafisici e Matematici: Edizione Critica Integrale Commentata a Cura di Luisa Cozzi e Libero Sosio*. Milano Napoli: Riccardo Ricciardi Editore, 1996, pp. 519-46. For Sarpi's original, see Bibliotheca Marciana (Venice), It. II, 129 (= 4914), ff. 303r-308v. The copy is on f. 303r.

[65] Magini, Giovanni Antonio. *Theorica Speculi Concavi Sphaerici*. Bononiae: Apud Ioannem Baptistam Bellagambam, 1602. There appears to be only one copy of this work preserved, in the Bibliotheca Comunale dell' Archiginnasio in Bologna. See Savelli, Roberto. 'Intorno al Rinvenimento della Theorica di Ettore Ausonio Pubblicata da Giovanni Antonio Magini.' *Atti della Fondazione G. Ronchi* 10 (1955): 3-11; Ventrice, Pasquale. *La Discussione sulle Maree tra Astronomia, Meccanica e Filosofia*, pp. 47-9.

[66] Part of this argument had been published in Dupré, Sven. 'Mathematical Instruments and the 'Theory of the Concave Spherical Mirror': Galileo's Optics beyond Art and Science.' *Nuncius* 15 (2000): 551-88, pp. 563-72.

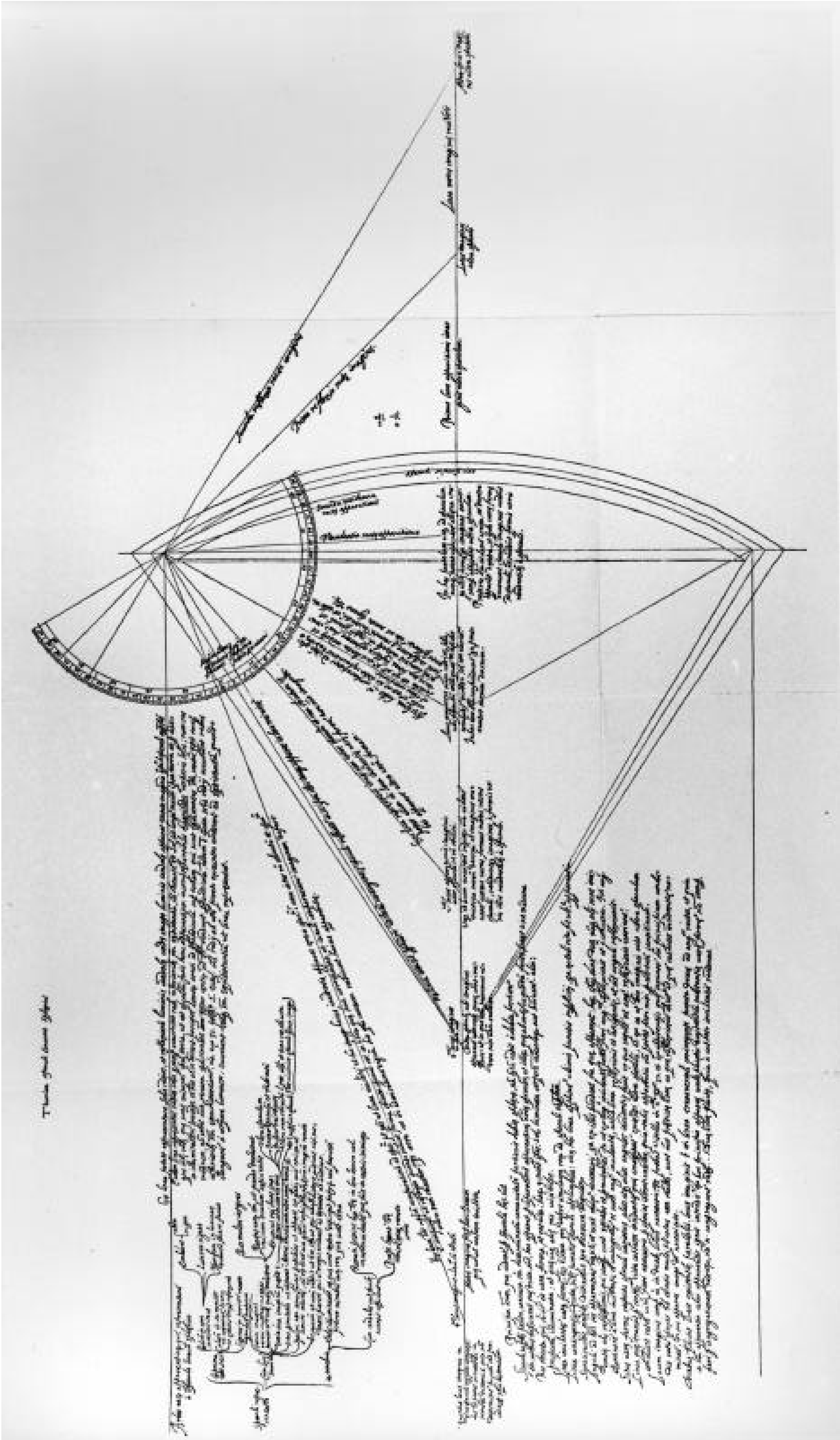


Figure 4.11

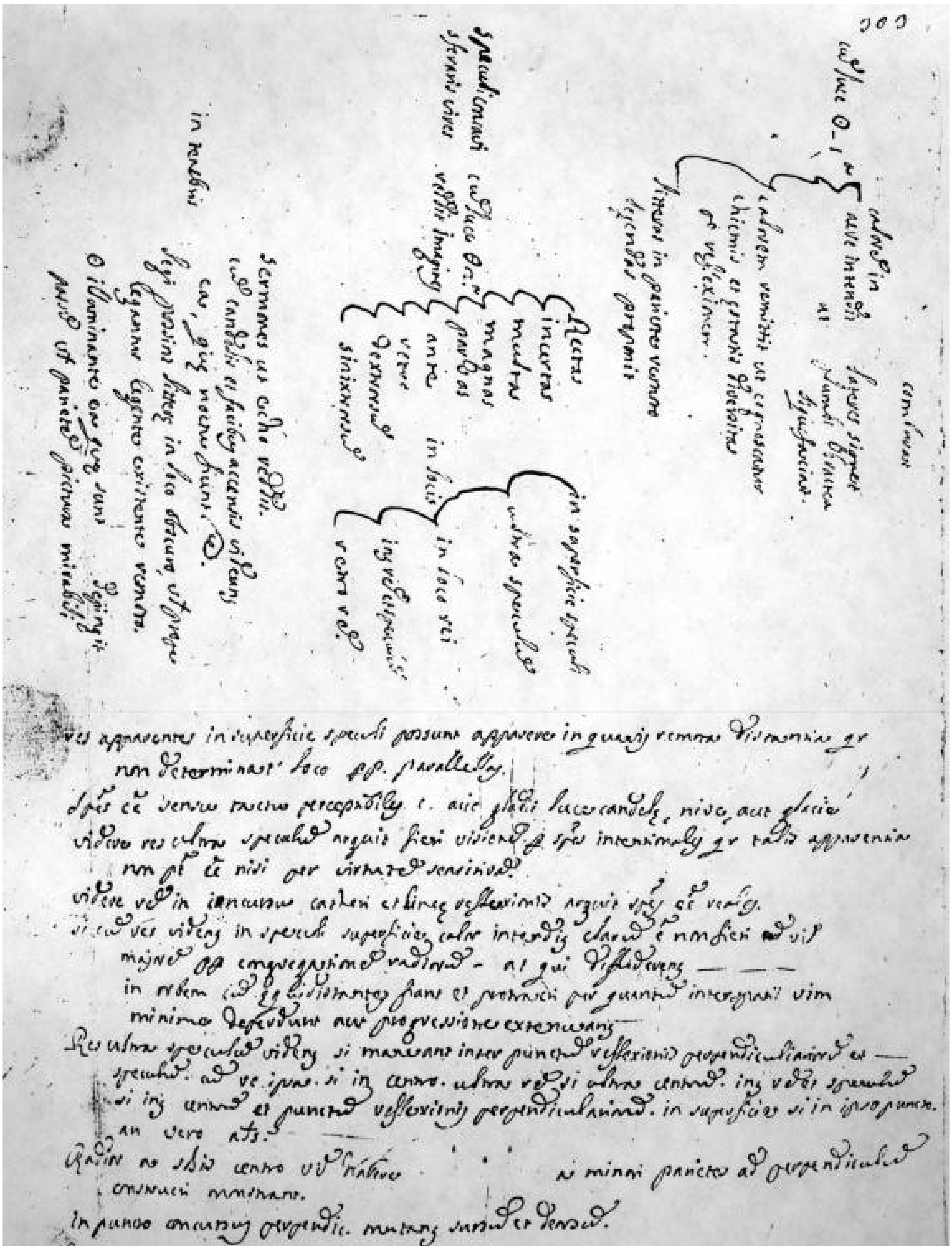


Figure 4.12

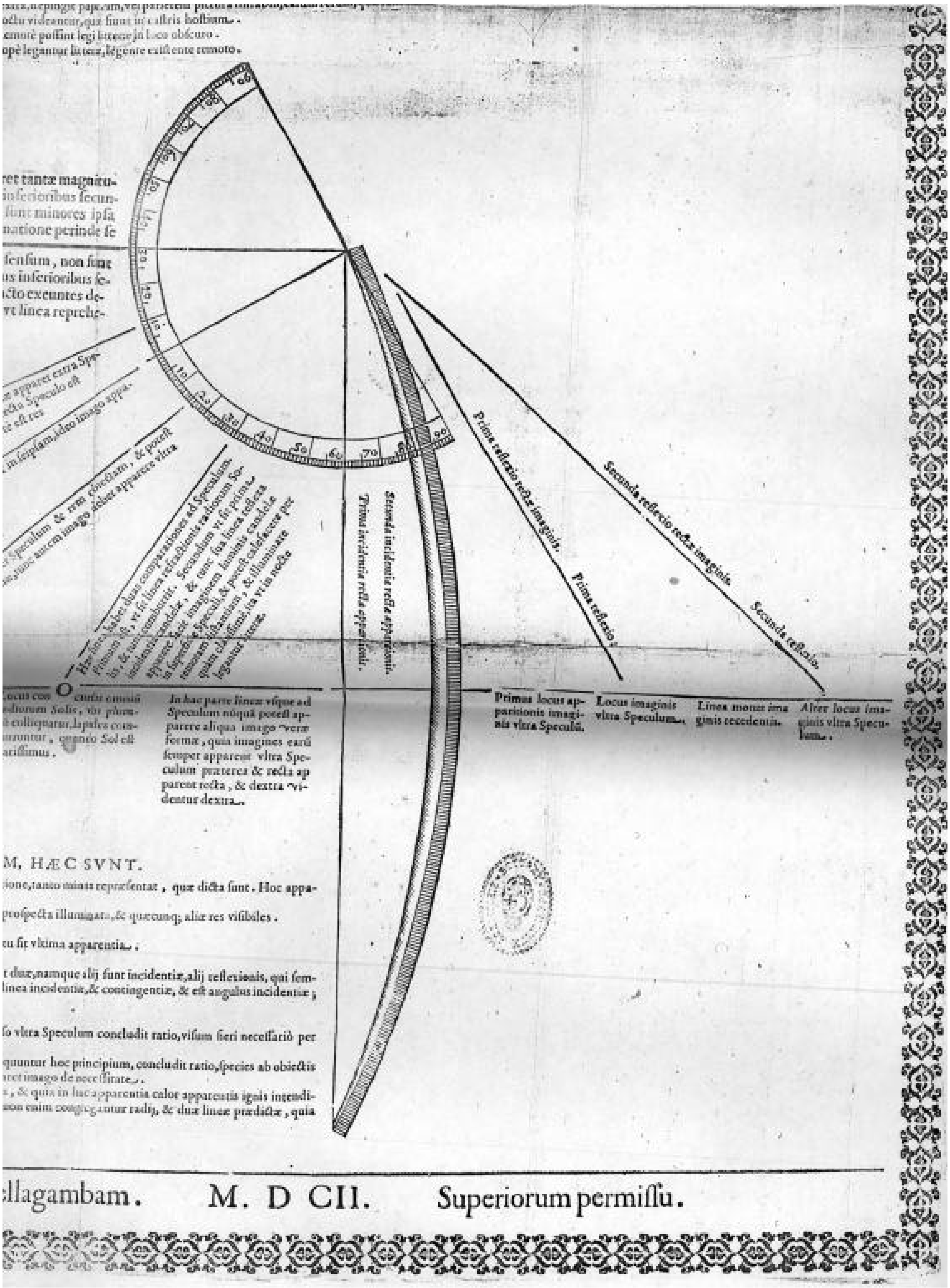


Figure 4.13

Galileo's copy of the 'Theorica' has long been considered to be an original work by Galileo. In the Galileiana 83, it is preserved among other fragments of uncertain date.[67] However, this was not its original position among Galileo's preserved notes. In the index of the manuscript, Antinori, who in the nineteenth century reorganized the Galilean collection, claimed that 'this autographical work of Galileo was found in a book, containing various recipes for lenses and telescopes' and that 'it was not known that Galileo ever dealt with this subject'.[68] Antinori has not always proven himself to be a reliable source beyond doubt. If such a work of Galileo on lenses and telescopes ever existed, it has, unfortunately, not been possible to locate it. However, Antinori's addendum shows that the 'Theorica' was considered an original work by Galileo already in the nineteenth century, which is, presumably, at the basis of later claims for originality.
Favaro continued the nineteenth century tradition to consider the 'Theorica' an original work by Galileo. In his introduction to the facsimile, he dated it to 1610, because he noted similarities with Magini's 'Breve instruttione sopra l' apparenze et mirabili effetti dello specchio concavo sferico', a work on concave mirrors only published in 1611.[69] That Favaro dated the 'Theorica' to 1610 was clearly meant to emphasize Galileo's priority with respect to Magini. Giovanni Antonio Magini (1555-1617) was a professor of astronomy at the University of Bologna, where he was appointed after winning the competition with Galileo. Beside astronomy, he was also active as a geographer and as a designer of mathematical instruments and very large concave mirrors, with circumferences up to 1m50 and a radius of curvature up to 50cm.[70] In this context of mirror design, Magini and Galileo indeed exchanged several letters between 28 September 1610 and 11 January 1611. Magini had sent one of his mirrors to Rudolf II, using Tengnagel, Tycho's son-in-law, as a broker, as early as 1604.[71] However, his attempts to be paid for it or to build some kind of patronage relationship with Rudolf II remained unsuccessful. As late as 1610, Magini was still waiting for his money. Therefore, he decided to change his strategy. Magini presumably informed Galileo about his work on mirrors as early as the latter's visit to Bologna in the summer of 1610 to show Magini his telescopic discoveries.[72] Magini's extant letters, shortly after

[67] These fragments are in Galileo, *Opere*, Vol. 8, pp. 559ff. See Procissi, Angelo. *La Collezione Galileiana della Biblioteca Nazionale di Firenze*. Vol. 1. Roma: Istituto Poligrafico dello Stato, 1959, p. 164.

[68] 'Questo lavoro autografo di Galileo fu ritrovato in un libro, che conteneva diverse ricette p' lenti e telescopii, non era noto che il Galileo si fosse occupato di questo soggetto'. Biblioteca Nazionale (Firenze), Gal. 83, index, presumably Antinori's. On Antinori, see Camerota, Michele. *Gli Scritti De Motu Antiquoria di Galileo Galilei: Il Ms. Gal. 71. Un' Analisi Storico-Critica*. Cagliari: CUEC Editrice, 1992, pp. 28-9.

[69] See Favaro's introduction to the facsimile, in Galileo, *Opere*, Vol. 3, p. 867.

[70] Bònoli, Fabrizio, and Daniela Piliarvu. *I Lettori di Astronomia presso lo Studio di Bologna dal XII al XX Secolo*. Bologna: CLUEB, 2001, pp. 143-7. See also Peruzzi, Enrico. 'Critica e Rielaborazione del Sistema Copernicano in Giovanni Antonio Magini.' In *La Diffusione del Copernicanismo in Italia 1543-1610*, edited by Massimo Bucciantini and Maurizio Torrini, 83-98. Firenze: Leo S. Olschki. For his geographical and cartographical studies, see Camerota, Filippo. *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*. Vol. 14, *Biblioteca della Scienza Italiana*. Firenze: Giunti Gruppo Editoriale, 1996, pp. 262ff.; Bònoli, Fabrizio, and Marina Zuccoli. 'On Two Sixteenth-Century Instruments by Giovanni Antonio Magini (1555-1617).' *Nuncius* 14 (1999): 201-12. On Magini's mirrors, see Favaro, Antonio. *Carteggio Inedito di Ticone Brahe, Giovanni Keplero e di Altri Celebri Astronomi e Matematici dei Secoli XVI e XVII con Giovanni Antonio Magini, Tratto dall' Archivio Malvezzi de' Medici in Bologna*. Bologna: N. Zanichelli, 1886, pp. 161-74.

[71] Ibid., pp. 163-5.

[72] On Galileo's visit to Magini in Bologna, see Drake, Stillman. *Galileo at Work: His Scientific Biography*. Chicago London: The University of Chicago Press, 1978, pp. 159-60.

Galileo's visit, show that he was anxious to engage Galileo as his broker in order to sell one of his large concave mirrors to the Grand Duke of Tuscany, Galileo's new patron, instead.[73] Also, Magini promised to send Galileo two copies of his newly published work to facilitate the deal.[74] It seems Galileo did indeed receive these copies, because in his last letter Magini acknowledged Galileo's praise for his 'Breve instruttione', and Galileo's library contained a copy of the book.[75] However, nothing in this correspondence between Magini and Galileo substantiates Favaro's claim for Galileo's originality or the dating of the 'Theorica' to 1610.

Drake also has considered the 'Theorica' to be an original work by Galileo. However, he disagreed with Favaro's dating it to 1610, because 'at that time Galileo was so busy day and night with the improvement and use of the refracting telescope that the elaborate calculations required for the Theorica are unlikely to have been carried out, nor would they have assisted him in any work he is known to have done at Padua'.[76] Drake's dating was based on what he considered to be the only time during Galileo's career that he dealt with mirrors. Drake argued that the 'Theorica' was written sometime mid-1626, because between 1624 and 1626 Galileo was engaged in discussions about the possibility of a concave mirror to function as a telescope with Marsili and about the focal point of a concave spherical mirror with Guazzaroni.[77] On 20 April 1624, Guazzaroni indeed referred to the determination of the locus of the focal point of a concave spherical mirror at half the radius of curvature of a concave spherical mirror by Magini and Della Porta, and 'spherical aberration' which makes the rays not converge exactly.[78] However, there is nothing in this correspondence to substantiate Drake's claim that Galileo actually wrote the 'Theorica' around 1626 as an answer to questions raised by Marsili and Guazzaroni on the telescopic properties of concave mirrors or on the locus of the focal point (or the point of inversion) of concave spherical mirrors. Without adding reasons of his own, Lindberg has accepted that the 'Theorica' was an original work by Galileo. Lindberg has not assigned a date to it. His comments were limited to an evaluation of its content as 'brief but competent'.[79]

The finding of other copies of the 'Theorica', to which a date can be assigned prior to 1610, has made the claim that it was an original work by Galileo untenable. One of these copies was first

[73] On the mirror for the Grand Duke, see the letters in Galileo, *Opere*, Vol. 10, pp. 437-9, 442-3. Magini and Galileo do not seem to have been on the bad terms suggested by the attack of Horky, a student of Magini, on Galileo's telescopic discoveries. See Baffetti, Giovanni. 'Il 'Sidereus Nuncius' a Bologna.' *Intersezioni* 11 (1991): 477-500.

[74] 'Hora dunque, gli ne mando due copie [of Magini's 'Breve instruttione' which just came off the press], acciò che possa con buona occasione farlo vedere al Ser.mo G. Duca; al quale haverei gusto che col mezo di V. S. toccasse quest' altro mio specchio grande, a confusione di sua Maestà Cesarea, ch' ha fatto starmi male sin hora con le sue vane promesse'. Magini to Galileo, 28 December 1610, in Galileo, *Opere*, Vol. 10, p. 496.

[75] 'le lodi che V.S. ha date al mio trattatello dello specchio concavo'. Magini to Galileo, 11 January 1611, in Galileo, *Opere*, Vol. 11, p. 19. For the copy of the Magini's 'Breve instruttione' in Galileo's library, see Favaro, Antonio. *La Libreria di Galileo Galilei*. Vol. 10, *The Sources of Science*. New York: Johnson Reprint Corporation, 1964, p. 263.

[76] Drake, *Galileo at Work*, p. 304.

[77] Guazzaroni to Galileo, 20 April 1624; Marsili to Galileo, 7 July 1626; Galileo, *Opere*, Vol. 13, pp. 172-4, 330-1.

[78] 'Passando poi alli specchi, et dimandandole io del luogo o punto dove s' accende il fuoco nello specchio concavo, et appartandole il parere del Sig.r Porta, che disse essere nel semidiametro o centro di quella sfera di cui è portione esse specchio, et il parere del Sig. Magino, che disse esser nella quarta parte del medesimo diametro; elle mi rispose che variamente si costituisce il detto luogo del fuoco, secondo la varietà delle sfere di cui li specchi sono segmenti; conseguentemente, che non è il luogo dell' incensione vero nel centro o quarta parte del diametro, ma vario; et che il luogo del fuoco non è punto, ma buono spatio'. Guazzaroni to Galileo, 20 April 1624, in Ibid., p. 172.

[79] Lindberg, 'Optics in sixteenth century Italy', p. 131.

located by Savelli in 1955, but Drake, Lindberg and others seem to have been unaware of it.[80] A firs copy was made by Paolo Sarpi (1552-1623), a friar and theologian, a collaborator and correspondent of Galileo during Galileo's stay in Padua between 1592 and 1610, and a regular visitor of the house of Pinelli, where Galileo and Sarpi must have first met, shortly after Galileo's arrival in Padua.[81] Sarpi's 'Manoscritto dell' Iride e del Calore', to which the copy of the 'Theorica' was added, is a collection of notes and drawings of the colors of the rainbow, reflection in mirrors and refraction through prisms and lenses. As Sosio has argued, Sarpi's 'Manoscritto' was written around 1587 or 1588, because Sarpi's drawings of the rainbow are dated to this time.[82] Also, Sarpi's 'Pensieri', some of them of 1578, others of 1588, show his interest in reflection in spherical mirrors, that is image formation in convex and concave mirrors as well as the heat and burning caused by reflected rays.[83] However, it cannot be excluded beyond doubt that the 'Theorica' was added at a later date to the 'Manoscritto'. Anyway, Galileo's copy of the 'Theorica' was independent of Sarpi's, because Sarpi only reproduced part of it. Sarpi's copy is limited to the chart dividing the phenomena observed in a concave spherical mirror in three distinct categories, presented at the top left corner in Galileo's copy.[84] Moreover, there is no exact word to word match between Sarpi's and Galileo's 'Theorica'. Sarpi often shortened or even paraphrased the wording as found in Galileo's copy, although the similarity is close enough not to doubt that it concerns the same source.[85] Thus, even if Sarpi's copy of the 'Theorica' was added at a later date to the 'Manoscritto', it is more likely that Sarpi copied from Galileo's copy, at a time of their colloboration around 1606, than the other way around.[86]

The third copy was published by Magini in 1602. Since Magini was involved with the design of concave mirrors, the publication must have been meant to serve as a manual on how to use these mirrors to see the 'miraculous effects' which Magini claimed could be obtained with them. It was

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[80] See the references in n. 65, n. 76, n. 79.

[81] On Sarpi, see Sarpi, *Pensieri Naturali, Metafisici e Matematici*, pp. xxiii-clxxi; Cozzi, Gaetano. *Paolo Sarpi tra Venezia e l' Europa*. Torino: Giulio Einaudi Editore, 1979; Sarpi, Paolo. *Lettere ai Gallicani*. Vol. 26, *Veröffentlichungen des Instituts für Europäische Geschichte Mainz*. Wiesbaden: Franz Steiner Verlag Gmbh, 1961; Muraro, Luisa. *Giambattista Della Porta Mago e Scienziato*. Milano: Feltrinelli, 1978, p. 17; Sosio, Libero. 'Galileo Galilei e Paolo Sarpi.' In *Galileo Galilei e la Cultura Veneziana*, 269-312. Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1995; Ferrone, Vincenzo. 'Galileo tra Paolo Sarpi e Federico Cesi: Premesse per una Ricerca.' In *Optics in Sixteenth-Century Italy*, edited by Paolo Galluzzi, 239-53. Firenze: Giunti Barberà, 1984.

[82] Sosio, Libero. 'Il Manoscritto dell 'Iride e del Calore.' In Sarpi, *Pensieri Naturali, Metafisici e Matematici*, 519-546, p. 520. The drawings of the rainbow are dated 12 December 1587, 9 January 1588 and 28 March 1588. See Bibliotheca Marciana (Venice), It. II, 129 (= 4914), f. 305r.

[83] Pensiero 61 deals with the law of equal angles; pensiero 63 with the 'left-right reversal' of an image in a mirror; pensiero 65 with the cathetus rule; pensieri 62, 67, 68, 70, 73 with image formation in plane, convex and concave mirrors. See Sarpi, *Pensieri*, pp. 74-84. Pensieri 53 and 124 deal with the burning mirror. See Ibid., pp. 65-66, 143. All of these pensieri are dated as early as 1578, but Sarpi returned to image formation in a concave mirror and to burning mirrors in 1588, respectively in pensieri 388 and 479. See Ibid., pp. 295, 358.

[84] Bibliotheca Marciana (Venice), It. II, 129 (= 4914), f. 303r. However, Sarpi also paraphrased some of the principles, not in the tree, but written under the large drawing of the mirror. For a transcription of Sarpi's text and a detailed comparative study, see Sosio, 'Il Manoscritto dell 'Iride e del Calore', pp. 521-3, 529-34.

[85] Ibid., pp. 529-34.

[86] Reeves has argued that Sarpi and Galileo did collaborative work on mirrors around 1606 or 1607. For a discussion of this collaboration, see Reeves, Eileen. *Painting the Heavens: Art and Science in the Age of Galileo*. Princeton: Princeton University Press, 1997, pp. 104-112.

Magini, who, in the short introduction, added by Magini to the 'Theorica', and not present in Galileo's copy, claimed that the author of the 'Theorica' was Ettore Ausonio, 'a physician in Venice and a mathematician of some stature'.[87] Magini's publication of the 'Theorica' in 1602 was, however, not the source of Galileo's copy of the 'Theorica', because there is an important difference between Magini's copy and Galileo's copy. Magini was a critical editor of Ausonio's 'Theorica'. He criticized Ausonio for identifying the focal point of the concave spherical mirror, at half the radius of the mirror, as will become evident, with its 'point of inversion'.[88] The 'point of inversion' refers to the point closest to the mirror vertex that produces an inverted image of an object. According to Magini, the point of inversion was the center of curvature, and he carefully corrected all references to the locus of the 'point of inversion' in Ausonio's notes adjacent to the large drawing of the concave mirror in the 'Theorica'. Since this point is the focal point of the mirror, Magini's changes with respect to Ausonio's 'Theorica' appear to be wrong. Even in his 'Breve instruttione', Magini was still convinced that the image of an object placed between the mirror vertex and the center of curvature was always right oriented (and virtual).[89] It is important for our purposes that Galileo's copy of the 'Theorica' does not have Magini's editorial change of the locus of the point of inversion. In Galileo's copy all references to the place of the 'point of inversion' are to the focal point of the concave spherical mirror.[90] Beside this editorial change, and some minor changes of spelling, Magini's and Galileo's copy are identical. Thus, Magini and Galileo both copied from the same source, Ausonio's original. Moreover, from the context of Magini's editorial changes, it is shown that Galileo's copy was closer to Ausonio's original. Since Magini's and Galileo's copy are otherwise identical, it can be concluded that Galileo's copy of the 'Theorica' was, beyond doubt, identical to Ausonio's original.

Unfortunately, Ausonio's original 'Theorica' appears to be not preserved. In the absence of this original, it is, of course, impossible to establish with certainty, but two markings in Galileo's copy are most likely his, and not Ausonio's. It concerns the underlinement of one word, that is 'magnitudo', in the text adjacent to the top ray in the drawing, and two numbers, 10 9/12 and 4 ½ on the right side of the mirror, more or less half way the folio. We will come back to the context and possible meaning of what Galileo added when discussing the content of the 'Theorica'. Notwithstanding the absence of the original, there is no reason to doubt Magini's

[87] 'Hector Ausonius, qui olim Venetijs medicinam exercuit haud infimi subsellij mathematicus' Magini, *Theorica speculi concavi sphaerici*, preface: dedication to Fachinetti.

[88] 'E quidem hanc Theoricam, ut multis partibus meliorem efficere me potuisse scio, sic unum tantum in ea praestitisse fateor, videlicet erratum quoddam non leve tamen sustulisse, quod fortasse per incuriam Ausonius admiserat, nempè in loco concursus radiorum Solarium, in quarta scilicet diametri parte inversionem fieri imaginu, ibiq, omnia confundi, quod sanè non hoc in loco contingit, sed in ipsius Speculi centro, idest in semidiametri termino'. Magini, *Theorica speculi concavi sphaerici*, preface: dedication to Fachinetti.

[89] 'Quando dunque si poneremo tra il centro, & la superficie dello specchio, vederemo sempre la nostra imagine maggiore del vero esser, la quale dal discostamento, che faremo dallo specchio andarà sempre crescendo fuor di modo, vedendosi sempre dietro allo specchio, o profondata in quello sino che pervenendo al centro vederemo gran confusione, & abbarbagliamento, perche l' imagine si rivolta sotto sopra, & ivi non potremo vedere se non il nostro occhio, & ritirandosi indietro oltre il centro comparisce di nuova la nostra imagine in forma grande, ma alla riverscia, & fuori dello specchio'. Magini, Gio. Antonio. *Breve Instruttione sopra l' Apparenze et Mirabili Effetti dello Specchio Concavo Sferico*, Bologna, Gio: Batt. Bellagamba, 1611, p. 23.

[90] In the notes additional to the drawing of the concave mirror in the 'Theorica', Magini added 'ad centrum Speculi omnia videntur inversa' to the locus of the center of curvature, and made Galileo's, 'in hoc loco omnia confunduntur, quia commutantur sursum deorsum' refer to the center of curvature instead of the focal point of the mirror. Compare Magini, *Theorica speculi concavi sphaerici*, and Galileo, *Opere*, Vol. 3, p. 869.

attribution of the 'Theorica' to Ausonio. Ausonio's manuscript notes show numerous similarities with the 'Theorica': a 'Speculum Sphaericum Vitellionis', almost identical, but without drawing, to the central part of the 'Theorica'; a 'Theorica eorum omnium quae apparent ex speculo'; a discussion, similar to the lower part of the 'Theorica', 'Principia omnium quae videntur per speculum'.[91] The most complete discussion is, however, a 'D' una nuova invenzione d' uno specchio', known through two different copies, the other one with a somewhat different title, 'Secreti d' alcune apparenze in uno specchio', similar in content to the 'Theorica' in general.[92] As the 'Theorica', the 'Invenzione' dealt with the five loci of images in a concave spherical mirror, its burning properties, the reflection of sound and heat, its use in combination with a candle to see far or read letters at nights, its use in combination with a camera obscura to project images and the identification of the focal point with the point of inversion, which are all discussed below.[93] Ausonio's 'D' una nuova invenzione d' uno specchio' is given in Appendix I. Since most of these catoptrical notes are among papers that, if dated, belonged to the end of the 1550s or the beginning of the 1560s, Ausonio's 'Theorica' was most likely written around 1560.

If Ausonio's 'Theorica' was written around 1560, when did Galileo copy it? As most of Ausonio's work it was most likely collected in the library of Gian Vincenzo Pinelli.[94] Pinelli arrived in Padua in 1558. He was famous in his own time for his library, but he also possessed a collection of mathematical and astronomical instruments. He had a network of correspondents all over Europe, who also purchased books for him, while his house in Padua functioned as an informal academy.[95] It is not precisely known how Ausonio's manuscripts came in the hands of Pinelli, but Ausonio's work fitted the collection policy of Pinelli. Pinelli had a collection of the manuscript notes of the local professors of mathematics. For example, Pinelli inherited the manuscripts of Galileo's predecessor at the chair of mathematics, Giuseppe Moletti.[96] Moreover,

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[91] *Speculum Sphaericum Vitellionis*, B. A. M., G 120 Inf., f. 37r; *Theorica eorum omnium quae apparent ex speculo*, B. A. M., G 120 Inf., f. 102r;*Principia omnia quae videntur per speculum*, B. A. M., G 120 Inf., f. 102r.

[92] *D' una nuova invenzione d' uno specchio*, B. A. M., A 71 Inf., ff. 20r-21v*; Secreti d' alcune apparenze in uno specchio*, MS D 246 Inf., B.A.M., ff. 1ff.

[93] Ausonio's 'D' una nuova invenzione d' uno specchio' presents also a discussion of burning mirrors as part of an optical explanation of the tides, which is not present in the 'Theorica'. Ausonio claimed to have performed an experiment with a burning mirror and a vessel of water to prove that the heat of the sun caused the waves of the water in the vessel. The tides would then be caused by the heat of the sun as a consequence of the different angles of incidence of the solar rays during the day. 'Si puo con un poco di tempo mostrare con il specchio la causa del flusso e reflusso del mare: ponendo in uno canale lungo del' acqua, et accommodando lo specchio al sole in modo che il punto, che abbrugia dia nell''acqua et in quel ponto ponendo delle pagliuzze, e lasciando cosi per un pezzo l' acqua in quel ponto incominciare di estuare, edi gonfiarsi e mandera le pagliuzze lontane dal punto riscaldato, eda poi movendosi il sole alla contraria parte la sera, poneremo lo specchio dell' altra parte, e riscaldaremo il luogo dove saranno ite le pagliuzze et ivi l' acqua ritornerà ad estuare e gonfiarsi e le pagliuzze ritornerano dove erano la matina, e da q:a apparenze si puo intendere qualm:te li lumi del cielo con il sito delli fondi del mare, e delli liti, montagne, e simil cose atte à far riflesso, et aggregationi de raggi possono in diversi luoghi del mare fare questi ponti, che per il caldo si gonfiano, e fanno il flusso, et il reflusso delli mari con tanta diversità quanta si vede'. B. A. M., A 71 Inf., f. 20v. See Appendix I. For discussion, see Ventrice, *La Discussione sulle Maree*, p. 43.

[94] Rivolta, *Catalogo dei Codici Pinelliani dell' Ambrosiana,* for several references to Ausonio's manuscripts.

[95] On Pinelli, see Stella, Aldo. 'Galileo, il Circolo Culturale di Gian Vincenzo Pinelli e la 'Patavina Libertas'.' In *Galileo e la Cultura Padovana*, edited by Giovanni Santinello, 307-25. Padova: CEDAM, 1992; Yates, Frances A. *Giordano Bruno and the Hermetic Tradition*. Chicago: The University of Chicago Press, 1964, pp. 293, 346.

[96] Giuseppe Moleto studied mathematics with Francesco Maurolico in Messina. After the death of Catena in 1576, he was appointed professor of mathematics at the University of Padua. After he died in 1588, he left his manuscripts to

Pinelli was connected to some members of the Accademia della Fama, of which, as shown, Ausonio was the chief mathematician.[97] Moreover, Pinelli's network, consisting of mathematicians interested in optics, like Contarini, Della Porta and Sarpi, was favorable toward optics. Pinelli, a man of wide diverging interests, had some interest in optics himself. Some of Moletti's notes on optics are in Pinelli's hand, as are notes on optics by Leonardo.[98]

That the 'Theorica' was part of the library of Pinelli appears to be confirmed by Magini's preface to his publication of the 'Theorica'. Magini claimed that 'it was important to publish this Theory, moreover, because it has ever been in danger, and it would certainly have perished, if a friend, for whom I once described it, would not have made a copy for me'.[99] When Pinelli died in 1601, his library was dispersed and part of it was later lost at sea during shipment.[100] Late in 1604, en route to Naples, the ship with the Pinelli collection on board was attacked by pirates, and several chests of books and manuscripts perished. The fate of the Pinelli collection explains why Magini was eager to publish Ausonio's 'Theorica' in 1602. After 1601, Pinelli's library would no longer be accessible to the public, and it appears that Ausonio's 'Theorica' was among the many books and manuscripts lost at sea. Magini was acquainted with Pinelli, if Aquilecchia's identification of Magini's informant in a letter to Porta of 27 July 1594 as Pinelli is correct.[101] A likely candidate for Magini's 'friend', who copied the 'Theorica' for him, is Ercole Bottrigaro. Bottrigaro, the translator of Fine's 'De speculo ustorio', thus, interested in burning mirrors, was a mutual connection of Magini and Galileo.[102] He was in contact with Pinelli around 1600.[103]

Gian Vincenzo Pinelli. For bio-bibliograhical information on Moleto or Moletti, see Carugo, Adriano. 'L' Insegnamento della Matematica all' Università di Padova Prima e Dopo Galileo.' In *Storia della Cultura Veneta: Il Seicento*, edited by Girolamo Arnaldi and Manlio Pastore Stocchi, 151-99. Vol. 4.2. Vicenza: Neri Pozza Editore, pp. 170-85. See also Favaro, Antonio. 'Giuseppe Moletti.' In *Amici e Correspondenti di Galileo*. Edited by Antonio Favaro and Paolo Galluzzi, 1585-1656. 3 vols. Vol. 3. Firenze: Libreria Editrice Salimbeni, 1983; Laird, W. R. *The Unfinished Mechanics of Giuseppe Moleti: An Edition and English Translation of His Dialogue on Mechanics, 1576*. Toronto Buffalo London: University of Toronto Press, 2000, pp. 18-20.

[97] Puliafito, Anna Laura. 'Due Lettere del Pinelli e l' Accademia della Fama.' *Studi Veneziani* 18 (1989): 285-95. See also Artese, Luciano. 'Una Lettera di Antonio Persio al Pinelli: Notizie Intorno all' Edizione del Primo Tomo delle 'Discussiones' del Patrizi.' *Rinascimento* 26 (1986): 339-48.

[98] *Ex libro Josephi Moleti*, B. A. M., R 94 Sup., ff. 145r, 149ff.; B. A. M., N 278 Sup., f. 21r shows Pinelli's notes on the 'Trattato della Pittura' of Leonardo da Vinci. See Rivolta, *Catalogo*, pp. LIII, LX.

[99] 'Mea porrò interesse dixi, hanc edere Theoricam, propterea quod in periculum aliquando veni; nè ea mihi periret, ut certè perijsset, nisi mihi ab amico quodam copia facta foret, cui olim describendam dederam'. Magini, *Theorica speculi concavi spherici*, preface: dedication to Fachinetti.

[100] On the fate of the Pinelli collection and its dispersal after Pinelli's death, see Grendler, Marcella. 'A Greek Collection in Padua: The Library of Gian Vincenzo Pinelli (1535-1601).' *Renaissance Quarterly* 33 (1980): 386-416, pp. 389-90. See also Grendler, Marcella. 'Book Collecting in Counter-Reformation Italy: The Library of Gian Vincenzo Pinelli (1535-1601).' *Journal of Library History* 16 (1981): 143-51.

[101] Aquilecchia, Giovanni. 'La Sconosciuta Metoposcopia di G.B. Della Porta, di una Differenziata del Cardano e di Quella del Magini Attribuita allo Spontoni. *Filologia e Critica* 10 (1985): 307-24, pp. 311-2. See also Aquilecchia, Giovanni. 'In Facie Prudentis Relucit Sapientia': Appunti sulla Letteratura Metoposcopica tra Cinque e Seicento.' In *Giovan Battista Della Porta nell' Europa del Suo Tempo*, edited by Maurizio Torrini, 199-228. Napoli: Guida Editori, 1990, p. 218; Aquilecchia, Giovanni. 'Nuovi Appunti sulla Metoposcopia di Giovanni Antonio Magini.' *Quaderni Veneti* 8 (1988): 109-30, p. 110.

[102] In a letter to Galileo of 11 January 1610, Magini wrote about a letter he had received on Galileo's observations of Venus that 'di che io sono restato a pieno sodisfattisimo, rallegrandomi molto seco di questo discoprimento, che gli apportarà molto honore per il lume che dà all' astrologia et alla filosofia. Ho a punto prestata la lettera di V. S. al Cav.re Botrigaro et ad altri, che l' hanno letta con molto gusto'. Galileo, *Opere*, Vol. 11, pp. 19-20.

Galileo was introduced to Pinelli's circle from the moment of his arrival in Padua in 1592, after a short stay as a professor of mathematics at the University of Pisa, by his patron Guidobaldo del Monte.[104] In fact, Pinelli had been instrumental in Galileo's obtaining of the chair in Padua and Galileo prepared for his inaugural speech in Pinelli's house. Thus, Galileo was introduced to the Pinelli circle in 1592, and the library became inaccessible after Pinelli's death in 1601. Consequently, Galileo must have copied Ausonio's manuscript, between 1592 and 1601. As Drake's quote at the beginning of this section shows, these dates seem not to coincide with any interest in optics of Galileo at that time. In the next chapter, the context of why Galileo copied Ausonio's 'Theorica' will be discussed. First, Ausonio's 'Theorica' will be discussed in the context of the mathematical practitioners' appropriation of medieval optics in the sixteenth century. The 'Theorica speculi concavi sphaerici', in Galileo's, but, as shown, identical to Ausonio's version, is given in Latin transcription and English translation in Appendix II.

3. Image Formation, Focal Point and Point of Inversion in Concave Spherical Mirrors

No formal mathematical proofs are to be expected in the 'Theorica'. In the 'Invenzione', Ausonio concluded that 'of all these things we have their demonstrative reasons which give the causes of all the said appearances, but because this would be a thing of the greatest speculation, it is enough for now, as I have said, to know the effects thereof'.[105] Neither in the 'Theorica', nor apparently anywhere else in Ausonio's optical work, he gave such mathematical demonstrations. Consequently, the 'Theorica' fits the pattern and style established in Ausonio's optical treatise discussed in chapter 3. As it was argued there, Ausonio reorganized Witelo's 'Perspectiva' to the extent of making his refractive dials and concave spherical burning mirrors the 'instrumental proof' from which the science of optics was to be built. Ausonio regarded the 'Theorica' as a reorganization of Witelo's catoptrics along the same lines. Moreover, nothing suggests that he would have considered himself contributing something beyond Witelo's heritage, although, as will become evident, in the process of appropriation of Witelo's catoptrics, he actually did.

Again, Ausonio organized the 'Theorica' along 'sensible' and 'intellectual' principles. They are discussed at the bottom of the folio, under the heading 'principles of all things which are seen through the mirror'.[106] The instrument consists of 'the polished concave surface of the mirror' and a 'semicircle divided into grades' to measure the varying angles of incidence and reflection.[107] Both are represented in Ausonio's drawing. The 'semicircle divided into grades' was clearly a schematised reference to Witelo's instrument to measure angles of incidence and

[103] See his two letters to Pinelli of July 1600 on ecclesiastical music in B. A. M., S 107 Sup., ff.106r-109v.

[104] For Galileo's introduction in Pinelli's circle, see Favaro, Antonio. *Galileo Galilei e lo Studio di Padova*. Padova: Editrice Antenore, 1964, pp. 1: 38-50, 2: 52-9.

[105] 'Di tutte queste cose havemo le sue ragioni demostrative che rendono le cause di tutte predette apparenze ma perche saria cosa di maggior speculatione basterà saperne gli effetti per adesso come vi ho detto'. B. A. M., A 71 Inf., f. 21v. See Appendix I.

[106] *Theorica*. See Appendix II.

[107] Ibid.

reflection.[108] It is interesting to note that the mirror in Ausonio's drawing is a cross-section of a real mirror, and, consequently, it suggests a real instrument. This is quite different from the flat mathematical diagrams, which represent the mirror as a single mathematically curved line, of the medieval tradition. (Figure 4.14)

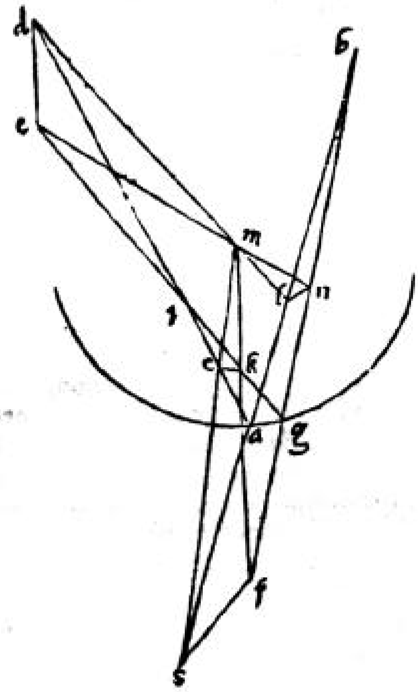

Figure 4.14

The purpose of Ausonio's 'Theorica' was the design of mirrors. This is also evident from one of his 'principles' in which he compared the higher quality of his design of mirrors with 'those that by the artificer are made without any certain measure'.[109] As in his optical treatise, optics is considered the science of mathematical angles, i.c. angles of incidence and angles of reflection. The instrument is said to establish the law of equal angles.

> The required angles of all these appearances. They are of the utmost necessity, because all things which appear are dependent upon them. There are two kinds of these, that is, on the one hand, the [angles of] incidence and, on the other hand, the [angles of] reflection. They are always equal and variable, so that the angle of incidence can be made into the angle of reflection, and the converse is also true. The angles are contained by the line of incidence and the tangent, and this is the angle of incidence; or contained by the angle of reflection and the tangent, and these are the angles of reflection.[110]

[108] Witelo, Perspectiva, book V, proposition 9, translation in Smith, *Witelonis Perspectivae Liber Quintus*, p. 92. For a description of Witelo's instrument to measure angles of reflection, see Ibid., p. 160.

[109] *Theorica*. See Appendix II.

[110] Ibid.

The 'intellectual' principles are the definitions of the line of incidence, which 'brings the images of things to the surface of the mirror', and the cathetus of incidence, which is 'the line through the center' of the mirror.[111] To locate the image, the cathetus rule is applied.

> The place of the image necessarily always is on these two lines, which Witelo demonstrates in proposition 37 of [book] five. … The lines named above are the [line of] reflection and the cathetus of the [line of] incidence. Where they join the image appears with necessity.[112]

The cathetus is the line running from left to right in the middle of the folio. In the text adjacent to this line, Ausonio noted that 'the movement of the image proceeds over this line, named the cathetus of [the line of] incidence'.[113] In the drawing, it is identical to the axis of the mirror. Other lines represent lines of incidence and lines of reflection, often both. It should be noted that the visual organization of Ausonio's 'Theorica' is unprecedented with respect to earlier optical treatises. In medieval optics, diagrams stand apart from the text of the mathematical theorem, with only cross-references between text and diagram denoted by letters of the alphabet. Unlike the medieval optical tradition, Ausonio's text is part of the overall visual organization of the drawing. Taken out of its visual spatial organization, the text becomes meaningless. There might be some influence of the visual and spatial organization of knowledge of Ramist inspiration. As discussed in chapter 2, the Accademia della Fama, of which Ausonio was a member, was highly influenced by the Ramist organization of knowledge. Bolzoni has shown that, also for the Accademia della Fama, the organization of knowledge was directed at 'rendere visibile il sapere', thus, at visualization in Ramist charts.[114] In the top-left corner of the folio, Ausonio organized his knowledge of catoptrics in such a Ramist chart. The integration of text and drawing accomplished a similar spatial organization of knowledge. As will become evident, this spatial visualization allowed the combination of the focal point of the mirror and image location in one drawing.

Where is the focal point of the concave spherical mirror according to Ausonio? It should be pointed out that, for ease of reference, I will occasionally use the terms 'focus' and 'focal point' in this sixteenth century context, while, of course, technically, it is not a focus, because it was not regarded as the locus of an image, until, as has been discussed, Kepler placed his pictures at this focal point. Ausonio referred to the focal point as the 'locus concursus'.[115] In his 'Breve instruttione', Magini argued that the 'Theorica' deserved publication, because Ausonio was the first to calculate the focal point of a concave spherical mirror to be at the 'fourth part of the diameter of the mirror', thus, at a point at half the radius of curvature of the spherical mirror.

> In Antiquity, everyone agreed that the focal point was the center of the mirror, or of the sphere, or the globe, of which it was part, like Euclid, Witelo, Alhazen and the others. However, this is certainly false, because nothing burns in the center of the mirror, nor is there any heat. It's amazing that Antiquity has made this mistake. Thus, the true focal point is in the fourth part of the diameter, as has been noted by Hettore

[111] Ibid.

[112] Ibid.

[113] Ibid.

[114] Bolzoni, ''Rendere Visibile il Sapere'', pp. 67-70; Bolzoni, 'L' Accademia Veneziana', pp. 144-6.

[115] *Theorica*. See Appendix II.

> Ausonio physician and excellent mathematician in Venice in his 'Theory of the concave spherical mirror', at another time published by us.[116]

Proposition 30 of the pseudo-Euclidean 'Catoptrics', considered to be Euclid's in the sixteenth century, confused the center of curvature of a concave spherical mirror with its focal point.[117] According to pseudo-Euclid, there were two modes of propagation, on the one hand, from one point on the surface of the sun to all points on the surface of the mirror, on the other hand, from every point on the surface of the sun through the center of curvature of the mirror falling perpendicularly on the mirror. If the first mode of propagation is assumed, with rays issuing from the point Δ on the sun EZ, the reflected rays from the mirror PH will intersect the axis of the mirror in a point K between the vertex of the mirror B and its center Θ. (Figure 4.15)

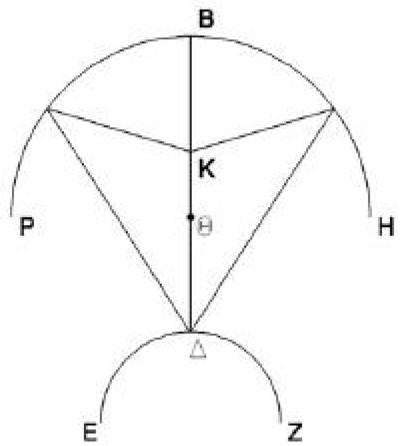

Figure 4.15

[116] 'Gl''antichi tutti concordemente tenero per fermo, che il punto dell' accensione fosse il centro dello specchio, o della sfera, o globo, di cui esso è portione, come Euclide, Vitellione, Alhazen & altri. Ma cio è manifestamento falso, perche nel centro non solo non accende, ma non riscalda punto, si che è stupore, che tutta l' antichità sia stata in questo errore. E dunque il vero sito d' abbruggiare nella quarta parte del diametro, come ho veduto notato da Hettore Ausonio medico & mathematico eccelente in Venetia nella sua Theorica dello specchio concavo da noi altre volte fatta stampare'. Magini, *Breve instruttione,* p. 13.

[117] Euclide. *L' Optique et la Catoptrique*. Translated by Paul Ver Eecke. Paris: Librairie Scientifique et Technique Albert Blanchard, 1959, pp. 122-3. For the problem of attribution, see Knorr, Wilbur R. 'Pseudo-Euclidean Reflections in Ancient Optics: A Re-Examination of Textual Issues Pertaining to the Euclidean Optica and Catoptrica.' *Physis* 31 (1994): 1-45, who has argued that the 'Catoptrics' is to be attributed to Euclid. On this problem of attribution, compare Simon, Gérard. 'Aux Origines de la Théorie des Miroirs: Sur l' Authenticité de la Catoptrique d' Euclide.' *Revue d' Histoire des Sciences* 47 (1994): 259-72; Lejeune, Albert. *L' Optique de Claude Ptolémée dans la Version Latine d' après l' Arabe de l' Émir Eugène de Sicile.* Vol. 31, *Collection de Travaux de l' Académie Internationale d' Histoire des Sciences*. Leiden: E. J. Brill, 1989, pp. 357-62.

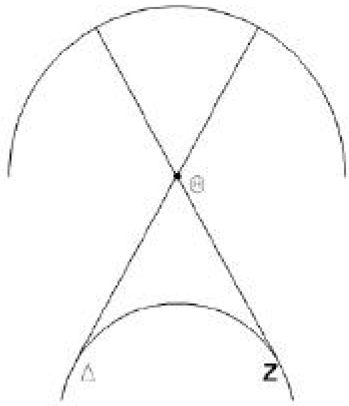

Figure 4.16

If the second mode of propagation is assumed, the rays from Δ and Z on the sun will go through the center of the mirror Θ. (Figure 4.16) These rays will be reflected on itself, and, consequently, intersect the axis of the mirror in its center Θ. Thus, the second part of the theorem is in an unsolved disagreement with the first part. Consequently, there is confusion between the focal point and the center of curvature of the concave spherical mirror.
Proposition 30 of pseudo-Euclid was corrected, already in antiquity, by Diocles in proposition 2 and 3 of his 'On Burning Mirrors', and, again, by Alhazen's 'Discourse on a concave spherical mirror'. Proposition 30 of pseudo-Euclid suggested that the sun was at a close distance. Diocles recognized the immense distance between the sun and the earth, and, consequently, that the solar rays were to be considered parallel. Once this was acknowledged, Diocles proceeded to prove that the reflected solar rays converge at the midpoint of the radius of the mirror.[118] Moreover, both Diocles and Alhazen showed that the solar rays reflected in a spherical concave mirror do not converge to the same point. Using an anachronistic term, they recognized the 'spherical aberration' of a concave spherical mirror. In his 'Discourse on a concave spherical mirror', Alhazen showed in two steps that rays do not have a common focus.[119] First, he showed that the rays which are reflected from a point Z of the mirror, the distance of which to B is equal to the side of a regular octagon inscribed in the circle of which the mirror is a part, is reflected to C, the center of the circle containing Z. (Figure 4.17) Second, Alhazen showed that the rays parallel to the axis, and closer to this axis than the side of a regular hexagon inscribed in the circle of which the mirror is a part are reflected inside the sphere. Moreover, as the angle of incidence decreases, the reflected ray will intersect the axis of the mirror always closer to the centre of the sphere.

[118] Toomer, *Diocles on Burning Mirrors*, pp. 54-63. For discussion, see Knorr, 'The Geometry of Burning Mirrors in Antiquity', pp. 55-60. See also Knorr, 'Archimedes and the Pseudo-Euclidean Catoptrics', pp. 78-83.

[119] Winter, H.J.J., and W. Arafat. 'A Discourse on the Concave Spherical Mirror by Ibn Al-Haitham.' *Journal of the Royal Society of Bengal* 16 (1950): 1-16, pp. 6-8. See also Wiedemann, E. 'Ibn Al Haithams Schrift über die Sphärischen Hohlspiegel.' *Bibliotheca Mathematica* 10 (1910): 293-307; Stiegler, Karl. 'Ibn Al Haythams Entdeckung der Sphärischen Longitudinalen Aberration.' *Physis* 13 (1971): 5-12; Wiedemann, Eilhard. 'Zur Geschichte der Brennspiegel.' *Annalen der Physik und Chemie* 39 (1890): 110-30, pp. 116-20.

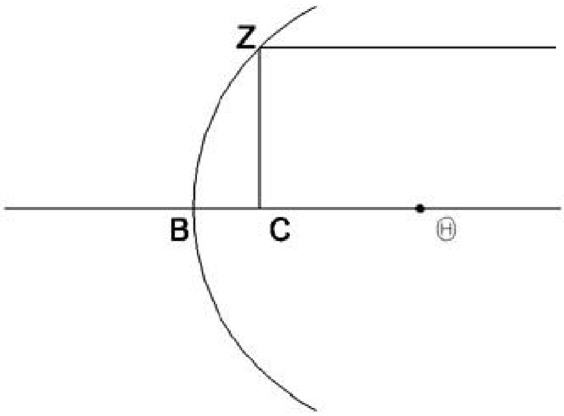

Figure 4.17

In medieval optics, pseudo-Euclid's proposition 30 was the point of departure of Roger Bacon's analysis of the rectilinear propagation of light in his 'On Burning Mirrors'. Bacon devoted his 'On Burning Mirrors' to the last proposition of pseudo-Euclid in order to analyse the modes of propagation of light.[120] Again, according to pseudo-Euclid, their were two modes of propagation, (1) from one point on the sun to all points of the surface of the mirror, and (2) from every point on the surface of the sun to one point on the mirror. Bacon argued that propagation took place according to the first mode. However, this argument was applicable not only to one particular point on the surface of the sun, but to all points on the surface of the sun. Thus, Bacon applied the principle of the punctiform analysis of light, that is, from every point on the surface of the sun a cone of rays radiates. Consequently, according to Bacon, proposition 30 of pseudo-Euclid's 'Catoptrics' was defective, because it did not take into account the infinity of rays actually reaching the point of combustion of the concave spherical mirror.

Next, Bacon only took into account the axes of these cones of radiation. Since the immense distance of the sun, these axes should be considered parallel, from which 'no sensible error results'.[121] Once this sensible parallelism established, he argued that the rays reflected in a concave spherical mirror converge near a point of combustion. Solar axes incident on points of one circle around the central axis converge to one point. He claimed that the furthest of these points of converge is not further from the mirror than half its radius of curvature.

> As for concave spherical mirrors, it is evident from the foregoing that all solar axes incident on one circle about the axis [of the mirror] are reflected to one point; and those incident on another circle are reflected to another point. This was demonstrated above, where it was supposed that multiplication takes place from a

[120] Roger Bacon, 'De speculis comburentibus', propositions 1-2. See Lindberg, *Roger Bacon's Philosophy of Nature*, pp. lxxi-lxxii, 272-89. For discussion, see also Lindberg, David C., 'Laying the Foundations of Geometrical Optics: Maurolico, Kepler, and the Medieval Tradition.' In *The Discourse of Light from the Middle Ages to the Enlightenment*, edited by Davic C. Lindberg, and Geoffrey Cantor, 3-65. Los Angeles: William Andrews Clark Memorial Library, University of California, 1985, pp. 14-5.

[121] Roger Bacon, 'De speculis comburentibus', translation in Lindberg, *Roger Bacon's Philosophy of Nature*, p. 333.

> single point. And if axial multiplication should be along sensibly parallel lines, as was stated, the same result would follow, as can easily be demonstrated. But the [point of] convergence of rays incident on the least possible circle about the axis [of the mirror], which converge at a point further from the surface of the mirror than do any others, cannot be further from the mirror than half the radius of the sphere of which the mirror is a portion. And this too can be easily demonstrated. And these two conclusions are demonstrated in the first two leaves [of this treatise], which were written before the little slips.[122]

Unfortunately, the first part of Bacon's argument that 'solar axes incident on one circle about the axis of the mirror are reflected to one point' was based on pseudo-Euclid's proposition 30, before the parallelism of the solar rays was introduced. In that context, trying to clarify pseudo-Euclid's proposition, Bacon had reflected rays converge up to the center of curvature of the mirror.[123] Moreover, both the mathematical demonstration that this was valid when parallel rays instead of rays diverging from one point, as in the pseudo-Euclidean proposition, were considered, as the mathematical demonstration that the reflected rays did not converge beyond the midpoint of the radius of the mirror are not present in Bacon's 'On Burning Mirrors', as it is known today. The little slips, presumably small leaves or cards, with additional information are missing.[124] If this was also the sixteenth century condition of Bacon's work, it might have been confusing.

In proposition 68 of the eighth book of Witelo's 'Perspectiva', Ausonio's primary source, Witelo borrowed proposition 30 of the pseudo-Euclidean 'Catoptrics'.[125] (Figure 4.18) Consequently, Witelo considered the center of curvature c of the concave spherical mirror abg as its focus or point of combustion. Solar rays dg and za, which go through the center of the mirror, will be reflected on itself, thus, to the center of the mirror. Parallel rays at an equal distance from the axis of the mirror bp will be reflected to the same point on the axis of the mirror between the mirror and its center of curvature, that is between b and c. Finally, 'the closer the radial lines are to the diameter [axis of the mirror], the more they are reflected to a point closer to the center c, and the farther the radial lines are from the diameter, and equidistant from them, the more they are reflected to a point more remote from the center, which is c'.[126] Witelo's failure to apply a punctiform analysis of light, and his location of the sun at a close distance from the mirror, is remarkable, in particular, since in proposition 35 of the second book, he had argued that the solar rays should be considered sensibly parallel, because of the immense distance of the sun from the earth.[127]

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[122] Ibid., pp. 334-7.

[123] Ibid., p. 275.

[124] Ibid., p. 398.

[125] Lindberg, *Opticae Thesaurus*, pp. 365-6.

[126] 'lineae radiales propinquiores diametro, reflectuntur ad punctum propinquius centro c: & lineae radiales remotiores à diametro, & aequidistantes illi, reflectuntur ad punctum remotius à centro, quod est c'. Witelo, Perspectiva, in Ibid., p. 366.

[127] Ibid., pp. 73-4.

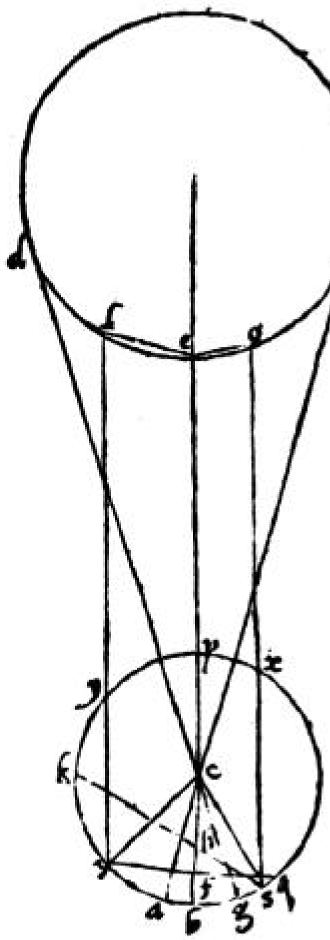

Figure 4.18

It was precisely to this proposition of Witelo that Ausonio referred in his 'Theorica' to show that that the solar rays should be considered parallel. In the text adjacent to the ray parallel to the axis,

> This same line represents the rays of the Sun, which always meet the earthly things parallel, although the Sun is much bigger than the Earth, and, therefore, bigger than all things which are smaller than the Earth, as a mirror is. The size of the solar body makes this appearance incomprehensible. All illuminated things behave the same way, also the walls of houses erected perpendicular on the Earth, which are parallel according to our sense because of the size of the Earth. However, they are not really perpendicular near the center, towards which they go perpendicularly. For the same reason the rays of the Sun meet the earthly things parallel, [or], as in proposition 35 of the second [book] of Witelo, all rays emitted out of one point approximate parallelism, when they move away from the luminous body. Thus, they meet [the earthly things] parallel, as the line represents.[128]

[128] *Theorica*. See Appendix II.

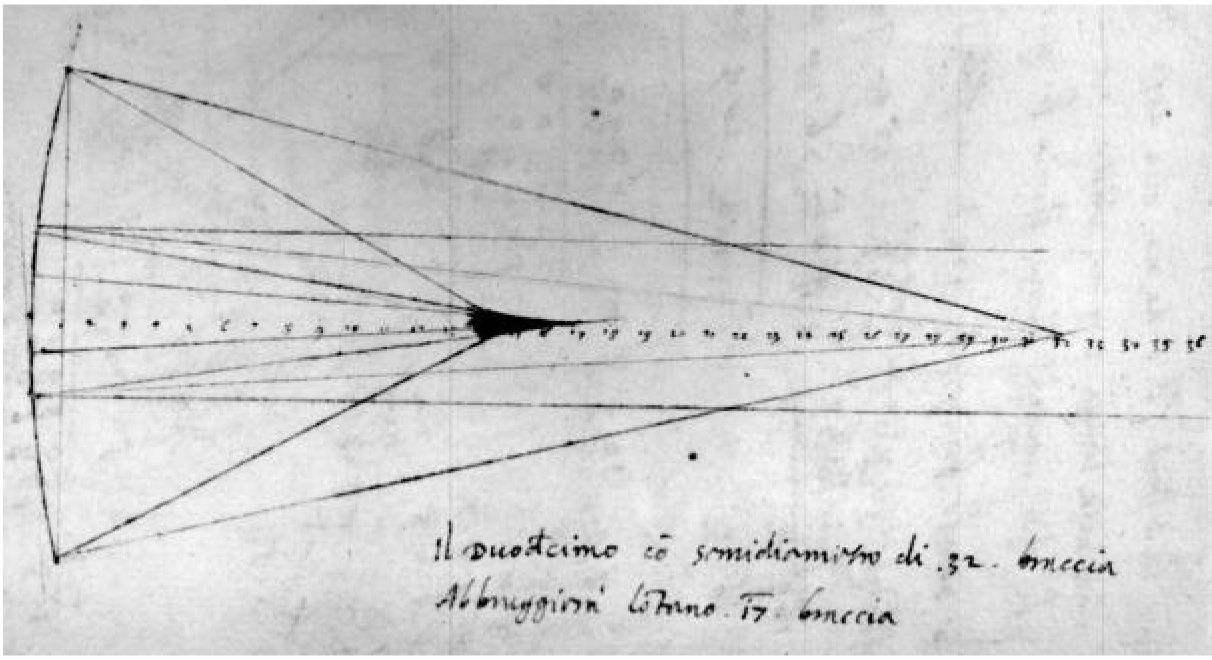

Figure 4.19

These parallel rays are reflected to a point which Ausonio identified as 'the place where all the rays of the Sun come together, where lead is melted and stones are burned, when the Sun is very clear', and which is, more or less, the midpoint between the surface of the mirror and its center of curvature.[129] (Figure 4.19) Thus, Ausonio went beyond Witelo's 'Perspectiva'. It has not been possible to identify Ausonio's source for his determination of the focal point of a concave spherical mirror. As shown, Magini was incorrect to claim that Ausonio was the first to locate the focal point at the midpoint of the radius of curvature of the mirror. However, neither Diocles' 'On burning mirrors' nor Alhazen's 'Discourse on a concave spherical mirror' appear to have been available in Latin in the Middle Ages and the Renaissance.[130] Bacon's 'On Burning Mirrors' might have been an important influence for Ausonio, although, as shown, Bacon did not present a mathematical demonstration of the locus of the focal point. At least, Ausonio's editor, Magini was acquainted with Bacon's optical marvels, in particular, with Claudio Celestino's 'De his quae mundo mirabiliter', as shown in chapter 2, published under the auspices of Fine in 1542, together with what was one of the earliest printed editions of Roger Bacon's 'Epistola de secretis operibus artis et naturae'. In a letter to Galileo of 23 October 1610, Magini complained that his former student Horky had stolen several books from his library. One of these books was Celestino's 'De his quae in mundo mirabiliter eveniunt et de mirabili potestate artis et naturae Rogerii Bachonis', 'in which book this author discussed some beautiful secret of the concave mirror'.[131] Also

[129] Ibid.

[130] Sabra, A.I. *The Optics of Ibn Al-Haytham.* Translated by A.I. Sabra. 2 vols. Vol. 2. London: The Warburg Institute University of London, 1989, pp. xlii-xlv.

[131] 'nel qual libro quest' auttore tocava qualche bel segreto dello specchio concavo, dicendo che si poteva, mediante quello, rappresentare nella luna un concetto da esser inteso da chi stava lontano; ma però non mi raccordo se proponeva cosi detto segreto. Però desidero che V. S. facia dire al S.or Roffeni nella sua Epistola qualche cosa dell'

Leonardo had identified the focal point at half the radius of curvature of the concave mirror but his work on mirrors did not have a wide circulation.[132] Consequently, it must have been unknown to Ausonio.

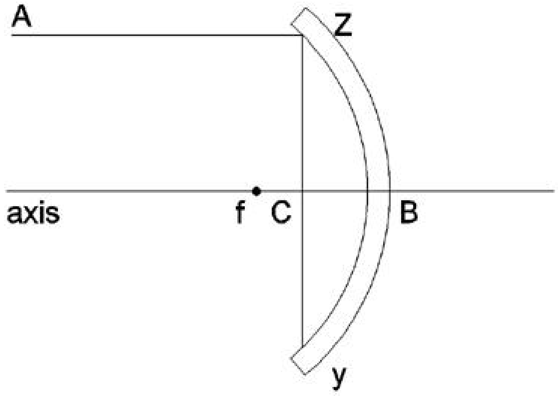

Figure 4.20

Anyway, Ausonio's 'Theorica' shows familiarity with the mathematics of the concave spherical burning mirror. Although at first sight the drawing suggests otherwise, Ausonio did not neglect spherical aberration. No text is added to the line joining the two end points z and y of the mirror's surface. What is the function of this line? (Figure 4.20) The ray az parallel to the axis of the mirror, with the already quoted adjacent text on Witelo's propostion on sensibly parallel solar rays, is to be considered reflected along this line, and not to the focal point of the mirror. Ausonio's drawing is reminiscent of Alhazen's already discussed mathematical demonstration that the rays reflected from a point of the mirror at a distance of the mirror's vertex equal to the side of a regular octagon inscribed in the circle of which the mirror is a part, is reflected to the center of the circle containing the mentioned point of the mirror. Thus, Ausonio's solar ray az parallel to the axis of the mirror hits the mirror's surface in a point z at a distance of the mirror's vertex b equal to the side of an inscribed regular octagon. Ausonio's 'Theorica' only considered paraxial rays within a distance equal to the side of an inscribed regular octagon of the mirror's

infidelità di costui [Horky], il qual so certo che m' ha rubbati questi libri, poi che mi fu scritto da Modena che egli si vantava d' aver tal segreto narrato dal Baccone'. Galileo, *Opere*, Vol. 10, p. 450.

[132] For discussion, see Stiegler, Karl. 'Leonardo's Graphische Methode zur Korrektion der Sphärischen Longitudinalen Aberration bei den Sphärischen Konkaven Spiegeln.' *Physis* 13 (1971): 361-75. Anywa y, around the turn of the sixteenth century, it was not evident that the focal point of a spherical mirror was at the fourth part of its diameter. Kepler criticized Magini for making this claim. Kepler was convinced that the focal point of a concave spherical mirror was in its center of curvature. See Kepler's letter to Brengger, 17 January 1605, in Kepler, Johannes. *Briefe 1604-1607*. Edited by Max Caspar. Vol. 15, *Gesammelte Werke*. München: C.H. Beck'sche Verlagsbuchhandlung, 1951, p. 106. Kepler's dissatisfaction with Magini's optics might explain the reluctance of Kepler's patron, the Emperor Rudolf II, to pay for the mirrors that Magini had sent him.

axis, which are reflected to points between the focal point f and the point c where the reflected ray at a distance equal to the side of an inscribed regular octagon intersects the axis of the mirror.
Ausonio's editors present evidential confirmation that this line perpendicular to the axis was not an accidental element of the drawing without any mathematical or optical significance. First, this line is also present on Magini's edition of 1602, although Magini did not hesitate to erase other lines like the lines of incidence just to the right of this line.[133] Second, the numbers that Galileo added to the drawing seem to be the values of the distances from the mirror's vertex to the intersection point of this line of reflection with the axis of the mirror and its focal point. Depending on the curved line chosen as the mirror's surface, which is ambiguous, the first distance is between 3.6 cm and 4.2 cm. The second distance is between 10.5 cm and 11.1 cm. Galileo's numbers presumably refer to measurements in punti (1 punto = 0.95 cm), the usual standard also used in his work on mechanics.[134] 4 ½punti equals ~ 4.3 cm and 10 9/12 equals ~ 10.2 cm, close enough to the measured values in the drawing. It is actually quite fortuitous to take measurements on the drawing, because the drawing is clearly schematised to fit text and drawing in a visually appealing way on the folio. For example, the focal point is not actually the midpoint of the radius. The distance from the vertex of the mirror to the center of curvature is between 25.6 cm and 26.3 cm, while the distance to the focal point is between 10.5 cm and 11.1 cm. However, that Galileo noted down the distance to the reflected ray with a point of reflection at a distance from the vertex equal to side of an inscribed octagon and to the focal point, shows that this line was not fortuitous and that Galileo understood 'spherical aberration'.
Although Ausonio appears to have been knowledgeable about the mathematics of the determination of the point of combustion and 'spherical aberration', in the 'Invenzione', he proposed experiential evidence of this point of combustion. Also, from experience, he identified this point of combustion with the point of inversion. For its mathematical demonstration, Ausonio referred to Euclid, but, as will become evident, the identification was Ausonio's.

> But if you want to know how this mirror works, hold the mirror toward the sun and make a bit of smoke in the room. By means of this smoke, it will clearly appear how the rays of the sun reflected in the mirror form a pyramid, in the vertex of which a luminous line is seen, …, and in this point it burns. … Beyond this point the images appear inversed, while within this point they are seen right, according to what Euclid in his optics and catoptrics has demonstrated with vivid reasons, and as one can also confirm by experience.[135]

With its identification of the focal point with the point of inversion, Ausonio discussed image formation in the same context as the propagation of light. However, Ausonio's 'Theorica' is not to be confused with Kepler's 'Paralipomena' that introduced the concept of pencils of light, the concept of refractive focus and the concept of a picture, as distinct from an image. Ausonio's conceptual account of image formation is along more traditional lines. In the chart of the 'Theorica', Ausonio made a distinction between primary light, which made the mirror act as a

[133] See the drawing in Magini's *Theorica speculi concavi sphaerici*.

[134] On Galileo's punti-system, see Drake, *Galileo at Work*, pp. 88-90, 129-31.

[135] 'Ma accioche sappiate come questo specchio operi tenerete il specchio al sole è nella camera farete fare un poco di profumo, e per questo fumo apparirà chiaramente come li raggi del sole percotendo nel specchio fanno una piramide nel sommo della quale si vede una linea luminosa, … et in quel punto abbrugia, … fuori di quel punto le imagini appiano riverscie, dentro di quel punto si vedono diritte, secondo che Euclide nella sua optica e catoptrica con vive ragioni ha dimostrato, e come per l' esperienza confirmare si puo'. B. A. M., A 71 Inf., f. 21v.

burning mirror, and indirect or secondary light, in the category of which image formation is discussed.[136] Consequently, in his account of image formation, Ausonio did not consider the Keplerian pencils of light, but the 'reflected object, which is said to be the real form as well as the visible thing, of which there are four kinds: the Sun, luminous bodies, other illuminated prospects of candles or likewise, and whatever other visible things'.[137] Ausonio's 'Theorica' consistently used 'forms' and their multiplication, optically identical with 'species' and their multiplication, in medieval optics after the Baconian synthesis.[138] As discussed in chapter 2, visible species were only one instance of a more general category. Consequently, Ausonio also discussed the reflection of sound and heat. He claimed that the concave spherical mirror 'reflects the heat, so that the difference between winter and summer is known through the reflection'.[139] In the 'Invenzione', again he argued that 'with this mirror one can clearly see the cause of summer and winter and of the seasons in between by holding it in diverse ways toward the sun according the altitude of the sun on the meridian in winter, spring and summer, because the variety of the heat is felt'.[140] Also, 'the species are perceptible through the sense of touch, so that the species emitted by the sharpness of a sword, by the light of a candle, and likewise by snow or freezing ice is felt from a distance through its image'.[141] Finally, the concave mirror reflects sound, that is it 'reflects conversations and voices, so that those who are very far away hear the echo'.[142]

Limiting the account to visible species, and image formation, Ausonio preferred the terminology of 'form', also used by Ausonio's primary source Witelo, who did not use 'species'.[143] There is one exception in Ausonio's discussion of virtual images as evidence for the reality of species. Ausonio argued that the cathetus 'is imagined to extend behind the mirror', and, consequently, 'from this and from the seen location of the image behind the mirror, reason concludes that vision with necessity takes place through species intentionales, because such an appearance cannot happen beside through our sensitive power'.[144] Moreover, 'from the appearances, which follow this principle [cathetus rule], reason concludes that the species multiplied by reflected objects are real'.[145] Virtual images as evidence for the reality of species was a standard argument of the optical tradition. The alternative was Ockham, who denied the reality of species, and, epistemologically absurd from the point of view of medieval optics, asserted that it was the thing itself and not its image that was viewed in the mirror.[146] Consequently, Ausonio accepted the

[136] See the chart of the *Theorica*. See Appendix II.

[137] Ibid.

[138] For the use of 'forms' and 'species', see Lindberg, David C. 'Alhazen's Theory of Vision and Its Reception in the West.' *Isis* 58 (1967): 321-41, pp. 331-41, reprinted in Lindberg, *Studies in the History of Medieval Optics*, 1983.

[139] *Theorica*. See Appendix II.

[140] 'Ancora con questo specchio si puo vedere chiaramente la causa dell' estate e dello verno e delle stagioni di mezo accommodandolo in diverso modo verso 'l sole secondo le altezze meridiane del sole, nel verno nella primavera, e nella state perche si sente la varietà delli calori'. B. A. M., A 71 Inf., f. 21r. See Appendix II.

[141] *Theorica*. See Appendix II.

[142] Ibid.

[143] Lindberg, 'Alhazen's Theory of Vision and Its Reception in the West', p. 333.

[144] *Theorica*. See Appendix II.

[145] Ibid.

[146] Tachau, *Vision and Certitude in the Age of Ockham*, pp. 144-5.

quasi-psychological definition of an optical image of the medieval perspectiva. Again, in medieval optics, the reality of an optical image was denied. It was only a misapprehension. For example, in one of the standard sources, Pecham's 'Perspectiva communis', an image was said to be 'merely the appearance of an object outside its place … it is the object that is really seen in the mirror, although is is misapprehended in position'.[147] On the other hand, Ausonio's identification of the focal point with the point of inversion brought catoptrics beyond medieval optics.

The Euclidean proposition to which Ausonio referred in his 'Invenzione' was proposition 28 of the pseudo-Euclidean 'Catoptrics' concerning image formation in a concave spherical mirror.[148] In this proposition, it was shown by use of the cathetus rule that when the point B is seen by the eye θ, both lying between the center Z of the concave mirror ΑΓΔ and the midpoint N of the radius ΓZ of the mirror, no image will appear, that is the image will be behind the observer at the intersection of BΓ and the cathetus θZ. (Figure 4.21)

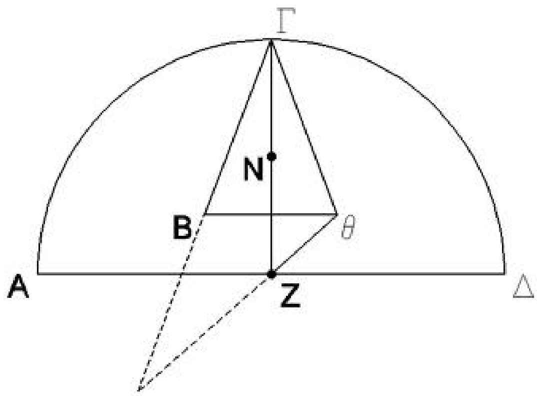

Figure 4.21

However, when the object B and the eye θ are between Γ and N, a magnified virtual image ΠΘ will appear behind the mirror. (Figure 4.22) Consequently, the midpoint N of the radius of the mirror is the point of inversion. Also, as we know, it is the focal point of the concave spherical mirror. However, that pseudo-Euclid was not aware of this, is shown by his discussion in proposition 30, as has been shown, of the center of curvature as the point of convergence of solar rays when reflected in the concave spherical mirror. Thus, in proposition 28 concerning image formation, pseudo-Euclid considered visual rays, and, as already pointed out, these visual rays are not to be confused with luminous rays.

[147] Pecham, Perspectiva Communis, propositio II.19. For translation and discussion of the concept of an image in medieval optics, see Lindberg, *John Pecham and the Science of Optics*, pp. 48, 171.

[148] Euclid, *L' Optique et la Catoptrique*, pp. 118-20.

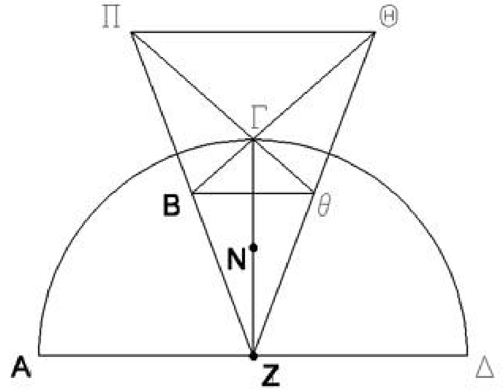

Figure 4.22

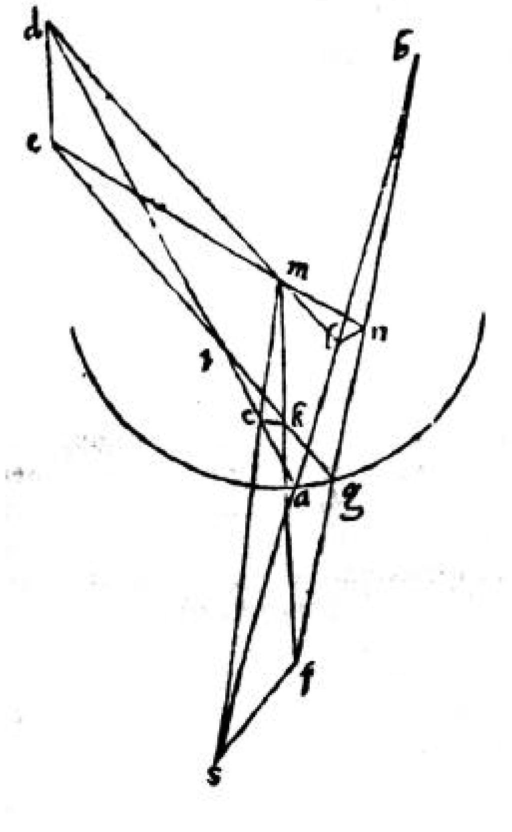

Figure 4.23

The pseudo-Euclidean proposition 28 was still the model for image formation in medieval optics. Ausonio's primary source Witelo again determined the loci of images with the cathetus rule.[149] (Figure 4.23) M is the center of curvature of the concave mirror, B is the eye, DE and CK are objects, both visible by the same rays. Using the cathetus rule to construct the images, LN is the inverted image of DE, and SF is the non-inverted virtual image of CK. Z is the place of intersection of the rays, consequently, it is the point of inversion. As we know, depending upon the position of the observer and the object, the locus of Z will change, but, in general, Z is in the focal plane located about halfway between the mirror and its center of curvature. As pseudo-Euclid, Witelo did not realize that Z was in the focal plane of the mirror. For Witelo, Z was not the point of combustion of the concave spherical mirror. As has been shown, when Witelo identified the point of convergence of the incident light rays, he returned to proposition 30 of pseudo-Euclid's 'Catoptrica', and located the focal point in the center of curvature of the concave spherical mirror. Consequently, the rays in the proposition on image formation are visual rays, while in the proposition on the concave spherical burning mirror the rays are luminous rays.

Ausonio took the analogy between luminous rays and visual rays more literally. The line going from and toward the 'locus concursus' is both a luminous line of incidence of a candle located at the focal point and a luminous line of reflection of the parallel solar rays.

> This line has two relations to this mirror. The first is that it is the line of refraction [reflection] of the rays of the Sun, and then it burns. The second is that it is the first incidence of the candle, and then its reflected line makes the image of the light of the candle appear on the surface of the mirror. Then, it is able to heat over an extended distance, and to illuminate so clearly that letters can be read at night.[150]

Nevertheless, different from Kepler, Ausonio did not consider a picture focussed at the 'locus concursus'. However, as Kepler did when he considered images (as opposed to pictures), Ausonio located the eye at this 'locus concursus'. When the eye is at the focal point, things will appear confused. Different from the optics of antiquity and the Middle Ages, however, Ausonio identified this focal point with the point where the luminous and the visual rays cross.

> The place where all the rays of the Sun come together, where lead is melted and stones are burned, when the Sun is very clear. In this place all things are confused, because they are turned upside down.[151]

Consequently, the focal point, where the luminous rays cross, is the point of inversion, where the visual rays cross. When the visible object is between the focal point or the point of inversion and the concave spherical mirror, a right virtual image will appear behind the mirror.

> In this part of the line until the mirror an image of a real form can never appear, because their images always appear behind the mirror. Moreover, all things are seen right, and things at right appear at right, and when the real forms approach the mirror, their images move towards the mirror, and when the real forms withdraw from the mirror, the images move back.[152]

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[149] Witelo, Perspectiva, book VIII, proposition 53. Lindberg, *Opticae Thesaurus*, p. 355. For discussion, see Lindberg, *John Pecham and the Science of Optics*, pp. 263-4.

[150] *Theorica*. See Appendix II.

[151] Ibid.

[152] Ibid.

Ausonio used the cathetus rule to find the locus of an image. Ausonio mentions double images, 'two images of one thing', and the size of images, 'smaller and larger', but they are not part of his account.[153] He is only interested in image location and orientation. Again, in Ausonio's drawing, the central line running from left to right over the folio was the cathetus, while the other lines served as lines of incidence and as lines of reflection. The drawing is to be understood as highly schematised, with little regard for the geometrical construction, in order to visualize the different loci of the images along the cathetus. Since Ausonio only considered paraxial rays, he applied the cathetus rule in an area where it is valid. From the point of view of modern geometrical optics, Ausonio's drawing can be understood as a discussion of points and images on the central axis of the concave mirror, identical to the cathetus. In the text of the 'Theorica', Ausonio considered 5 different loci of images, (1) 'behind the mirror', (2) 'between the mirror and the reflected object', (3) 'on the place of the reflected object', (4) 'behind the reflected object, that is at its farther side, and then the reflected object is closer to the mirror than its image', and (5) 'on the surface of the mirror'.[154] However, since the loci are more spatially defined in the drawing, the loci can be more refined as (1) if the object is between the focal point and the surface of the mirror, the image is behind the mirror, (2) if the object is beyond the center of curvature, the image is between the center of curvature of the mirror and the focal point, (3) if the object is at the center of curvature of the mirror, the image will also be at the center of curvature, (4) if the object is between the focal point and the center of curvature, the image is beyond the center of curvature, and (5) if the object is at the focal point of the mirror, the image will be judged to be on the surface of the mirror. Consequently, this visual way of describing the loci of objects and images spatially with respect to the focal point was of course absent from medieval optics, but equivalent to the geometrical construction of the loci of images following the ancient cathetus rule.

In the first case, two lines of incidence 'of a right appearance' between the focal point and the surface of the mirror correspond with two lines of reflection and the locations of images behind the mirror.[155] In the second case, a line of incidence beyond the center of curvature, with the adjacent text 'the image of this [line of] incidence appears outside the mirror in the air between the reflected object and the mirror', corresponds with a line of reflection between the focal point and the center of curvature, with the text 'this line represents the image between the mirror and the reflected object', which intersects the cathetus in the 'second location of an image between the mirror and the reflected object'.[156] Ausonio again emphasized that images beyond the focal point are inverted, that is 'to the place where the rays come together all things are seen inverted and upside down'.[157] In both cases, there is no difference between the way Ausonio located images, and the application of the cathetus rule in medieval optics, with the exception that Ausonio's locations are more precise since he had located the focal point of the concave mirror.

In the fifth case, a line of incidence from the focal point, with the adjacent text that 'it is the first incidence of the candle, and then its reflected line makes the image of the light of the candle appear on the surface of the mirror', corresponds with a line of reflection parallel to the cathetus, which represents 'the reflection of the light of a candle, where the image of the light of a candle

153 See the chart, Ibid.

154 Ibid.

155 Ibid.

156 Ibid.

157 Ibid.

appears of the same size as the surface of the mirror'.[158] A fifth location is given on the left side of the folio as 'the fifth location of an image appearing on the surface of mirror. It can be extended over a large distance, because it is not limited by a point, but it proceeds through equidistant [rays]'.[159] This case presented a particular problem to medieval perspectiva and to Ausonio. The line of reflection and the cathetus, at the intersection of which the image is according to the cathetus rule, are parallel. If ΓZ is the radius of the concave mirror $A\Gamma\Delta$ and N is the midpoint of the radius, then the image of the point B seen by the eye Θ, will be on the intersection of the line of reflection ΓB and the cathetus ΘZ, which are parallel. (Figure 4.24)

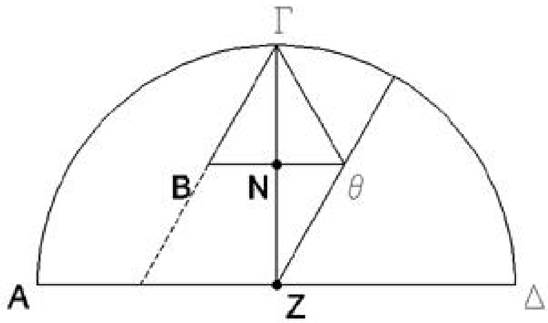

Figure 4.24

As Ausonio explained in the last of his principles at the bottom of the folio of the 'Theorica',

> If the two lines named above are parallel, the location of the image is on the transverse line through their meeting point at right angles [to the two lines]. Because the heat of the appearing fire is directed towards this appearance, it appears from this principle that the multiplication of the accidents of the substance of the real form does not always take place because the rays come together. The rays do not come together, because the two lines named above recede from the cathetus of the [line of] incidence.[160]

Then, where is the image? Since the cathetus and the line of reflection are parallel, in proposition 28 of the 'Catoptrics', pseudo-Euclid concluded that in this case there is no image.[161] However, in agreement with medieval optics, Ausonio argued that the image is perceived on the surface of

[158] Ibid.

[159] Ibid.

[160] Ibid.

[161] Euclid, *L' Optique et la Catoptrique*, p. 119.

the mirror. The same problem arises in modern geometrical optics.[162] Modern geometrical optics predicts that an observer will perceive an infinitely large image behind the mirror at an infinite distance. However, when an observer actually looks at the image of candlelight placed in the focal point of the concave mirror, he will see the image not at an infinite or, for practical purposes, very large distance behind the mirror, but at only a short distance behind the mirror. Consequently, both the cathetus rule of medieval optics and modern geometrical optics are very bad predictors of the perception, including psychological factors, of images.[163] However, since medieval optics was interested in images, as opposed to Kepler's pictures, they corrected the cathetus rule for the actually perceived image in the concave spherical mirror. This is the key to understanding Ausonio's third and fourth case of image location in the concave mirror.

In the third case, from the line of incidence from the center of curvature, Ausonio said that 'when this line is perpendicular to the line tangent to the mirror, it is reflected on itself, therefore, the image appears on the place of the observed thing'.[164] The center of curvature is the 'third location of an image on the place of the object', and 'in the centre of the mirror, where the images appear, together with the things of which they are the images, and there we are not able to see anything beside one eye'.[165] In the fourth case, a line of incidence between the focal point and the center of curvature, with the adjacent text that 'the image must appear behind the real form' corresponds to a line of reflection that 'brings the image behind the reflected object'.[166] Ausonio added that 'it is certain that this deserves the admiration of those, who do not know the cause of this appearance'.[167] This line of reflection intersects the cathetus in the 'fourth location of an image behind the reflected object'.[168] In Ausonio's drawing, this is a location beyond the center of curvature. This is the location of the 'images in the air', which, as discussed in chapter 2, feature prominently in the prefaces of sixteenth century editions of optical works of antiquity and the Middle Ages of Pena, Tanstetter and Apianus, Risner and John Dee. Why?

Case 4 is precisely what the cathetus rule predicts. It is also what modern geometrical optics predicts. However, modern geometrical optics argues in terms of real images, or Kepler's 'pictures', thus, in terms of a real image appearing on piece of paper on a location beyond the center of curvature of the concave mirror. Medieval optics did not. They were interested in what the eye sees. Consequently, medieval perspectivists were reluctant to accept what their geometry, that is the cathetus rule, predicted, because if an actual eye is between the focal point and the center of curvature, the images appear 'behind the eye'. Thus, it is physically impossible to see this image. Henceforward, medieval perspectivist did not accept this image. For example, Pecham claimed that these images 'do not have their appearance certified as does an object [directly] visible, because it is not natural for sight to perceive forms unless they are opposite the

[162] For example, see Pecham, Perspectiva communis, proposition II.39. 'Since T [equivalent to the focal point] is a divisible point, it should appear beyond the mirror according to its higher part, but within the mirror according to its lower part. However, since form is one, it must appear in the intermediate place, namely on the mirror itself at point E'. Translation in Lindberg, *John Pecham and the Science of Optics*, p. 193.

[163] See Ronchi, Vasco. *L' Optique, Science de la Vision*. Paris: Masson et Cie, 1966, pp. 66-79.

[164] *Theorica*. See Appendix II.

[165] Ibid.

[166] Ibid.

[167] Ibid.

[168] Ibid.

face'.[169] It is interesting to note that Ausonio, as opposed to medieval perspectivists, did not have second thoughts about these images. For Ausonio, there is an image 'in the air' beyond the center of curvature, when the object is between the focal point and the center of curvature. Consequently, did Ausonio analyse image formation not exclusively in terms of vision? However, as already discussed, Ausonio's concept of an image was not different from medieval optics, and, consequently still opposed to Kepler's, who, again, was the first to make a distinction between an 'imago', as seen by the eye, and a 'pictura', an image on a piece of paper. Then, how is Ausonio's 'Theorica' to be understood in the absence of this conceptual innovation?

The ambiguity resolves when the drawing in Ausonio's 'Theorica' is understood as the graphical representation of hands-on experience with a concave spherical mirror.[170] If the candlelight O_1O_2 is placed between the focal point F and the center of curvature C of a concave mirror, geometrical optics predicts that a magnified real image I_1I_2 will appear beyond the center of curvature C. (Figure 4.25)

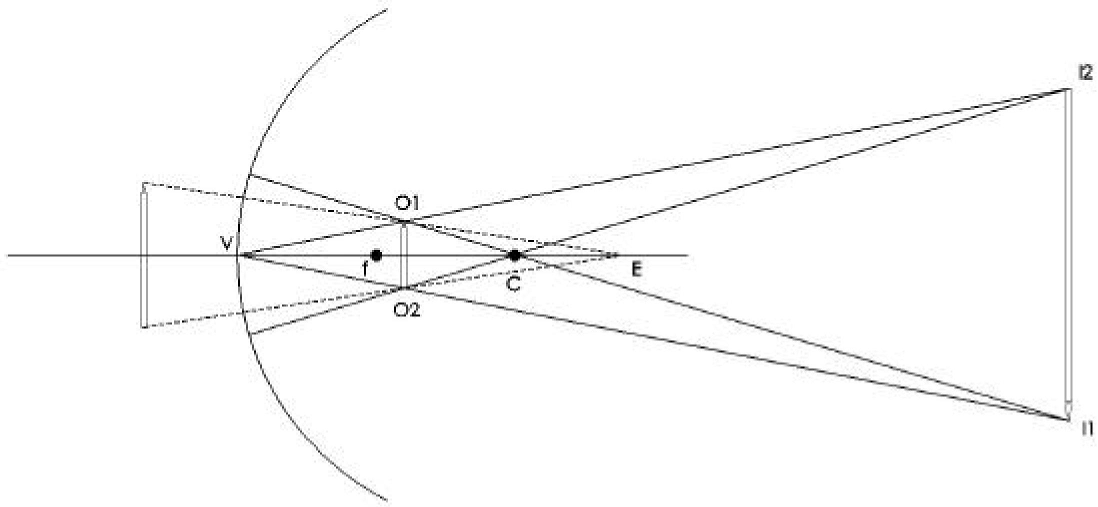

Figure 4.25

[169] 'Unde que apparent in ipso oculo vel retro caput non apparent cum certificatione rei visibilis, quoniam visus non est natus acquirere formas nisi faciliater obiectas'. Lindberg, *John Pecham and the Science of Optics*, pp. 192-3.

[170] In replicating this experience, Ronchi's discussion has been very helpful. See Ronchi, *L' Optique*, pp. 73-6.

Figure 4.26

However, modern geometrical optics does not predict what you will actually see when you apply your eye E at a point beyond the center of curvature C. If the candlelight is placed against the mirror, almost at the vertex V, a virtual and a little magnified image of the same orientation as the candlelight is seen behind the mirror. When the candlelight is removed farther from the mirror, toward the focal point, the right image is still seen behind the mirror, at a growing distance and of a growing size. (Figure 4.26)

Figure 4.27

This is still the case when the candlelight is at the focal point of the mirror. As in the discussion of Ausonio's case 5, the image is seen behind the mirror, not at an infinite distance, and of an infinite size, as modern geometrical optics predicts, but at a little, though still growing, distance behind the mirror and of still growing size. If the candlelight is at a little distance beyond the focal point, the image will collapse in a complete confusion and be spread out over the whole mirror. (Figure 4.27)

If the candlelight is still further removed from the mirror, the image will gradually become less confuse and more distinct. However, the image will now also become progressively smaller, but it is still seen behind the mirror. Moreover, it will now always be inversed with respect to the orientation of the candlelight. (Figure 4.28)

Figure 4.28

Consequently, the locus of the candlelight, a little beyond the focal point of the concave mirror, where the image collapses and which is the point of division between non-inverted and inverted images, is Ausonio's point of inversion, where, as he noted, 'all things are confused, because they are turned upside down'.[171] Moreover, he also noted that, when the candlelight is placed at this point, 'the image of the light of a candle appears of the same size as the surface of the mirror'.[172] Moreover, this practical determination of the point of inversion also allows finding the image of the candlelight 'behind the candlelight'. If a piece of paper is placed at the location of the eye, a magnified inverted image will appear, as modern geometrical optics and the cathetus rule predict. Where the practical experience diverges a little from Ausonio's 'Theorica' is in the exact identification of this point of inversion and confusion with the focal point. As seen, the point of inversion is actually a little beyond the focal point. Consequently, Ausonio has somewhat reorganized his experience to take the analogy between light rays and visual rays literally. Thus, he also made no conceptual distinction between an image as seen by the eye and the image as it appears on paper. As already discussed, it was left to Kepler to make this conceptual distinction.

Henceforward, Ausonio's 'Theorica' is to be understood as the graphical representation of skill.[173] The point of inversion, as the point of maximum confusion, cannot as such be found in the optics of antiquity and the Middle Ages. It was only possible to find it by handling a real concave spherical mirror, as opposed to geometrically construct it with the cathetus rule in medieval optics, and by looking into a mirror. When observing your own face in a concave

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[171] *Theorica*. See Appendix II.

[172] Ibid.

[173] For the background of this notion of skill, see the discussion in chapter 1. See De Mey, Marc. *The Cognitive Paradigm: An Integrated Understanding of Scientific Development*. Chicago London: The University of Chicago Press, 1992, pp. 230-6.

spherical mirror, the effects on the image, from a magnified right-oriented image over complete confusion and the collapse of the image to an inverted progressively smaller image, as you move your face away from the mirror, are easily noticeable. Since Ausonio took the analogy between visual rays and light rays literally, he observed the effects on the image of the candlelight as he moved the candlelight to and fro with respect to the concave mirror. This action of movement of the candlelight and its observed effects on the image are abstracted, not so much in the verbal description, as, visually, in the graphical representation of the drawing of Ausonio's 'Theorica'. Once the effects are graphically represented, it became possible for sixteenth century opticians after Ausonio to obtain the imaging effects without taking recourse to a trial and error procedure.

The difficulty of understanding Ausonio's 'Theorica' is in the fact that such a qualitative notion as the point of inversion or confusion is introduced within a conceptual framework that he appropriated from medieval optics. Ausonio himself never suggested that he introduced the qualitative notion of the point of inversion or confusion within the framework that he appropriated from Witelo, and which linked, apparently unproblematically, the geometry of image location with 'instrumental proofs' claimed to involve measurement, for example, of the law of equal angles or the tabulations of refraction. The difference with the case of the refractive dials is that Ausonio's involvement with refraction as an instrument designer did not result in the introduction of a qualitative notion similar to the point of inversion in the case of his involvement as an instrument designer with concave mirrors. Ausonio's diagram of image location presents itself as a diagram similar to the geometrical diagrams of medieval optics, but, as the schematised organization of lines of incidence and reflection and loci of images suggests that it is not, the visual location of the point of confusion along the cathetus, makes that it is not. On the one hand, the general framework of Ausonio's appropriation of Witelo made the spatial location of the focal point – point of inversion – point of confusion possible, but, on the other hand, it allowed obscuring the conceptual difference between the different optical images that Ausonio used.

It was not until Kepler that Ausonio's reflexive abstraction from his hands-on experience with the concave mirror resulted in the conceptual reorganization of the distinction between 'imago' and 'pictura' and the latter concept would eventually allow in the course of the seventeenth century to show how images are formed by an optical system as the telescope without any recourse to an eye. However, in chapter 6, it will be discussed how the introduction of the notion of point of inversion was connected with sixteenth century instrumental practices in the field of optics that eventually evolved into the invention of the telescope. It will be argued that the invention of the telescope only became possible after Ausonio's introduction of the notion of the point of inversion or confusion. However, in the following chapter, first Galileo's early background in optics will be discussed. As has been shown, Galileo copied Ausonio's 'Theorica' between 1592 and 1601. What did he do with the 'Theorica' of Ausonio and what did Galileo know about optics ten to twenty years before he constructed his first refracting telescope?

V. Galileo, Mathematical Practitioner: His Early Sources on Optics

1. Galileo's Early Sources on Optics: From the Middle Ages to the Renaissance

After having introduced the telescope in his 'Sidereus Nuncius' (1610), Galileo concluded with promising a discussion of the optics of the newly invented instrument in the near future.

> Let it suffice for the present, however, to have touched on this so lightly and to have, so to speak, tasted it only with our lips, for on another occasion we shall publish a comp lete theory of this instrument.[1]

About two months after the publication of the 'Sidereus Nuncius', Galileo wrote to Belisario Vinta in an eventually succesfull attempt to enter the service of Cosimo II, the Grand Duke of Tuscany, as his court mathematician and philosopher, that he had 'several little works on natural philosophical subjects, like *De sono et voce*, *De visu et coloribus*, *De maris estu*, *De compositione continui*, *De animalium motibus*, and still others'.[2] However, no such work of Galileo on vision and colors appears to have been preserved. Could it be that Galileo was only trying to impress the Grand Duke? Crombie and Carugo have argued that Galileo's 'De visu et coloribus' must have been about primary and secondary qualities, while, more to the point, Reeves has speculated that it most likely dealt with the secondary light of the moon.[3] Anyway, there is no suggestion that it contained the 'complete theory' of the optics of the telescope that Galileo had promised in the 'Sidereus Nuncius'. Galileo never published an optical theory of the telescope. The absence of evidence has led to widely diverging opinions on Galileo's knowledge of optics, his acquaintance with the optical tradition and his own understanding of the telescope.

The most widespread scholarly opinion is that Galileo was not acquainted with the medieval optical tradition and, consequently, had little or no knowledge of optics. Ronchi has portrayed Galileo as an outsider in the field of optics, who, precisily because he was an outsider, thrusted the telescope.[4] On the basis of what appears Galileo's reluctance to answer optical questions raised by his correspondent Sagredo and his poor knowledge of the optical work of Kepler, in

[1] 'Haec tamen sic leviter tetigisse, et quasi primoribus libasse labiis, in praesentarium sit satis; per aliam enim occasionem absolutam huius Organi theoriam in medium proferemus'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 62, translation in Galilei, Galileo. *Sidereus Nuncius or the Sidereal Messenger*. Translated by Albert Van Helden. Chicago London: The University of Chicago Press, 1989, p. 39.

[2] 'Ho anco diversi opuscoli di soggetti naturali, come *De sono et voce*, *De visu et coloribus*, *De maris estu*, *De compositione continui*, *De animalium motibus*, et altri ancora'. 7 May 1610, in Galileo, *Opere*, Vol. 10, p. 352.

[3] Carugo, Adriano, and Alistair C. Crombie. 'The Jesuits and Galileo's Ideas of Science and of Nature.' *Annali dell' Istituto e Museo di Storia della Scienza* 8 (1983): 3-68, p. 57; Reeves, Eileen. *Painting the Heavens: Art and Science in the Age of Galileo*. Princeton: Princeton University Press, 1997, pp. 113-8.

[4] Ronchi, Vasco. 'Two Thousand Years of Struggle between Reason and the Senses.' In *Science and History: Studies in Honor of Edward Rosen*, edited by Edward Rosen, 63-81. Wroclaw: Zoklad narodowy im. Ossolinskich, 1978, in particular, p. 77; Ronchi, Vasco. 'The Influence of the Early Development of Optics on Science and Philosophy.' In *Galileo, Man of Science*, edited by Ernan McMullin, 195-206. New York London: Basic Books, Inc., 1967, pp. 199-202; Ronchi, Vasco. *Il Cannocchiale di Galileo e la Scienza del Seicento*. Torino: Edizioni Scientifiche Einaudi, 1958, pp. 111-229.

particular, his 'Dioptrice' (1611), Pedersen has argued for 'Galileo's patent lack of interest in his [Sagredo's] studies in the theoretical field [of optics]', and in optical theory in general.[5] Recently, Hamou has been convinced that 'Galilée manifestement considère que la science de la vision et de la lumière est encore à créer, et caresse certainement le projet de reprendre lui-même l' ensemble de ces problèmes *ab initio*, sans trop tenir compte de ce qu' a pu dire la tradition sur ces sujets mais en s' appuyant sur des recherches expérimentales et mathématiques nouvelles, un peu à la manière de ce qu' il parvient à faire au même moment pour la science du mouvement'.[6]
Lindberg's account appears to have been balanced. On the one hand, he has argued that 'Galileo has never been credited with mastery of the science of optics', on the other hand, without giving any arguments to sustain the claim, he appears to have been convinced that 'Galileo encountered the science of optics'.[7] Machamer has argued, against Feyerabend, that Galileo was familiar with the tradition of medieval optics of Alhazen, Pecham and Witelo, and, above all, the optical work of Maurolico, unpublished until 1611, but maybe known to Galileo via Clavius.[8] However, as also Hamou has pointed out, Machamer's claims on the transmission of Maurolico's manuscript to Galileo are not entirely convincing and rest, eventually, on speculation.[9] Finally, most recently, Zik has been convinced that 'Galileo used a model from one body of theoretical knowledge, namely *Perspective*, and applied its principles into a practical field of knowledge that he acquired from craftsmen', however, without giving the arguments to fully sustain this claim.[10]
Consequently, the arguments pro and contra Galileo's acquaintance with and/or mastery of optics have been based on a distinction between scholars and craftsmen. The former have been associated with the medieval optical tradition, while the latter have been granted only 'rules of thumb' and trial and error procedures to make, for example, the lenses of a telescope. Henceforward, the argument has been that, if Galileo is to be considered knowledgeable on optics, it is the medieval optical tradition with which he should have been acquainted. There is no apparent reason to suggest that he was not. The books in Galileo's library, as compiled by Favaro, show that Galileo, to some extent, had the printed heritage of the medieval perspectiva at his disposal.[11] Galileo owned a copy of Reisner's edition of Alhazen and Witelo (1572). Since there are references to Witelo's 'Perspectiva' in Galileo's work as early as 1606, as will become evident, Galileo must have been acquainted with the work quite early in his intellectual career.[12]

[5] Pedersen, Olaf. 'Sagredo's Optical Researches.' *Centaurus* 13 (1968): 139-50, p. 142.

[6] Hamou, Philippe. *La Mutation du Visible: Essai sur la Portée Épistémologique des Instruments d' Optique au XVIIe Siécle*. 2 vols. Vol. 1: Du Sidereus Nuncius de Galilée à la Dioptrique Cartésienne. Villeneuve d' Ascq: Presses Universitaires du Septentrion, 1999, p. 85.

[7] Lindberg, 'Optics in Sixteenth-Century Italy', pp. 131, 148.

[8] Machamer, Peter K. 'Feyerabend and Galileo: The Interaction of Theories, and the Reinterpretation of Experience.' *Studies in the History and Philosophy of Science* 4 (1973): 1-46, pp. 13-24. For the transmission of Maurolico's optical work to Clavius, see Scaduto, Mario. 'Il Matematico Francesco Maurolico e i Gesuiti.' *Archivum Historicum Societatis Jesu* 17 (1948): 126-41, p. 134.

[9] Hamou, *La Mutation du Visible*, pp. 82-3.

[10] Zik, Yaakov. 'Galileo and the Telescope: The Status of Theoretical and Practical Knowledge and Techniques of Measurement and Experimentation in the Development of the Instrument.' *Nuncius* 14 (1999): 31-67, p. 63.

[11] Favaro, *La Libreria di Galileo*, p. 262.

[12] See the reference to Witelo in his 'Considerations of Alimberto Mauri' (1606), in Drake, Stillman. *Galileo against the Philosophers in His Dialogue of Cecco Di Ronchitti (1605) and Considerations of Alimberto Mauri (1606)*. Translated by Stillman Drake. Los Angeles: Zeitlin & Ver Brugge, 1976, p. 91.

As shown in chapter 2, Reisner's edition is to be considered a sixteenth century appropriation of medieval optics. Moreover, Galileo's references to the Reisner edition show that he endorsed the sixteenth century appropriation of medieval optics. One of the few instances he referred to Alhazen and Witelo, in his annotations to Sizzi's 'Dianoia astronomica, optica, physica' (1611), he cited them precisely for their discussion of the construction of parabolic burning mirrors.[13]

Of other works on optics originally published prior to 1610, and present in Galileo's library, it is less certain whether he was acquainted with them prior to 1609, when he started working on the construction of a telescope. Della Porta's 'De refractione' was published in 1593, but, although, as will become evident in the next chapter, there are good reasons to assume that Galileo was acquainted with it at an early date, no conclusive evidence in terms of early references of Galileo to the 'De refractione' have been found.[14] There is less doubt about two other works on optics present in Galileo's library. Galileo most likely did not acquire copies of those before 1610. Although there had been several earlier editions of Della Porta's 'Magiae naturalis', Galileo owned the Italian edition of 1611.[15] Galileo also owned a 1604 edition of Kepler's 'Paralipomena'.[16] However, Galileo did not see this book before, at least, the end of 1610. In October 1610, he asked Giuliano de' Medici, his contact with Kepler in Prague, to send him a copy of the 'Paralipomena', because, so far, he had not been successful in obtaining a copy in Venice as well as in Florence, to where he had just moved.[17] Thus, the only thing to be concluded from the evidence is that Galileo was not acquainted with Kepler's optical work prior to 1610.

In the light of the scarce evidence as concerns Galileo's early sources on optics, it is surprising that Galileo's copy of Ausonio's 'Theorica', readily available in Favaro's edition of the collected works of Galileo, has not been used as evidence with consequences for this discussion. What Galileo's copy of Ausonio's 'Theorica' suggests is that Galileo was acquainted with medieval optics, not so much directly through an extensive study of the medieval authors on optics, for which conclusive evidence is lacking, but, indirectly, through the appropriation of medieval optics by Renaissance mathematical practitioners. Then, like he saw Alhazen and Witelo through Reisner's sixteenth century appropriation, as has been shown, he would have been, by extension, acquainted with the optics of Renaissance mathematical practitioners. As has been argued, Renaissance mathematical practitioners transcend the historiographical distinction between scholars and craftsmen. One of the cognitive consequences of their appropriation of medieval optics was that they reorganized it as being about instruments. In such a context, it makes little sense to make a strong juxtaposition of medieval optical theory and instruments and limit Galileo's interest to the latter, while denying him knowledge of the former. The gap between those two areas, as will be shown, also as concerns Galileo's telescope, was bridged by Renaissance optics.

In this chapter, it will be argued that Galileo was trained as a mathematical practitioner, with consequences for his knowledge of optics not so different from any of his sixteenth century

[13] 'Alaz. et Vitel. docent constructionem speculi parabolici'. Sizzi, Dianoia astronomica, optica, physica, in Galileo, *Opere*, Vol. 3, p. 239. See also Ronchi, Vasco, ed. *Scritti di Ottica.* Milano: Edizioni Il Polifilo, 1968, p. 409.

[14] Favaro, *La Libreria di Galileo*, p. 263.

[15] Ibid., p. 263.

[16] Ibid., p. 263.

[17] 'Io prego V. S. Ill.ma a favorirmi di mandarmi l' Optica del S. Keplero, e il trattato sopra la Stella Nuova, perchè nè in Venezia nè qua [Firenze] gli ho potuti trovare.' 1 October 1610, in Galileo, *Opere*, Vol. 10, p. 441.

colleagues. Considering Galileo a sixteenth century mathematical practitioner is hardly to be considered without precedent, but as concerns optics, the consequences for his knowledge have been limited to assessing his acquaintance with perspective of painters.[18] First, the evidence of Galileo's mastery of painter's disegno will be reviewed.[19] Second, it will be argued that Galileo was also acquainted with perspective through his knowledge of the design of mathematical instruments, an activity in which he was engaged more deeply than has so far be assumed. Third, it will be shown that, as a mathematical practitioner, Galileo became acquainted with optics, not only perspective, but also catoptrics, as early as 1585, and precisily through the study of the optical work of a Renaissance mathematical practitioner. Finally, Galileo's copy of Ausonio's 'Theorica' between 1592 and 1601 will be set against his early background in optics.

2. Galileo, Perspective and *Disegno*

In 1957, in his paper 'The Role of Art in the Scientific Renaissance', Giorgio de Santillana argued that the artistic tradition of the Renaissance was at the origin of the 'scientific revolution' of the seventeenth century.[20] The research programme that Santillana outlined was overtly ambitious, because its aim was to find the critical element that caused the take off of the scientific revolution of the seventeenth century. In particular, he criticized the then widespread view, inititiated by Koyré, of identifying the 'mathematical method' as the cause of the scientific revolution by arguing that this was nothing more but begging the question.[21] However, Santillana also believed that no accumulation of progress in the crafts could bring the scientific revolution forth. According to Santillana, the gap between the crafts and philosophy was bridged by Brunelleschi's architecture, in particular, by what Alberti understood Brunelleschi's architecture to be. Santillana argued that 'the label that he [Alberti] puts on it is the neoplatonic one. Neoplatonism was orthodox enough to be safe, strong and speculative enough to be new. The mathematical bent of the whole school was so marked, that such a label is hardly unexpected. Here again, we are faced with the fact so well marked by Professor Koyré, that the new thought arising against the old logic is of necessity under the invocation of Plato, and will stay so until and after Galileo'.[22] Thus, Santillana pointed to the theoretical core of neoplatonism and the 'mathematical method' involved in the arts to make the link with Koyré's identification of Galileo with platonism and the 'mathematical method'. Where Santillana disagreed with Koyré was the latter's ahistorical bias, and Santillana identified the arts, in particular architecture, as the cultural factor that introduced Galileo to the revolutionary 'mathematical method'.

[18] For one of the earlier claims, see Olschki, Leonardo. *Galilei und Seine Zeit*. Vol. 3, *Geschichte Der Neusprachlichen Wissenschaftlichen Literatur*. Leipzig: Olschki, 1927.

[19] See also Dupré, Sven. *De Optica van Galileo Galilei: Interactie tussen Kunst en Wetenschap*. Vol. 5 Nieuwe Reeks, *Verhandelingen van de Koninklijke Vlaamse Academie van België voor Wetenschappen en Kunsten*. Brussel: Koninklijke Vlaamse Academie van België voor Wetenschappen en Kunsten, 2001.

[20] Santillana, Giorgio de. 'The Role of Art in the Scientific Renaissance.' In *Critical Problems in the History of Science*, edited by Marshall Clagett, 33-65. Madison: The University of Wisconsin Press, 1959.

[21] Ibid., p. 38.

[22] Ibid., p. 51.

Santillana was much indebted to Panofsky's study 'Galileo as a critic of the arts' of 1954.[23] Panofsky brought to light that Galileo's was a literary and artistic critic. He wrote a 'Considerazioni al Tasso', and a 'paragone' of painting and scultpure for his painter and friend Lodovico Cigoli.[24] Panofsky argued that Galileo's personal involvement with the arts determined his esthetic preference of circles above ellipses, and, consequently, he never accepted the elliptical planetary orbits introduced by Kepler.[25] As discussed in chapter 1, Santillana's and Panofsky's account, based on Koyré's, of Galileo's method and, by extension, of the 'scientific revolution' of the seventeenth century, is no longer tenable. As most clearly seen in the case of Santillana, their choice of art as the origin of the scientific revolution was dependent upon their view that art was able to bridge the gap between the low-level crafts and the high-level philosophical tradition that they considered the core of the scientific revolution. Consequently, their choice of art, above other practical mathematical fields, as the origin of science is ultimately biased by their ignorance of the contributions of Renaissance mathematical practitioners.

Panofsky's innovative study of Galileo and the arts has given rise in more recent years to several studies that have concentrated on the influence of Galileo's drawing skills on his astronomical observations.[26] As has become evident from the analysis in chapter 2 of Dee's 'Mathematicall Praeface', drawing was one of the practical mathematical disciplines in which any mathematical practitioner needed to be skilled. As Cigoli wrote to Galileo, 'a mathematician without *disegno* is not only a mediocre mathematician, but also a man without eyes'.[27] In sixteenth century art theory, disegno was considered to be the conception of a drawing in the artist's mind as well as its material realization on paper.[28] Consequently, disegno also had theoretical connotations, in particular, with the science of perspective. As already discussed in chapter 2, the sixteenth century saw the absolute primacy of perspective. In his preface of the 'Perspectiva libri sex' (1600), of which Galileo received something of a preprint in 1593, Guidobaldo del Monte again stressed this primacy by claiming that 'of these sciences, the arts obtain norm and rule in their manifestations, and they show most clearly that the acquired merit of so many discoveries needs to be attributed to it, and that it is to be considered fully pertinent to the mathematical sciences'.[29]

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[23] Panofsky, Erwin. *Galileo as a Critic of the Arts*. The Hague: Martinus Nijhoff, 1954. See also Panofsky, Erwin. 'Galileo as a Critic of the Arts: Aesthetic Attitude and Scientific Thought.' *Isis* 47 (1956): 3-15; Panofsky, Erwin. 'More on Galileo and the Arts.' *Isis* 47 (1956): 182-85.

[24] For discussion, see Panofsky, *Galileo as a Critic of the Arts*, pp. 4-11. See also Damianaki, Chrysa. *Galileo e le Arti Figurative*. Roma: Vecchiarelli Editore, 2000, pp. 63-93.

[25] Panofsky, *Galileo as a Critic of the Arts*, pp. 20-8.

[26] Edgerton, Samuel Y., Jr. *The Heritage of Giotto's Geometry: Art and Science on the Eve of the Scientific Revolution*. Ithaca London: Cornell University Press, 1991, pp. 223-53; Kemp, Martin. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*. New Haven London: Yale University Press, 1990, pp. 93-8; Reeves, Eileen. *Painting the Heavens*. Princeton: Princeton University Press, 1997.

[27] 'un matematico … trovandosi senza disegnio, sia non solo un mezzo matematico, ma anche un huomo senza ochi'. Cigoli to Galileo, 11 August 1611, in Galileo, *Opere*, Vol. 11, p. 168.

[28] For discussion, see Damianaki, *Galileo e le Arti Figurative*, p. 65.

[29] 'arti che da queste scienze ottennero norma e regola nelle loro manifestazioni, e che chiarissimamente mettono in evidenza che il merito acquisito di tante loro scoperte debba attribuirsi, a pieno merito, e da considerarsi pertinente alle scienze matematiche'. Translated from Sinisgalli's Italian translation. See Sinisgalli, Rocco, ed. *I Sei Libri della Prospettiva di Guidobaldo dei Marchesi del Monte dal Latino Tradotti Interpretati e Commentati da Rocco Sinisgalli*. Roma: "L' Erma" di Bretschneider Editrice, 1984, p. 35. For Galileo's 'preprint', see the letter of Guidobaldo del Monte to Galileo, 10 January 1593, in Galileo, *Opere*, Vol. 10, p. 54.

Figure 5.1

Figure 5.2

The claims for Galileo’s training in disegno and perspective go back to what is known about his education under Ostilio Ricci in the early 1580s, after Galileo had left the University of Pisa. As Settle has shown, Ricci introduced Galileo not only to the study of Euclid and Archimedes.[30] Galileo also studied perspective under Ricci, together with Cigoli, in the house of the artist and engineer Bernardo Buontalenti. Viviani, Galileo’s first biographer, claimed that Galileo busied himself ‘with great delight and marvelous success in the art of drawing, in which he had such great genius and talent that he would later tell his friends that if he had possessed the power of choosing his own profession at that age, … he would absolutely have chosen painting’.[31] Bredekamp has brough some unpublished drawings of female figures by the young Galileo to light that confirm that Galileo was a skilled draughtsman.[32] (Figure 5.1-Figure 5.2) However, their quality is, as will become evident, far surpassed by Galileo’s later drawings of the moon. Galileo’s connection with artistic circles continued, also after he became a famous natural philosopher at the Tuscan court. After 1610, there is not only the continuation of his correspondence with Cigoli, about art as well as astronomical observations, but around 1613, Galileo, like his teacher Ricci, also taught at the Accademia del Disegno of Florence.[33]

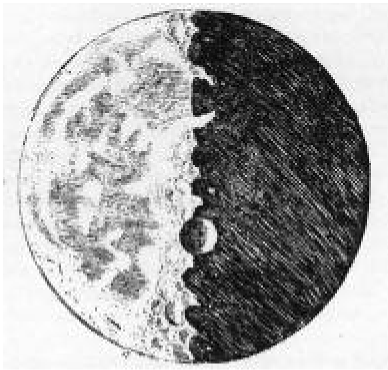

Figure 5.3

Galileo’s drawing skills were not without consequences for his astronomical work. In the ‘Sidereus Nuncius’, Galileo published his engravings of the lunar surface as he had recently observed it through his telescope. (Figure 5.3) They are to be considered the beginning of the acquisition of a new visual language for astronomy, because there appear to have been made very

[30] Settle, Thomas B. ‘Ostilio Ricci, a Bridge between Alberti and Galileo.’ In *XIIe Congrès International d' Histoire des Sciences*, 122-26. Paris: Librairie Scientifique et Technique Albert Blanchard, 1968.

[31] ‘con gran diletto e con mirabil profitto nel disegnare; in che ebbe così gran genio e talento, ch’ egli medesimo poi dir soleva agl’ amici, che se in quell’ età fosse stato in poter suo l’ eleggersi professione, averebbe assolutamente fatto elezione della pittura’. Racconto Istorico di Vincenzio Viviani, in Galileo, *Opere*, Vol. 19, p. 602.

[32] For discussion, see Bredekamp, Horst. ‘Gazing Hands and Blind Spots: Galileo as Draftsman.’ *Science in Context* 13 (2000): 423-62, pp. 430-8. Bredekamp has argued that these drawings should be dated prior to 1584.

[33] Biagioli, Mario. ‘New Documents on Galileo.’ *Nuncius* 1 (1988): 157-69, pp. 166-9.

few naked eye drawings of the moon prior to Galileo.[34] Notable exceptions are the lunar drawings of Jan Van Eyck and Leonardo.[35] (Figure 5.4A – Figure 5.4B)

Figure 5.4A

Figure 5.4B

On these naked eye drawings only the large dark spots are seen, as they are the only dark spots that can be perceived on the moon with the naked eye. However, Galileo pointed out that with the telescope many little bright spots in the dark part of moon and many little dark spots in its bright part were perceived.[36] Edgerton has argued that Galileo perceived these light and dark spots on the moon as evidence for the moon's mountainous surface, because he was familiar with 'chiaroscuro', in particular with the depictions of shadows casted by very complex illuminated regular bodies, which are found in contemporaneous manuals on painter's perspective, for example, of Sirigatti.[37] (Figure 5.5)

[34] See Winkler, Mary G., and Albert Van Helden. 'Representing the Heavens: Galileo and Visual Astronomy.' *Isis* 83 (1992): 195-217. For a broader seventeenth century perspective on the visual language of astronomy, see also Winkler, Mary G., and Albert Van Helden. 'Johannes Hevelius and the Visual Language of Astronomy.' In *Renaissance and Revolution: Humanists, Scholars, Craftsmen and Natural Philosophers in Early Modern Europe*, edited by J.V. Field and Frank A. J. L. James, 97 - 116. Cambridge: Cambridge University Press, 1993.

[35] Montgomery, Scott L. 'The First Naturalistic Drawings of the Moon: Jan Van Eyck and the Art of Observation.' *Journal for the History of Astronomy* 25 (1994): 317-20; Reaves, Gibson, and Carlo Pedretti. 'Leonardo Da Vinci's Drawings of Surface Features of the Moon.' *Journal for the History of Astronomy* 18 (1987): 55-58. I would like to thank Marc De Mey for pointing the image of moon in Jan Van Eyck's 'Chancellor Rolin'.

[36] Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, pp. 62-9.

[37] Edgerton, *The Heritage of Giotto's Geometry*, in particular, pp. 226-33. See Sirigatti, Lorenzo. *La Pratica di Prospettiva*. In Venetia: Per Girolamo Franceschi Sanese, 1596, plates 54-65. For placing Edgerton's claim in a broader perspective of art and science, see also Lüthy, Christoph. 'Die Kunst der Renaissance als Voraussetzung für die Moderne Wissenschaft?' *Zeitschrift für Ästhetik und allgemeine Kunstwissenschaft* 37 (1992): 215-24.

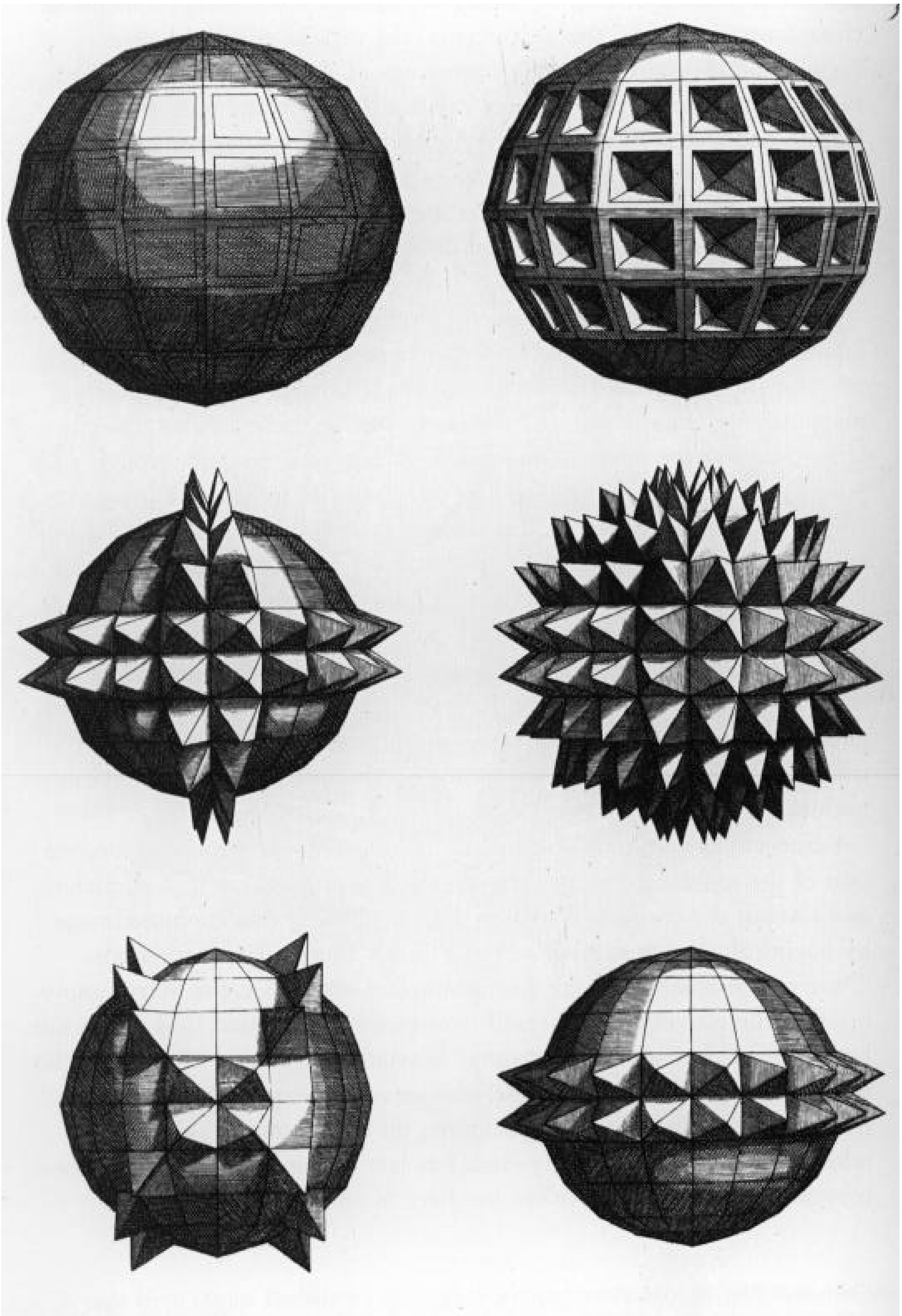

Figure 5.5

Contemporaneous topographical maps used similar 'chiaroscuro' techniques. It was a pictorial convention to use little 'molehills' lit from one side, mostly the left side as at sunset, and casting their shadow to the other side to represent a mountainous area. In the 'Sidereus Nuncius', Galileo used the circular valley of Bohemia as a terrestrial analogy for the lunar Albategnius crater, whose dimensions, in comparison to his watercolors of the moon, he greatly exaggerated to show the division of light and shadow inside the lunar crater.[38] (Figure 5.6)

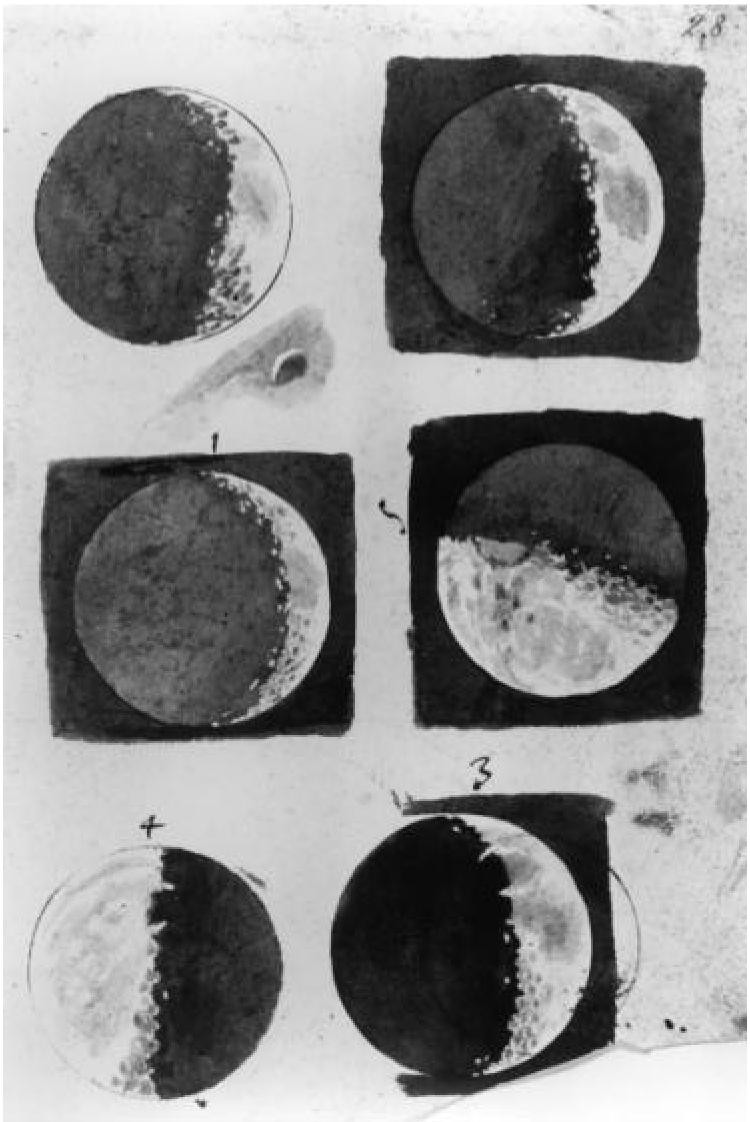

Figure 5.6

[38] For Galileo's exaggaration of the Albategnius crater, see Gingerich, Owen. 'Dissertatio cum Professore Righini Sidereo Nuncio.' In *Reason, Eexperiment and Mysticism in the Scientific Revolution*, edited by M.L. Righini-Bonelli and W.R. Shea, 77-88. London: MacMillan, 1975, p. 84; Shea, William R. 'Galileo Galilei: an Astronomer at Work.' In *Nature, Experiment and the Sciences: Essays on Galileo and The History of Science in Honour of Stillman Drake*, edited by T.H. Levere & W.R. Shea, 51-76. Dordrecht: Kluwer Academic Publishers, 1990, p. 56.

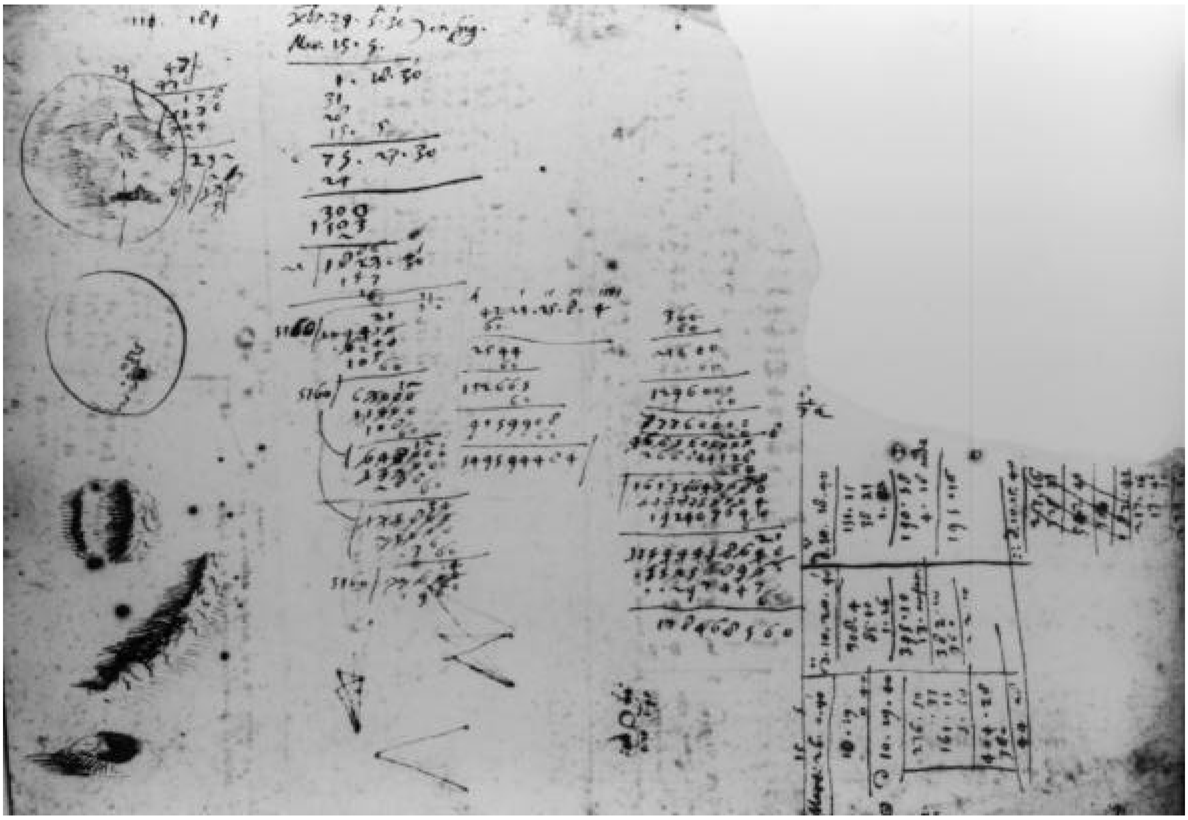

Figure 5.7

Another unpublished drawing shows Galileo's intense study of the crater's shadows from a bird-eye perspective as if he was flying closely over the lunar surface instead of looking at it from the earth. (Figure 5.7)

> The area around the middle of the Moon is occupied by a certain cavity larger than all others and of a perfectly round figure. … It offers the same aspect to shadow and illumination as a region similar to Bohemia would offer on Earth, if it were enclosed on all sides by very high mountains, placed around the periphery in a perfect circle. For on the Moon it is surrounded by such lofty ranges that its side bordering on the dark part of the Moon is observed bathed in sunlight before the dividing line between light and shadow reaches the middle of the diameter of that circle. But in the manner of the other spots, its shaded part faces the Sun while its bright part is situated towards the dark part of the Moon, which is to be esteemed as a very strong argument for the roughness and unevennesses scattered over the entire brighter region of the Moon.[39]

[39] 'medium quasi Lunae locum a cavitate quadam occupatum esse reliquis omnibus maiori, ac figura perfectae rotunditatis; … eundem, quo ad obumbrationem et illuminationem, facit aspectum, ac faceret in terris regio consimilis Bohemiae, si montibus altissimis, inque peripheriam perfecti circuli dispositis, occluderetur undique; in Luna enim adeo elatis iugis vallatur, ut extrema ora tenebrosae Lunae parti contermina, Solis lumine perfusa spectetur, priusquam lucis umbraeque terminus ad mediam ipsius figurae diametrum pertingat. De more autem reliquarum macularum, umbrosa illius pars Solem respicit, luminosa vero versus tenebras Lunae consituitur; quod tertio libenter observandum admoneo, tanquam firmissimum argumentum asperitatem inaequalitatumque per totam Lunae clariorem plagam dispersarum'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 68, translation in Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 47.

Figure 5.8

The moon's crater showed the same appearances as a valley on earth at sunrise. Montgomery has argued that Galileo must have seen the basin of Bohemia depicted like this on the corresponding map in Ortelius' atlas 'Theatrum Orbis Terrarum', of which he owned a copy.[40] (Figure 5.8) Other types of maps might have influenced Harriot, who contemporaneous to Galileo, observed the moon through a telescope. Harriot's maps do not show the 'chiaroscuro' rendering of the lunar surface. Alexander has argued that Harriot's representations of the moon, which appear to show coastal outlines much like a division of water and land, reflect his practice of mapping the coast of Virginia, together with the painter John White, rather than show his ignorance of chiaroscuro.[41] (Figure 5.9) However, as soon as Harriot became acquainted with Galileo's engravings of the 'Sidereus Nuncius', he started drawing the lunar surface in the same way.[42]

[40] Montgomery, Scott L. *The Scientific Voice*. New York London: The Guilford Press, 1996, pp. 223-9. See Ortelius, Abraham. *The Theatre of the Whole World*, edited by R.A. Skelton. Amsterdam: Theatrum Orbis Terrarum Ltd., 1968. For Galileo's copy, see Favaro, *La Libreria di Galileo Galilei*, p. 261.

[41] Alexander, Amir. 'Lunar Maps and Coastal Outlines: Thomas Hariot's Mapping of the Moon.' *Studies in the History and Philosophy of Science* 29 (1998): 345-68. England might not have been the barren land as concerns perspective for which Egerton took it. On contemporary English topographical maps, for example, Saxton's, there is a 'chiaroscuro' rendering of the mountains. However, these topographical maps were often engraved by Flemish engravers. Also the engraving by De Bry, the Flemish publisher of Harriot's 'A briefe and true report', of White's map shows a 'chiaroscuro' rendering of the mountains. However, White's map does not show as much land inwards, and other drawings show that he did not master perspective. For White's drawings, see Hulton, P. H., and Quinn, D. B. *The American drawings of John White, 1577-1590*, Chapel Hill: University of North Carolina Press, 1964; for De Bry's engravings, see Harriot, Thomas. *A Briefe and True Report of the New Found Land of Virginia: The Complete 1590 Theodor De Bry Edition*. New York: Dover Publications Inc., 1972.

[42] Bloom, Terrie F. 'Borrowed Perceptions: Harriot's Maps of the Moon.' *Journal for the History of Astronomy* 9 (1978): 117-22, pp. 117-8. On Cigoli's moon, also derivative of Galileo's, see Booth, Sara, and Albert Van Helden. 'The Virgin and the Telescope: The Moons of Cigoli and Galileo.' *Science in Context* 13 (2000): 463-88.

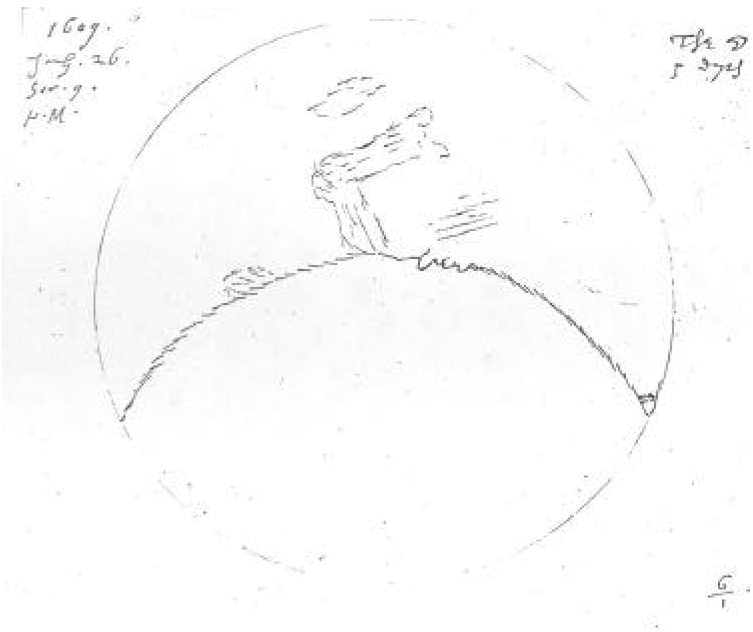

Figure 5.9

Other lunar features were also used by Galileo to argue for the roughness of the moon's surface. Already in 1606, in his 'Considerazioni d' Alimberto Mauri', Galileo used as proof for the roughness of the lunar surface the naked eye observation that the terminator of the moon was irregular. He argued that 'for proof of this [the roughness of the lunar surface] I shall adduce an easy and pretty observation that can be made continually when she is in quadrature with respect to the sun; for then the semicircle is not smooth and clean, but always has a certain boos in the middle'.[43] However, in the 'Sidereus Nuncius', the problem arised that, if the lunar surface was considered rough, then the moon's circumference should apparently have to look like a jagged wheel, while no such irregularity was noted when looked at it through the telescope. To solve this contradiction, Galileo postulated in the 'Sidereus Nuncius' that the moon is surrounded by an atmosphere, just like the earth.[44] To reach the surface of the moon, our visual rays have to traverse more of this atmosphere at the moon's periphery, than in the middle of it, because they intersect the atmosphere more obliquely. Galileo argued that, inhibited by the moon's atmosphere, the mountains at its periphery were not seen. Thus, the moon did not appear with an irregular circumference. The argument was convincing, if one was willing to accept a lunar atmosphere. However, Galileo soon learned that adversaries of lunar irregularity used the lunar atmosphere to accommodate Galileo's moon observations to Aristotelian cosmology by proposing a mountainous lunar surface surrounded by a perfectly regular crystal sphere, invisible, but completely smooth.[45] Consequently, Galileo tried to account for the regular circumference of the moon, without postulating a lunar atmosphere in a letter to Grienberger of September 1611.

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[43] Galileo, Considerazione d' Alimberto Mauri. Translation in Drake, *Galileo against the Philosophers*, p. 104.

[44] Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3, p. 70. Translation in Galileo, *Sidereus Nuncius*, p. 50.

[45] For discussion, see Reeves, *Painting the Heavens*, pp. 155-8.

Hallyn has argued that Galileo's argument in this letter was based on his familiarity with anamorphoses.[46] In his 'Considerazione al Tasso', Galileo noted that anamorphoses 'show a human figure when looked at sideways and from a uniquely determined point of view but, when observed frontally as we naturally and normally do with other pictures, display nothing but a welter of lines and colors which we can make out, if we try hard, semblances of rivers, bare beaches, clouds, or strange chimerical shapes'.[47] Galileo's description resembled the circulating images of powerful men, whose anamorphic images were hidden in what is a landscape-like picture when looked at under normal viewing conditions. For example, in Florence, Accolti claimed to have made such an anamorpic picture of Cosimo II, Grand Duke of Tuscany.[48] Galileo's argument in his letter to Grienberger was based on the same differentiation between a non-lateral 'in faccia' and a lateral 'in scorcio' or 'delineate' viewpoint as in his description of anamorphoses. He observed a difference between the terminator of the moon when in quadrature and when in a gibbous phase. In the former case, the terminator appeared 'in faccia', while in the latter case it appeared 'in scorcio'. (Figure 5.10)

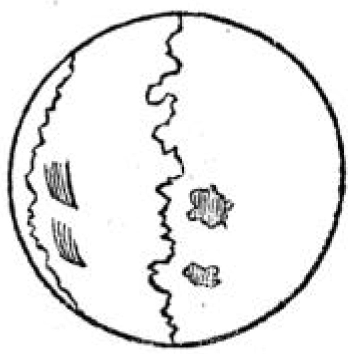

Figure 5.10

Due to the effects of foreshortening from a lateral point of view, near the circumference, the sinuous terminator lost some of its irregularity. The terminator 'loses much of its width and appears, although long, compressed and narrow, because the visual ray is little elevated above the boundary'.[49] The closer to the circumference of the moon, the more the terminator will lose its irregularity, until, at the visible circumference of the moon, it would appear circular without any irregularity at all. Thus, by exploiting the well-known anamorphic effect, Galileo had an

[46] Hallyn, Fernand. 'Le Regard Pictural de Galilée sur la Lune.' *Créis* 2 (1994): 25-41, pp. 33-4.

[47] 'riguardate in scorcio da un luogo determinato mostrino una figura umana, sono con tal regola di prospettiva delineate, che, vedute in faccia e come naturalmente e comunemente si guardano le altre pitture, altro non rappresentano che una confusa e inordinata mescolanza di linee e di colori, dalla quale anco si potriano malamente raccapezare imagini di fiumi o sentier tortuosi, ignude spaggie, nugoli e stranissime chimere'. Galileo, Considerazioni al Tasso, in Galileo, *Opere*, Vol. 9, pp. 129-30, translation in Panofsky, 'Galileo as a critic of the arts: Aesthetic attitude and scientific thought', p. 6.

[48] Accolti, Pietro. *Lo Inganno de gl' Occhi*. Edited by Theodore Besterman. Vol. 14, *The Printed Sources of Western Art*. Portland, Oregon: United Academic Press, 1972, p. 49.

[49] 'perdono assai della larghezza, et appariscono lunghe sì, ma strette et sottili, perchè pochissimo si gli eleva il raggio visuale' Galileo to Grienberger, 1 September 1611, in Galileo, *Opere*, Vol. 11, p. 188.

argument for a circular circumference, without having to postulate a lunar atmosphere, and still allowing a lunar surface fully covered with mountains.

Kemp has argued that Galileo used a similar argument, when, shortly thereafter, Galileo became involved in a controversy on sunspots with Scheiner.[50] In 1611, the Jesuit Christoph Scheiner sent three letters to Welser about his observations and interpretation of the sunspots, recently observed by several astronomers all over Europe.[51] In Scheiner's interpretation, the spots were satellites revolving around the sun, similar to the satellites recently discovered by Galileo to revolve around Jupiter. If these spots were 'stars', opaque like Venus and the moon, they would show changing phases to an observer on Earth, with their illuminated part oriented towards the sun. (Figure 5.11)

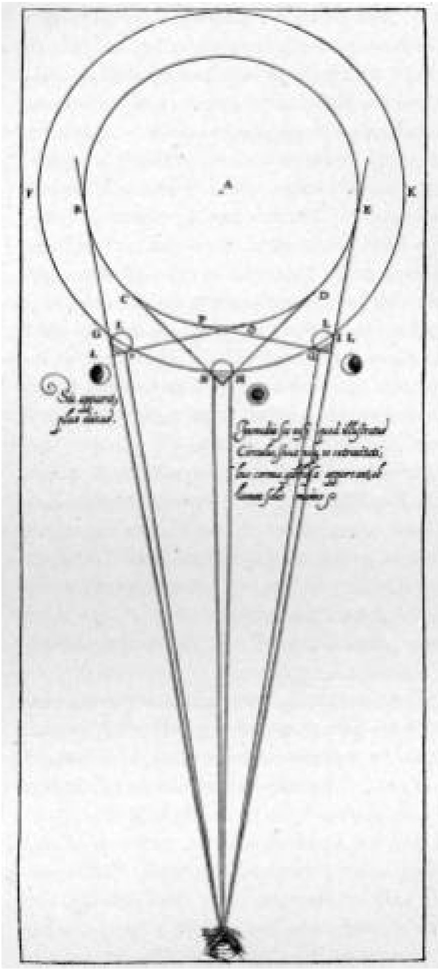

Figure 5.11

[50] Kemp, *The Science of Art*, p. 97.

[51] On the sunspot controversy, see Van Helden, Albert. 'Galileo and Scheiner on Sunspots: A Cas e Study in the Visual Language of Astronomy.' *Proceedings of the American Philosophical Society* 140 (1996): 358-95.

Also, the spots would be darker and larger towards the center of the sun.[52] However, when Welser forwarded Scheiner's letters to Galileo, asking him about his opinion about the sunspots and Scheiner's interpretation, he found Galileo disagreeing with Scheiner. In his first letter to Welser, Galileo pointed out to Scheiner, that it was, in fact, the darker side of the spots, and not the more luminous part, that was oriented towards the sun, an observation that contradicted Scheiner's interpretation of the spots as satellites.[53] According to Galileo, it was much more likely that the spots were in the immediate vicinity of the sun, that is on the surface of the solar body or so little removed from it that the distance between the spots and the solar surface could not be perceived by an observer on earth. Again, one of his arguments was based on a distinction between the appearance of the spots when near the circumference of the sun and when in the middle of the solar body. Three differences were manifest to the observer: (1) the spots become thinner at the periphery of the sun, (2) they seem to travel greater distances in the center of the sun, and (3) they seem to move farther apart nearer to the center of the sun and to come closer to each other farther from the center. The difference of appearance of the sunspots is explained by realizing that the spots at the periphery of the sun are seen foreshortened, while the spots in the center of the sun are seen 'in faccia'. (Figure 5.12)

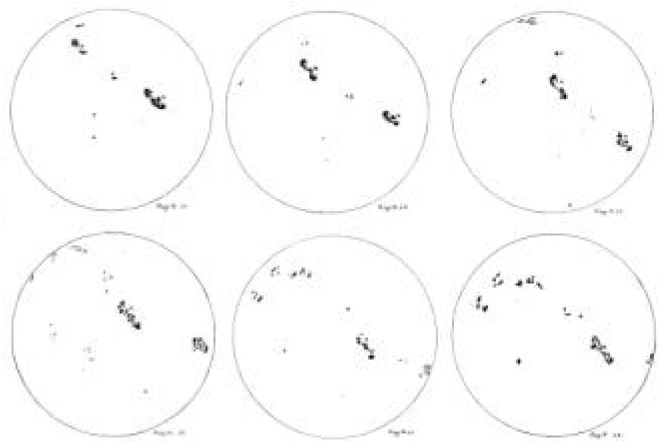

Figure 5.12

Galileo concluded that 'for those who understand, in virtue of perspective, what is meant by a spherical surface retreating near the periphery of the observed hemisphere, this will be a manifest argument that the sun is a globe, that the spots are close to the solar surface, and that as they are carried on that surface toward the center a growth in length is always discovered, while they preserve the same breadth'.[54]

[52] Christoph Scheiner, Tres epistolae de maculis solaribus, in Galileo, *Opere*, Vol. 5, p. 30.

[53] Galileo, Istoria e dimostrazioni intorno alle macchie solari e loro accidenti, in Galileo, *Opere*, Vol. 5, p. 104.

[54] 'a quelli che intenderanno, in virtù di perspettiva, cio che importi lo sfuggimento della superficie sferica vicino all' estremità dell' emisfero veduto, sarà manifesto argomento si della globosità del Sole, come della prossimità delle macchie alla solar superficie, e del venir esse poi portate sopra la medesima superficie verso le parti di mezo, scoprendosi sempre accrescimento nella lunghezza e mantenendosi la medesima larghezza'. Galileo, Istoria e dimostrazioni intorno alle macchie solari e loro accidenti, in Galileo, *Opere*, Vol. 5, p. 119.

From Galileo's abundantly obvious mastery of disegno, it is often concluded that Galileo succeeded in bridging the gap between the 'two cultures' of art and science. However, it should be realized that the disjunction of these 'two cultures' is a modern concept that, for reasons of anachronism, is not simply to be applied to the Renaissance. The Renaissance considered all practical activities such as surveying, navigation, dialling, etcetera 'arts'. Consequently, disegno did not have the priviliged status that it has been assigned by modern scholarship. It is only one among many other arts.[55] As shown in chapter 2, when discussing the prefaces to sixteenth century editions of optical works, the science of perspective was considered to have applications in a whole range of practical activities, such as surveying, map-making, dialling, navigation, building and painting. Bennett has argued that the history of perspective has too often been considered the exclusive domain of painters due to the idea-centered history of science inherited from Koyré, which, as has been shown, was behind Santillana's choice of a priviliged status of art in his account of the scientific revolution.[56] However, the cognitive core that provided arts like map-making, dialling and painting their disciplinary unity were projection techniques to represent space. Moreover, these projection techniques were embodied in mathematical instruments.[57]

It has been argued that the invention of perspective had its origin in techniques used in any of these practical mathematical arts. Aiken has proposed that Alberti's perspective construction was an adaptation of graphic techniques already employed by instrument-makers, while Kemp has shown that a surveying technique was at the basis of the perspective panels of Filippo Brunelleschi.[58] However, for our purposes, the argument for the ubiquity of perspective is limited to the sixteenth century. As will become evident, the projection techniques applied in the several practical activities with which the sixteenth century mathematical practitioner was involved were considered projections by sight, and, consequently, these projection techniques were not yet considered to be projective geometry, what they would only become in the seventeenth century, but optics. Consequently, if the background in optics of Galileo, as a mathematical practitioner, is to be understood, it should be realized that Galileo, not unlike other contemporaneous mathematicians, was involved with mathematical instrument design, and that, therefore, he was acquainted with the projection techniques embodied in mathematical instruments in sixteenth century Italy. Here, I will focus on the orthographic projection, because it shows that Galileo's

[55] Bennett, Jim. *The Measurers: A Flemish Image of Mathematics in the Sixteenth Century*. Oxford: Museum of the History of Science, 1995, pp. 15-6.

[56] Bennett, Jim. 'Practical Geometry and Operative Knowledge.' *Configurations* 6 (1998): 195-222, pp. 197-8.

[57] Ibid., pp. 209-19; Bennett, Jim. 'Projection and the Ubiquitous Virtue of Geometry in the Renaissance.' In *Making Space for Science: Territorial Themes in the Shaping of Knowledge*, edited by Crosbie Smith and Jon Agar, 27-38. London: Macmillian Press Ltd, 1998, pp. 29-37.

[58] Aiken, Jane Andrews. 'Truth in Images: From the Technical Drawings of Ibn Al-Razzaz Al-Jazari, Campanus of Novara, and Giovanni De' Dondi to the Perspective Projection of Leon Battista Alberti.' *Viator* 25 (1994): 325-59; Kemp, Martin. 'Science, Non-Science and Nonsense: The Interpretation of Brunelleschi's Perspective.' *Art History* 1 (1978): 134-61. On the origins of linear perspective, see also Johannsen, Birgitte Boggild, and Marianne Marcussen. 'A Critical Survey of the Theoretical and Practical Origins of Renaissance Linear Perspective.' *Acta ad Archaeologiam at Artium Historiam Pertinentia* 1 (1981): 191-227; Edgerton, Samuel Y., Jr. *The Renaissance Rediscovery of Linear Perspective*. New York Evanston San Francisco London: Harper & Row Publishers, 1975, pp. 91-123; Aiken, Jane Andrews. 'The Perspective Construction of Masaccio's Trinity Fresco and Medieval Astronomical Graphics.' In *Massacio's Trinity*, edited by Rona Goffen, 90-107. Cambridge: Cambridge University Press, 1998; Veltman, Kim H. 'Ptolemy and the Origins of Linear Perspective'. In *La Prospettiva Rinascimentale: Codificazioni e Trasgressioni*, edited by Marisa Dalai Emiliani, 403-7. Vol. 1. Firenze: Centro Di, 1980.

discussion of the sunspots is not only evidence of his mastery of disegno, but also of his acquaintance with a projection technique embodied in mathematical instruments.[59]

First, the orthographic projection itself, and its use in drawing and mathematical instrument design will be discussed. It will be shown that, although widely used as a drawing method, the concept of an orthographic projection as a projection with the center of projection at infinity was only acquired in the discussion of the design of universal astrolabes and sundials in the sixteenth century. Next, it will be argued that Galileo was involved with mathematical instrument design by training, reading and the supervision of his own workshop. Finally, it will be argued that Galileo's use and interpretation of an orthographic projection in his discussion of sunspots reflects his close acquaintance with the optics of mathematical instrument designers.

3. Galileo, Mathematical Practitioner: Mathematical Instruments and Perspective

3.1. Perspective Embodied in Mathematical Instruments

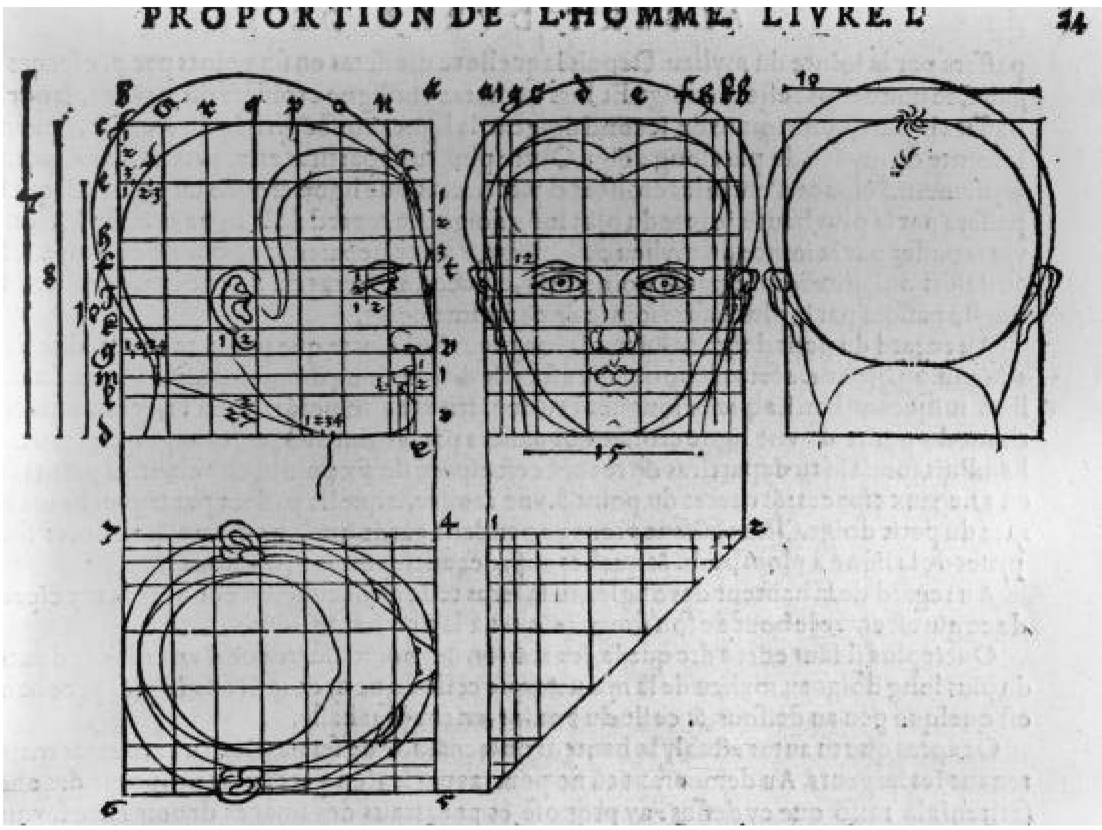

Figure 5.13

[59] For a similar argument, see Dupré, Sven. 'Galileo, Mathematical Instruments and Orthographic Projection.' *Bulletin of the Scientific Instrument Society* 69 (2001): 10-20.

Painters were well acquainted with the representation of a three-dimensional object by three two-dimensional drawings, in each of which the object is viewed along parallel lines that are perpendicular to the plane of the drawing (plan, front and side elevations). For example, a projection by means of parallel lines was used by Albrecht Dürer in his 'Von Menschlicher Proportion' in 1528. Dürer used an orthographic projection to foreshorten the heads of humans, showing a top, front and side view (Figure 5.13), a method already used by Piero della Francesca in his unpublished manuscript 'De Prospectiva Pingendi'.[60] Thus, as a drawing method, a projection along parallel lines must have been common practice in the painter's workshop. However, it was left to the designers of mathematical instruments around the middle of the sixteenth century to understand the orthographic projection as a projection along parallel lines perpendicular to the plane of projection with the center of projection at infinity.

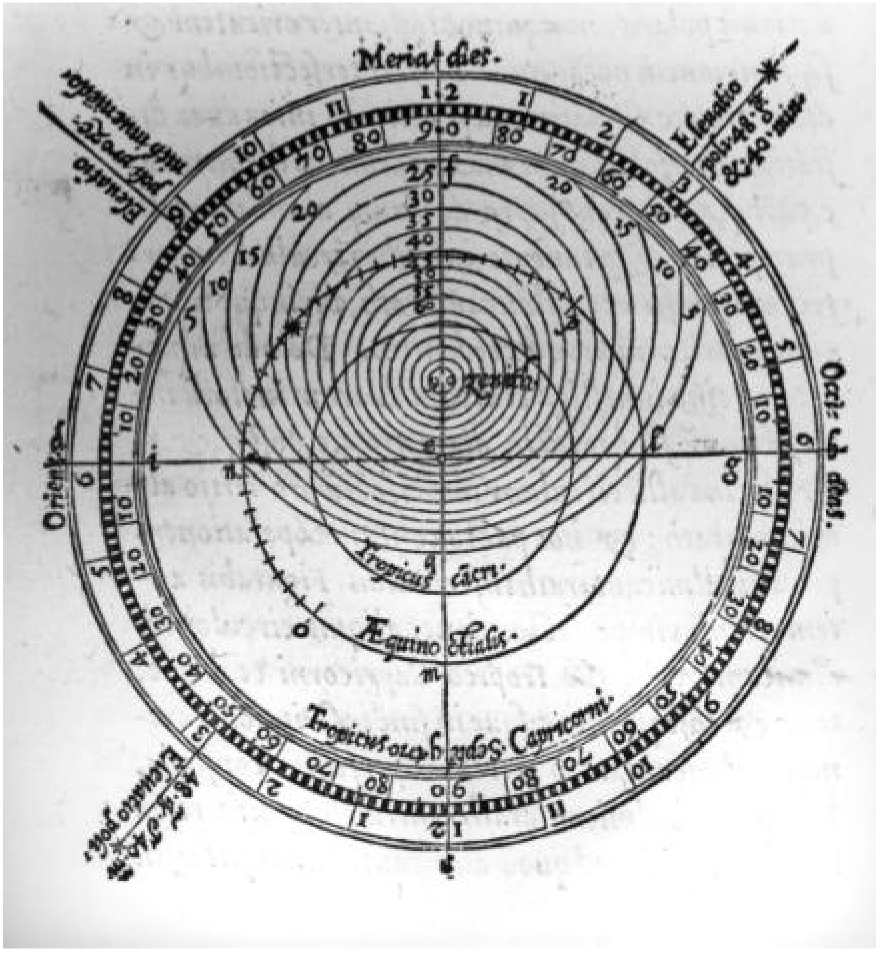

Figure 5.14A

[60] Dürer, Albrecht. *Von Menschlicher Proportion: Faksimile-Neudruck der Originalausgabe Nürnberg 1528.* Nördlingen: Uhl, 1980, p. Eiii. See Sinisgalli, Rocco. *Per la Storia della Prospettiva (1405-1605): Il Contributo di Simon Stevin allo Sviluppo Scientifico della Prospettiva Artificiale ed i Suoi Precedenti Storici*. Roma: 'L' Erma' di Bretschneider, 1978, pp. 151-3, for similar drawings in Piero della Francesca's 'De prospectiva pingendi'.

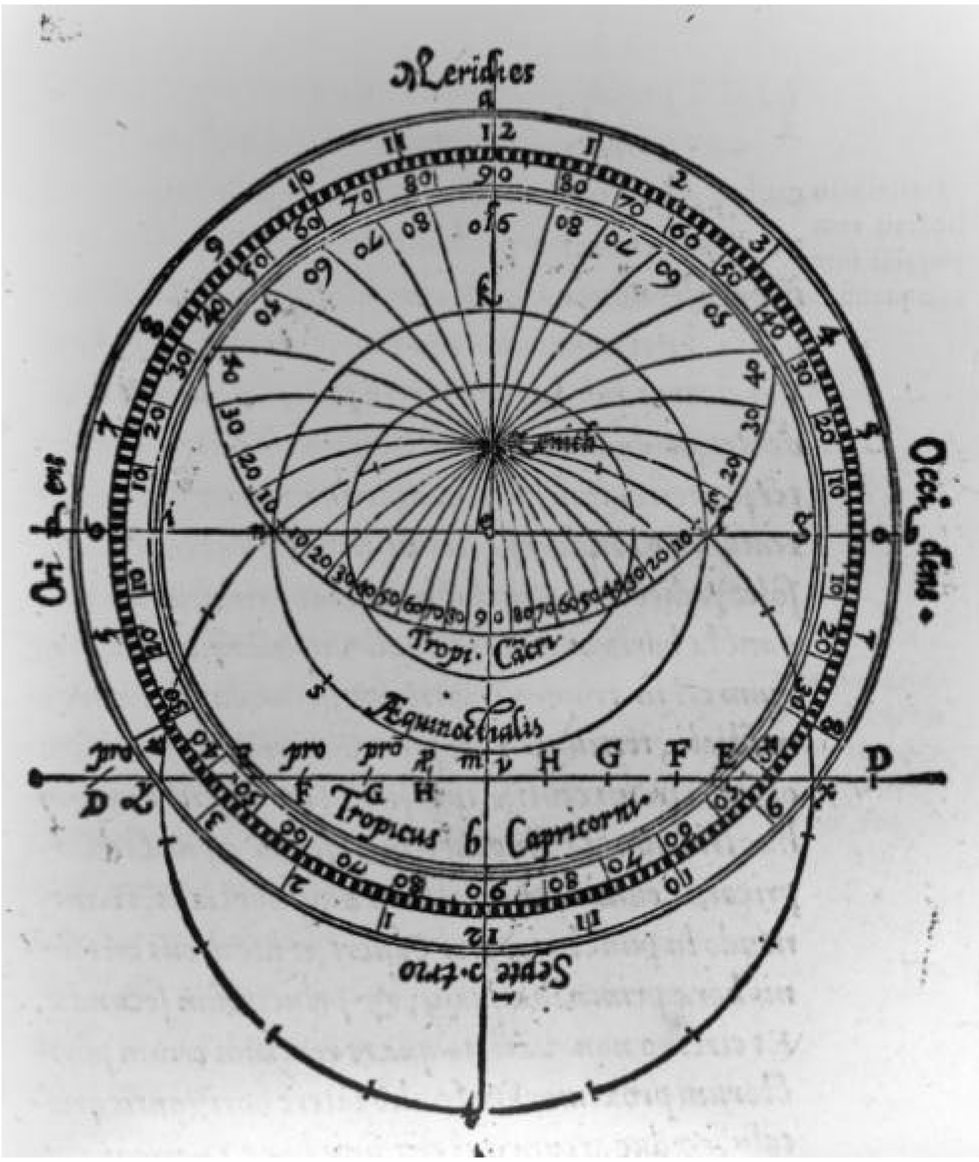

Figure 5.14B

Astrolabes had a plate showing a map of the celestial sphere. The equator, the tropics, the ecliptic, the lines of equal azimuth and the almucantars were projected onto the astrolabe plate. Most common, also in the sixteenth century, was the use of a stereographic projection, with the south celestial pole as center of projection and the plane of the equator (or a plane parallel to it) as the plane of projection.[61] This type of stereographic projection was used by Stöffler in his 'Elucidatio fabricae ususque astrolabii'.[62] (Figure 5.14A – 5.14B)

[61] North, J.D. 'The Astrolabe.' *Scientific American* 230 (1974): 96-106. See also Michel, Henri. *Traité de l' Astrolabe*. Paris: Gauthier-Villars, 1947, pp. 15-8, 27-9.

[62] Stöffler, Johannes. *Elucidatio Fabricae Ususque Astrolabii*. Parisiis: Hier. de Marnef, 1585, pp. 2-13. On this work, see also Simcock, A. V. 'Elucidatio Fabricae Ususque: Rambling among the Beginnings of the Scientific

However, a major drawback was that the astrolabe plate made with this type of stereograhic projection could only be used at the particular latitude it was designed for. Each latitude asked for a different plate. The sixteenth century saw the publication of two different types of projection that were considered to be a solution to this problem. These projections allowed a 'universal' application of the astrolabe, that is, only one plate was needed, whatever the latitude of the observer or user of the astrolabe.

In 1556, the 'astrolabum catholicum' of Gemma Frisius was posthumously published. Gemma revived a universal astrolabe type known as the 'sapheae arzachelis', originally invented in the eleventh century by Ibn az-Zarqellu, an astronomer of Toledo.[63] Gemma Frisius used a stereographic projection, but shifted the center of projection to one of the equinoxes. The colure of the solstices functioned as plane of projection. In this projection, the parallels and meridians became arcs of circles. Gemma's 'astrolabum catholicum' was presumably meant as an alternative to another type of universal astrolabe, published by one of his students at Louvain, Juan De Rojas. With the help of Hugo Helt, De Rojas designed a universal astrolabe that used the orthographic projection to map the celestial sphere.[64] The plane of projection was the solstitial colure, while the center of projection was considered to be at infinity and perpendicular to the plane of the solstitial colure. In this projection, the parallels became straight lines and the meridians were elliptical arcs, although the latter were only recognized as such by Guidobaldo del Monte in his 'Planisphaeriorum Universalium Theorica', published in 1579.[65]

The De Rojas astrolabe became popular in Florence around 1570, when Egnazio Danti had several of them made in the workshop of Giusti and devoted a chapter to it in his 'Trattato dell' uso e fabbrica dell' astrolabio' (1567).[66] Typically, the map of the celestial sphere on the orthographic projection appeared on the back of an 'ordinary' astrolabe with a map of the celestial sphere on the stereographic projection designed for a specific latitude on the front of the instrument (Figure 5.15).

Instrument Bookshelf.' In *Learning, Language and Invention: Essays Presented to Francis Maddison*, edited by W. D. Hackmann and A. J. Turner, 273-96. Aldershot Paris: Variorum Societé Internationale de l' Astrolabe, 1994.

[63] Frisius, Gemma. *De astrolabo catholico liber latissime patentis instrumenti multiplex usus explicatur.* Antverpiae: Joan. Steelsius, 1556. See also Michel, *Traité de l' Astrolabe*, pp. 18-20, 93-102.

[64] De Rojas, Juan. *Illustris viri D. Ioannis de Roias Commentariorum in Astrolabium, quod Planisphaerium vocant, libri sex nunc primum in lucem edit*. Paris, 1550. For discussion, see Maddison, Francis. 'Hugo Helt and the Rojas Astrolabe Projection.' *Revista do Faculdade de Ciencias Coimbra* 39 (1966), 5-61, in particular, pp. 12-37.

[65] Guidobaldo del Monte. *Planisphaeriorum Universalium Theorica*. Pisauri: Apud Hieronymum Concordiam, 1579. For discussion, see Sinisgalli, Rocco, and Salvatore Vastola. *La Teoria sui Planisferi Universali di Guidobaldo Del Monte*. Firenze: Edizioni Cadmo, 1994. Guidobaldo discussed both the projection used by Gemma Frisius in his 'De astrolabo catholico' and the De Rojas projection. In contrast to Guidobaldo, Gemma Frisius had considered the meridians, orthographically projected, to be anomolous curves without recognizing them to be ellipses. 'Meridiani vero lineis curvis anomalis, quae neq, circuli sunt, neq, certa designatione constitutae, sed tantum per puncta adsignata manu diligenti traductae'. Gemma Frisius, *De astrolabo catholico*, p. 9.

[66] Danti, Egnatio. *Trattato dell' uso, e fabbrica dell' astrolabio di M. Egnatio Danti del' ord. di S. Domenico. Con il Planisferio del Roias*. Firenze: Giunti, 1578, pp. 114-45. For discussion, see Turner, Gerard L'E. 'The Florentine Workshop of Giovan Battista Giusti, 1556 - c. 1575.' *Nuncius* 10 (1995): 131-72.

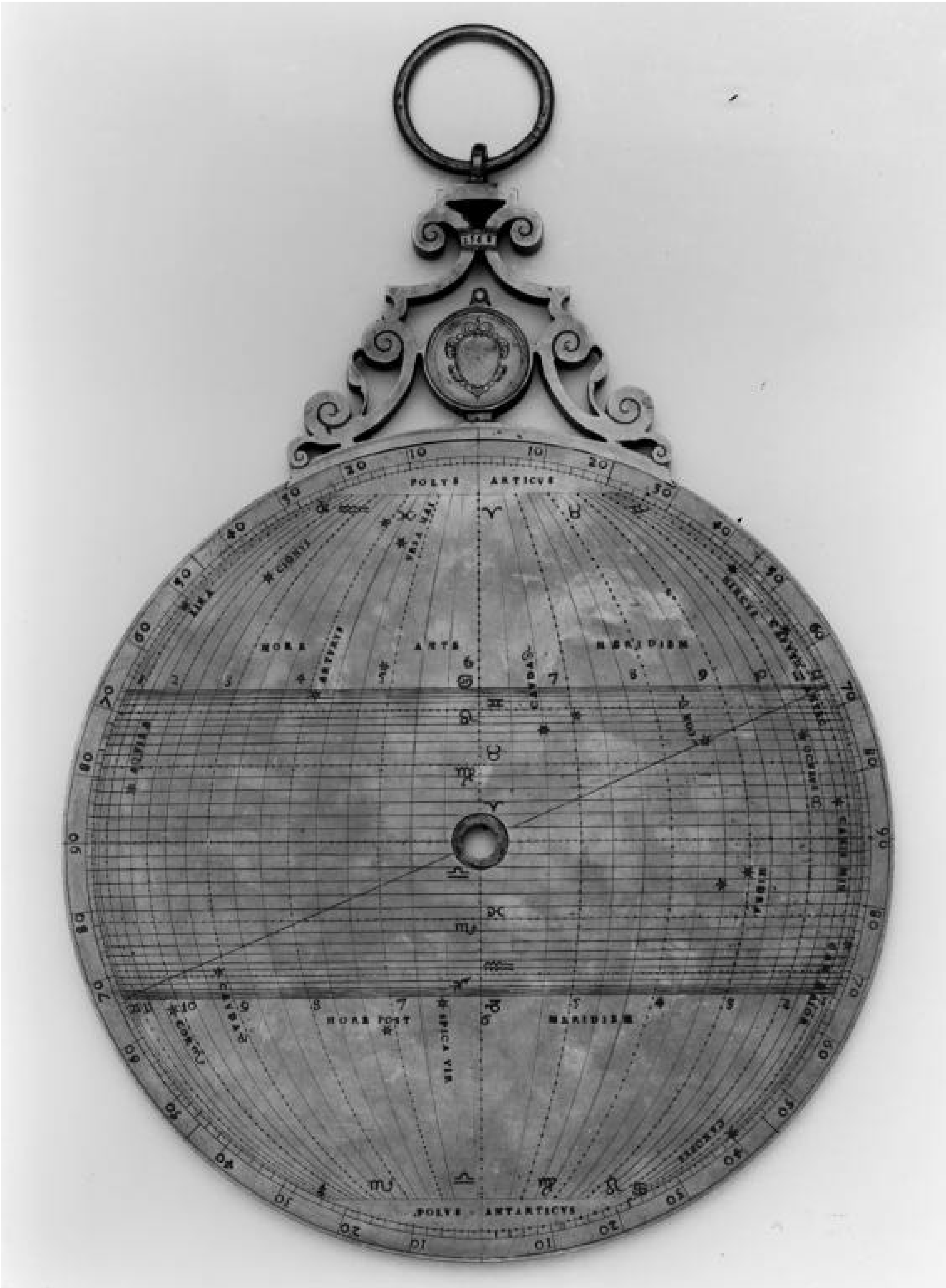

Figure 5.15

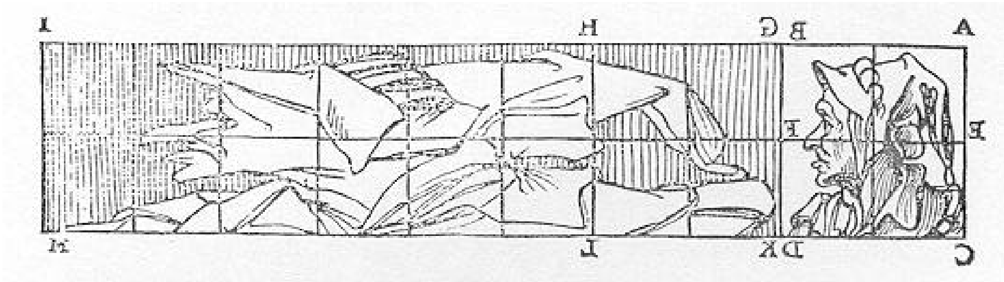

Figure 5.16

When Danti published his edition and commentary of Vignola's perspective treatise, he applied the orthographic projection to a painterly drawing problem, that is the construction of an anamorphic picture, in Danti's case, of a human head.[67] (Figure 5.16) As shown in chapter 2, Danti incorrectly thought that an anamorphosis would result from projecting an ordinary picture along parallel lines. Although certainly reminiscent of Dürer's human heads, of which some are distorted, in his 'Von Menschlicher Proportion', Danti's acquaintance with the orthographic projection from his work on the design of astrolabes might have biased him towards using the orthographic projection in trying to solve this puzzling painterly drawing problem. The use of the orthographic projection to design astrolabes did certainly not originate with Juan De Rojas. As Maddison has noted, several earlier astrolabes using the orthographic projection are preserved.[68] Moreover, the orthographic projection was also used to design sundials. In this context, it was known as the analemma. It was discussed in the nineth book of Vitruvius' 'De Architectura', who considered gnomonics to be a part of architecture, and in one of Ptolemy's lesser known works, 'De analemmate'.[69] The construction of an analemma was only the first step in the construction of a sundial. It is the orthographic projection of the path of the Sun on the meridian plane for a place of given latitude and for the different seasons of the year (Figure 5.17). In Barbaro's drawing of the analemma of Vitruvius, FIPQ is the celestial meridian, QP the axis of the universe, FX the equator, and EI the horizon with the Earth at A. LG is the tropic of Cancer, KR the tropic of Capricorn, with a semicircle of both folded into the plane of the celestial meridian. A gnomon AP is erected from a base line perpendicularly to the horizon. This baseline represents the ground on which the shadows of the gnomon are to be projected. For example, when the sun is at N, that is, at noon on the equinox, the gnomon will throw a shadow BC.

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[67] Danti, *Le Due Regole della Prospettiva Practica*, p. 96. See Baltrusaitis, *Anamorphic Art*, pp. 28-30.

[68] Maddison, 'Hugo Helt and the Rojas Projection', pp. 29-30, mentions three astrolabes. One is attributed to Regiomantanus (now in Greenwich) and two are atrributed to Hans Dorn (IC 492 now in Florence, the other in Cracow), all dated to the second half of the 15th century, and one 16th century manuscript drawing that shows an orthographic projection. The sources of this orthographic projection are unknown. Some knowledge of the analemma in the circle of Gemma Frisius is possible, as Maddison has suggested, but since Gemma Frisius did not understand the analemma, as will be shown, it is unlikely that it was influential on the design of the De Rojas astrolabe.

[69] Barbaro, Daniele. *I dieci Libri dell' Architettura di M. Vitruvio*. Venetia: Francesco Marcolini, 1556; Commandino, Federico. *Claudii Ptolemaei Liber de Analemmate, a Federico Commandino Urbinate instauratus, et commentariis illustratus. Qui nunc primum eius opera e tenebris in lucem prodit. Eiusdem Federici Commandini Liber de Horologiorum Descriptione*, Romae: Apud Paulum Manutium Aldi F., 1562. For discussion, see Sinisgalli, Rocco, and Salvatore Vastola. *L' Analemma di Tolomeo*. Firenze: Edizioni Cadmo, 1992.

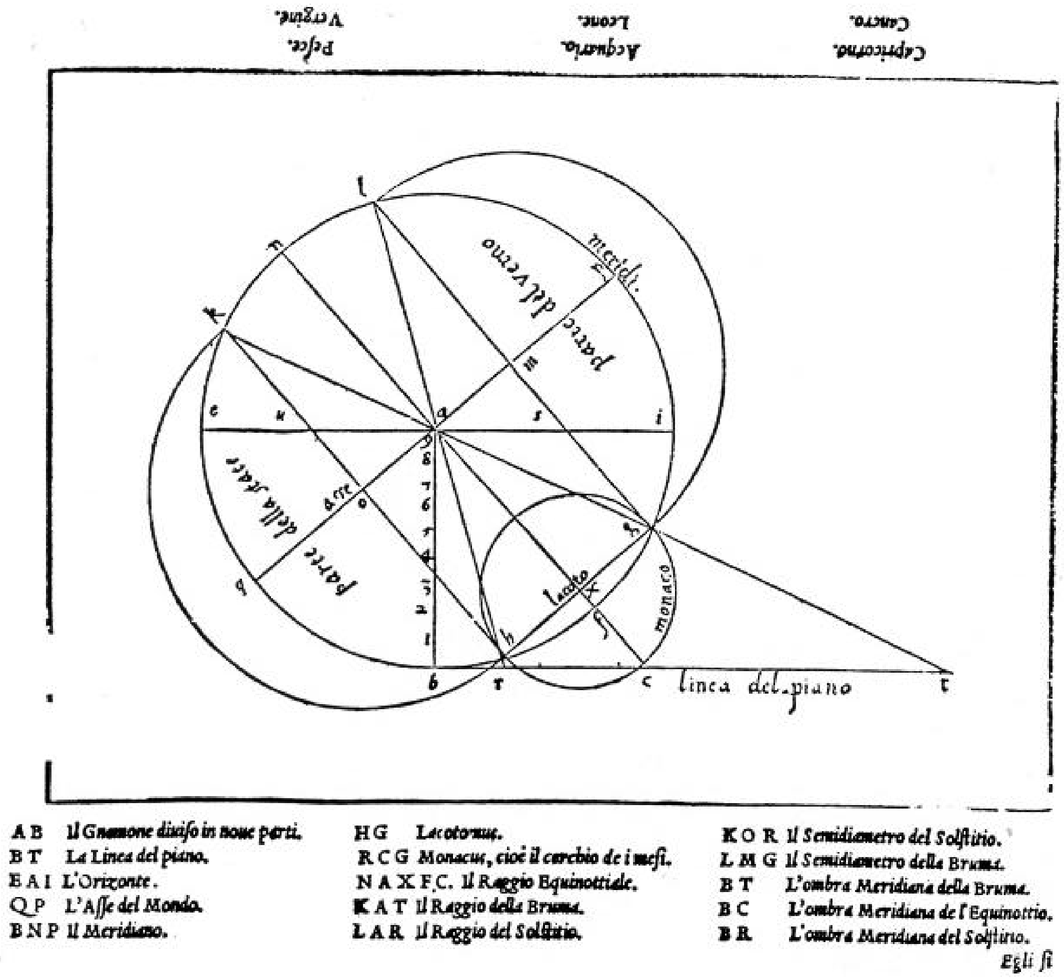

Figure 5.17

The only difference between Vitruvius' and Ptolemy's analemma was that Vitruvius used the 'menaeus', a kind of graphical shortcut to trace the parallel of the Sun during the year once given the ecliptic longitude of the sun.[70] In Barbaro's drawing of Vitrivius' analemma, the circle RCG is the menaeus. It is divided in twelve parts, corresponding to the entry of the sun in the twelve signs of the zodiac. These signs were used to contruct the diurnal path of the sun, parallel to equator and the tropics in the analemma, on its day of entry in any sign of the zodiac. It is important to note that the practical application of the analemma to the construction of sundials, not evident in Vitruvius and Ptolemy, was only restored in the sixteenth century.

[70] Ibid., p. 43; Neugebauer, O. *A History of Ancient Mathematical Astronomy*. 3 vols. Vol. 2. Berlin Heidelberg New York: Springer-Verlag, 1975, pp. 839-54; Ronca, Luigi. 'Gnomonica sulla Sfera ed Analemma di Vitruvio.' *Problemi Attuali di Scienza e di Cultura* 373 (1976): 3-17; Evans, James. *The History & Practice of Ancient Astronomy*. New York Oxford: Oxford University Press, 1998, pp. 132-41; Drinkwater, Peter I. 'The Analemma of Vitrivius.' *Bulletin of the British Sundial Society* 93 (1993): 18-20.

Although reconstructions of the Vitruvian analemma had appeared in the works of Fra Giocondo and Giovan Battista da Sangallo, it was Daniele Barbaro's edition of Vitruvius' 'De architectura' in 1556 and, again, in 1567, that was most important in the study of the analemma. This second edition was already informed by Commandino's first edition of Ptolemy's 'De analemmate' in 1562. Commandino's edition was published together with the 'Liber de Horologiorum Descriptione' of his hand to show how the analemma could be used to construct any horizontal, vertical or inclined sundial.[71] Losito has shown that Barbaro was much indebted to Commandino for his reinterpretation of the Vitruvian analemma and its practical application to the construction of sundials in his second edition of Vitruvius' 'De architectura' of 1567.[72] Consequently, interest in the analemma from sixteenth century mathematicians was connected with its practical application in the design of sundials. However, for our purposes, it is not so much this practical application that is important, but more the way the analemma was optically interpreted.

Daniele Barbaro was a major patron of mathematics and a central figure in the Venetian mathematical culture of the sixteenth century. The owner of an important library and of a collection of mathematical instruments, he showed an interest in mathematical instruments, and sundials in particular, not only in his edition of Vitruvius' 'De Architectura', but also in his 'La Pratica della Perspettiva', primarily intended for painters, and in an unfinished manuscript 'De horologiis describendis libellus', presumably containing the preparatory work for his edition of Vitruvius.[73] In fact, the construction of the analemma became an important focus in the work of his correspondents and the direct heirs of the mathematical culture he established in the Veneto. At the end of the sixteenth century this culture centered around Pinelli. As will become evident, Galileo was acquainted with many of these mathematicians dealing with the analemma.

Contarini, a priviliged correspondent of Barbaro, was famous for his design of sundials on the analemma, and his interest in the development of a compass to draw ellipses was, without doubt, related to this activity.[74] Also, Contarini's correspondent and a close friend of Pinelli, Giuseppe Moleto, left a construction of the analemma in a manuscript devoted to the construction of mathematical instruments.[75] Also, another intimitate of this circle, Guidobaldo del Monte tried to come to grips with Commandino's edition of Ptolemy's 'De analemmate', as shown in his

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[71] For the application of the analemma to the design of sundials, see Sinisgalli, Rocco, and Salvatore Vastola. *La Rappresentazione degli Orologi Solari di Federico Commandino*. Firenze: Edizioni Cadmo , 1994.

[72] Losito, Maria. 'La Gnomonica, il IX Libro dei Commentari Vitruviani di Daniele Barbaro e gli Studi Analemmatici di Federico Commandino.' *Studi Veneziani* 18 (1989): 177-237; Losito, Maria. 'Il IX Libro del De Architectura di Vitruvio nei Commentari di Daniele Barbaro (1556-1567).' *Nuncius* 4 (1989): 3-42.

[73] On Barbaro, see Alberigo, G. 'Daniele Barbaro.' In *Dizionario biografico degli italiani*. Vol. 6, pp. 89-95. Barbaro owned a library and a collection of instruments, 'both purchased and made at home', see Boucher, Bruce. 'The Last Will of Daniele Barbaro.' *Journal of the Warburg and Courtauld Institutes* 42 (1979): 277-82. For Barbaro's discussion of sundials elsewhere than in his commentary of Vitruvius, see his 'De horologiis describendis', Biblioteca Marciana (Venice), Lat. VIII, 42=3097; Barbaro, *La Pratica della Perspettiva*, pp. 187-90.

[74] Contarini's studies of the analemma are mentioned in Girolamo Porro's description of Contarini in the dedication of Vincenzo Scamozzi's 'Discorsi sopra le antichità di Roma' (Venetia, 1582). 'Intendente di tutte le belle cose, ò sia d' architettura, pittura, scoltura ò stromenti bellici, armonci et analemmatici, havendo de gli uni, et degli altri un numero quasi infinito, che da tutte le parti sono meritatamente appresentati. La onde V.S. quasi moderno Archimede, è suplicato ogno giorni da numero grandissimo d' uomini virtuosi, et astretta a rendere ragione, et discorrere d' intorno d' essi stromenti'. On Contarini's elliptical compass, see Rose, 'Jacomo Contarini', p. 124; Camerota, *Il compasso di Fabrizio Mordente*, pp. 76-7.

[75] Moleto, 'Quomodo analemma describendum sit ad datam sphaeram positionem', B. A. M., S 100 Sup., ff. 37r-37v.

unpublished notes.[76] Finally, another correspondent of several of the major Venetian mathematicians, including Barbaro, the jesuit Christoph Clavius, who taught mathematics at the Collegio Romano, considered the analemma in his discussion of sundials and astrolabes.[77]
In the sixteenth century, the stereographic projection was considered perspective, but this was less clear when it came to the orthographic projection. Gemma Frisius described what he did to the 'ordinary' stereographic projection in his 'De astrolabo catholico' as moving the 'eye' from the southpole to the spring equinox, thus considering his mapping procedure as a projection by sight.[78] Also, he recognized that the projection of De Rojas, that is the orthographic projection, was a projection by sight with the eye considered to be at infinity.[79] However, Gemma Frisius was confused about the analemma. He used the word 'analemma' both for the De Rojas projection and the projection he himself used when constructing his 'astrolabo catholico'.[80] Only Guidobaldo recognized that the projection used by De Rojas was an orthographic projection like the analemma of Ptolemy that had become available through Commandino's edition.[81] In his 'Planisphaeriorum Universalium Theorica', he also disagreed with Gemma Frisius, claiming that a projection by sight with an eye at an infinite distance went against what perspective is.[82] As will become evident, Galileo must have been familiar with this discussion, as he came to rescue with a different interpretation of the orthographic projection. It will be shown that Galileo was the direct heir of this mathematical culture, due to his training and his personal contacts.

[76] Guidobaldo del Monte, 'Meditatiunculae Guidi Ubaldi Marchionibus Montis S. Mariae de rebus mathematicis', Bibliothèque Nationale de France, Lat. 10246, , ff. 1-5, 13-9, 23-6, 153-4.

[77] Clavius, Christophorus. *Gnomonices libri octo*. Romae: Fr. Zannettus, 1581, pp. 11-9, 528-74, on Ptolemy's 'De analemmate'. For his 'Astrolabium', see Clavius, Christophorus. *Opera Mathematica*. Moguntiae: Reinh T. I., 1611. Vol. 3, pp. 34-7, 125-205.

[78] 'Verum eo solum differt, quod oculus non in polo, sed in aequinoctiali constituitur, atq, ita oppositum oculo hemisphaerium in planum per centrum extensum, oculoq, ad perpendiculum obiectum visu describitur'. Gemma Frisius, *De astrolabo catholico*, p. 8.

[79] 'Oculus vero in infinitum (si fieri potest) adsistat, radiosq, per haemisphaerium in planum subiectum fundat, ita ut puncta aequinoctialia in rectum oculo opponantur'. Ibid., p. 9.

[80] On his own 'astrolabo catholico', Gemma Frisius wrote that 'hoc igitur Analemma, haec inquam Sphaera plana omnium est commodissima atq, universalissima, innumerabilis habens usus, ad omnem coeli inclinationem aeque accommoda'. However, he also referred to the projection of De Rojas as analemma: 'Restat & alius modus Analemmatis, Sphaera scilicet plana circulos sphaerae cotinens, sic ut circuli paralleli lineis rectis designentur. Meridiani vero lineis curvis anomalis, quae neq, circuli sunt, neq, certa designatione constitutae, sed tantum per puncta adsignata manu diligenti traductae. De huius sphaerae & compositione & usu cum diligentur tum elegatur & erudite scripsit illustris viris D. Ioannes de Rojas libris sex de Planisphaerio editis'. Ibid., pp. 8-9.

[81] Sinisgalli and Vastola, *La Teoria sui Planisferi Universali di Guidobaldo del Monte*, pp. 141-2.

[82] 'nam quo pacto fieri potest, aliquid ex perspectiva ortum ducere, oculum vero infinita distantia absistere? hoc nimirum ipsi perspectivae repugnat'. Ibid., p. 140. For discussion, see Ibid., pp. 85-8.

3.2. Galileo and Mathematical Instruments: Training, Library and Workshop

Biagioli has pointed that, in particular before obtaining the title of 'mathematician and philosopher' of the Grand Duke of Tuscany in 1610, Galileo was a mathematician, trying to live up to the social role, that was expected from a sixteenth century mathematician.[83] Already Rose had emphasized that Galileo was the direct heir of the 'Italian Renaissance of Mathematics', in particular of its Archimedean component in the Urbino tradition of Commandino and Guidobaldo del Monte.[84] However, in these studies, a well known fact, that is, that mathematical instrument design was central to the shaping of the social role of the mathematician as well as one of his central cognitive preoccupations, has received less attention, at least when it comes to Galileo's case in the period before 1610 and his move to Florence. Any mathematian at the end of the sixteenth century would have been familiar with mathematical instrument design, and, consequently, with the projections embodied in this instruments. As will be shown, at the beginning of his career, Galileo was no exception with respect to this aspect of what it meant to be a mathematician. Moreover, from the beginning of his career, and up to 1610, Galileo was connected with circles that were particularly interested in mathematical instrument design.
When Galileo studied perspective with Ricci in the early 1580s, not only disegno, but also an introduction to mathematical instrument design would have been an important part of Ricci's course on perspective. Ricci's course notes have not been preserved. However, a good insight in the content of such a course on perspective in Florence around 1600 is provided by the course notes of Giulio Parigi, Buontalenti's successor after his death in 1608, heir of his instruments and machines for stage design, and engineer at the Tuscan court. This course discussed several intruments to measure by sight, mostly for topographical purposes, an instrument to draw in perspective, that can be used to convert a ground plan of a fortification into a perspective drawing, aspects of military fortification, clocks, fountains, artillery and architecture. (Figure 5.18 - Figure 5.19) Indeed, the general outlook of the course shows it to have been designed for students interested in the military applications of mathematical instruments and geometric techniques.[85]

[83] Biagioli, Mario. *Galileo, Courtier: The Practice of Science in the Culture of Absolutism*. Edited by David Hull, *Science and Its Conceptual Foundations*. Chicago London: The University of Chicago Press, 1993.

[84] Rose, *The Italian Renaissance of Mathematics*, pp. 280-91.

[85] For Parigi's course, see Bartoli, Leandro Maria. 'Argomenti di Architettura e Arte Militare nei Trattati di Giulio Parigi.' In *Atti del Convegno di Studi: Architettura Militare nell' Europa del XVI Secolo*, edited by Carlo Cresti, Amelio Fara and Daniela Lamberini, 399-406. Siena: Periccioli, 1988. Two manuscripts containing notes for this course have been preserved. See Bibliothèque Nationale de France (Paris), Italien 486; Italien 1292. MS It. 486 discusses simple mathematical problems, for example, dividing a line into equal parts (ff. 1r-12r), measuring by sight distances and heights, using instruments like the 'squadra zoppa' (ff. 12v-20r), techniques of levelling (ff. 20v-22r), surveying (ff. 22v-28r), mining (ff. 28r-30r), artillery (ff. 45v-51r), and an instrument to convert a plan to a drawing in perspective, attributed to 'Messer Bernhardino delle Girandole Ingignero del Ser.mo Gran Duca di Toscana' (ff. 57v-58r). MS It. 1292 discusses the same topics, but also includes water-pumps and fountains (ff. 27r-38r), clocks (ff. 41v-43r), machines (ff. 44v-55r), different architectural orders (ff. 66v-79r), and the architecture of Buontalenti (ff. 81ff). Thus, Parigi's course must have included the study of architecture, military fortification, mechanics, clocks, measuring by sight, and perspective. For the latter subject, examples were taken from a military context. On Buontalenti and Parigi, see also Mamone, Sara. 'Le Miroir des Spectacles: Jacques Callot à Florence (1612-1622).' In *Jacques Callot, 1592-1635*, edited by Paulette Choné, 183-87. Paris: Réunion des Musées Nationaux, 1992.

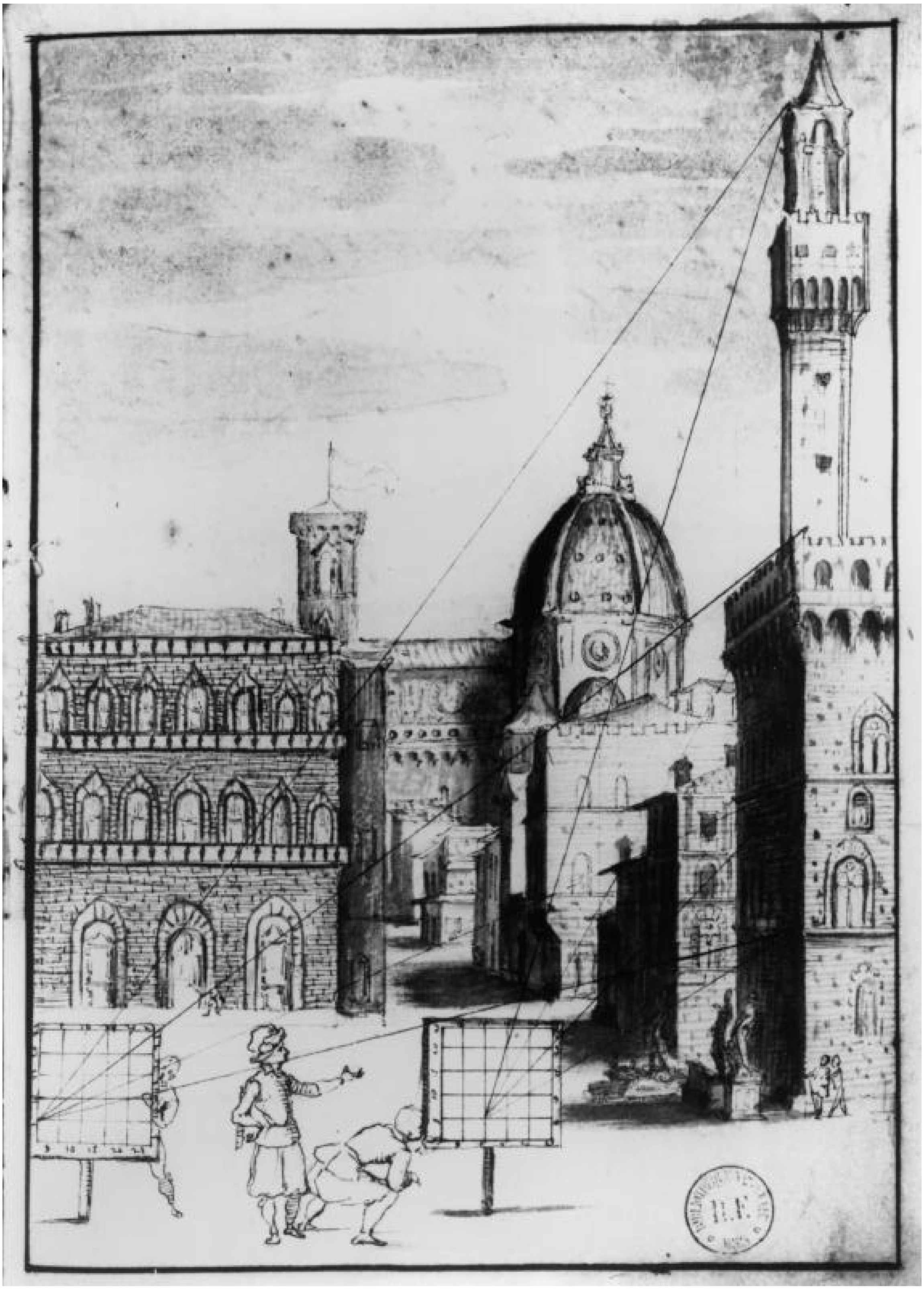

Figure 5.18

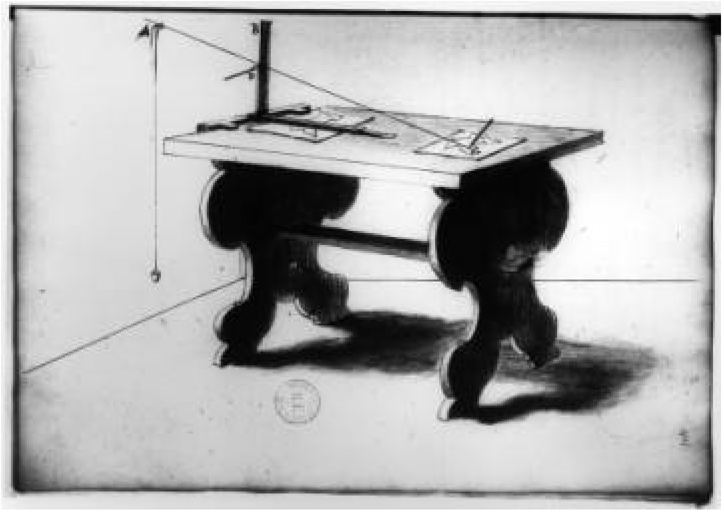

Figure 5.19

Parigi's course is highly reminiscent of Alberti's 'Ludi matematici'. Indeed, some drawings of Parigi are copied directly from Alberti's 'Ludi matematici'.[86] (Figure 5.20)

Figure 5.20

[86] For example, compare the construction of a fountain in Giulio Parigi, 'Modo per fare una fonte che schizzi in alto l' acqua da se', Bibliothèque Nationale de France (Paris), Italien 1292, ff. 30v-31r, with Leon Battista Alberti, 'Ludi rerum mathematicarum' in Grayson, *Opere volgari*, Vol. 3, pp. 147-8. In general, there is a Florentine orientation in Parigi's class notes. See, for example, the reference to the use of Danti's 'radio latino' to measure the height of a tower, Bibliothèque Nationale de France (Paris), Italien 1292, ff. 2v-3r.

Alberti's 'Ludi matematici' was most likely used as a textbook in Ricci's class. Settle has convincingly argued that the MS Galileiana 10, the 'Problemi Geometrici', attributed to Ricci, is a copy of the 'Ludi Matematici' of Alberti, available in several manuscript copies and in a published edition of 1568 by Cosimo Bartoli.[87] The 'Ludi matematici' was an exemplary treatise on measuring heights, depths and distances using nothing more than a stick or a plane mirror, but it also introduced more advanced instruments, for example, the 'orrizonte', for topographical purposes. Moreover, also Ricci's own interest in the development of mathematical instruments, for example, his 'archimetro', a foldable instrument useful for surveying, suggests that Ricci introduced Galileo to mathematical instrument design at an early stage of his life.[88] Recently, Gerard Turner has identified a Hartmann astrolabe, modified in the Florentine workshop of Giusti, that belonged to Galileo around 1580.[89] Although an astrolabe was a useful instrument to a typical student of medicine, it is possible that Galileo's possession of such an instrument was related to his somewhat later mathematical training in Ricci's class on perspective in Florence.

After Galileo's graduation from Ricci's class, his interest in mathematical instrument design did not fade away. On the contrary, an interest in mathematical instrument design was central to the preoccupations of the circles Galileo frequented during the years immediately following. After graduating from Ricci's class, Galileo took up the job of professor of mathematics at the University of Pisa. At the same time, around 1590, Galileo familiarized himself with the teaching of the Jesuits at the Collegio Romano. While Galileo was particularly interested in the courses of the Jesuits on logic, presumably already preparing for a job as philosopher, it is most likely that Galileo was also familiar with the specific kind of mathematical education provided at the Collegio Romano.[90] From its inception, the mathematical programme of the Jesuits focussed on mathematical instruments. They were considered a most useful didactic tool for mathematical education. From the 1580s onwards, Clavius provided the Collegio Romano with didactic material, by publishing numerous textbooks on mathematical instruments.[91]

Galileo was well aware of Clavius' publications in the field. In a letter of 18 December 1604, Clavius told Galileo that he had tried to send him his 'Astrolabium' immediately after its publication in 1593.[92]

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[87] Settle, 'Ostilio Ricci, a Bridge between Alberti and Galileo', pp. 122-6.

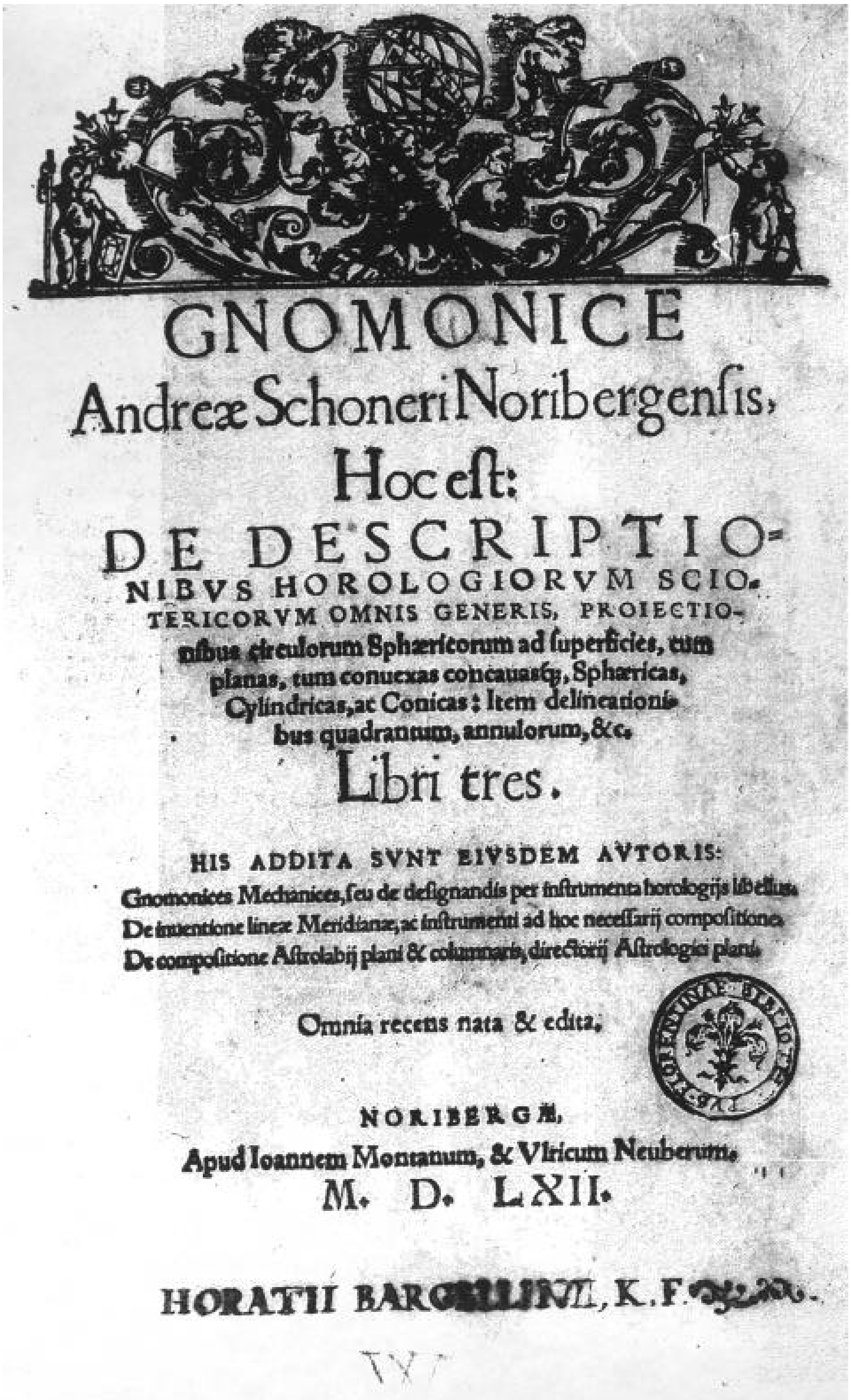
[88] For Ostilio Ricci's 'L' uso dell' archimetro', including a transcription of the manuscript, see Vinci, Federico. *Ostilio Ricci da Fermo, maestro di Galileo Galilei*. Fermo, 1929, pp. 23-9.

[89] Turner, Gerard L'E. 'An Astrolabe Belonging to Galileo?' *Nuncius* 12 (1997): 87-92.

[90] For Galileo's use of jesuit courses on logic and methodology, see Wallace, William. *Galileo and His Sources: The Heritage of the Collegio Romano in Galileo's Science*. Princeton: Princeton University Press, 1984.

[91] Romano, Antonella. *La Contre-Réforme Mathématique: Constitution et Diffusion d' une Culture Mathématique Jésuite à la Renaissance (1540-1640)*. Roma: Ecole Française de Rome, 1999, pp. 1-180.

[92] 'Mi vergono quasi della mia negligentia in fare a saper V. S. come molti anni sono, almeno 11, che finito di stampare il mio Astrolabio, l' anno 1593, mandai subito uno a lei … trovai che 'l libro non era mandato a V. S.; perchè s' era partito da Pisa senza sapere io niente di questo; et un gentilhuomo Sanese l' haveva usurpato per sè, et pregandomi gli lo donai. Hora, perchè mi pare molto probabile che già V. S. l' haverà visto; et se non, m' avisi, che gli manderò uno, che a punto mi restò. Interim gli mando la Geometria Prattica, stampato adesso, benchè non è degna di lei; ma lo fo per continuare l' amicitia tra noi. … Vegga V. S. se posso niente per lei; et se non havesse havuto il libro della nova descrittione d' horivoli per via delle tangenti, insieme con un compendio brevissimo, me lo significhi, che non mancarò di mandargli lo.' Galileo, *Opere*, Vol. 10, pp. 120-1.



GNOMONICE
Andreæ Schoneri Noribergensis,
Hoc est:
DE DESCRIPTIONIBVS HOROLOGIORVM SCIOTERICORVM OMNIS GENERIS, PROIECTIONIBVS circulorum Sphæricorum ad superficies, cum planas, tum conuexas concauasq́, Sphæricas, Cylindricas, ac Conicas: Item delineationibus quadrantum, annulorum, &c.
Libri tres.

HIS ADDITA SVNT EIVSDEM AVTORIS:
Gnomonices Mechanices, seu de designandis per instrumenta horologijs libellus.
De inuentione lineæ Meridianæ, ac instrumenti ad hoc necessarij compositione.
De compositione Astrolabij plani & columnaris, directorij Astrologici plani.

Omnia recens nata & edita.

NORIBERGÆ,
Apud Ioannem Montanum, & Vlricum Neuberum.
M. D. LXII.

HORATII BAR[illegible], K. F.

Figure 5.21

Apparently, the book never arrived at that time, as Clavius had just learned, but, now, Clavius was prepared to send him the book, although he was sure that Galileo would have seen it in the meantime. Also, Clavius thought it most likely Galileo had seen his 'Horologiorum nova descriptio' and his 'Compendium brevissimum describendorum horologiorum horizontalium ac declinantium', but, in case not, he would likewise send them to Galileo. Anyway, with the letter already came a copy of Clavius' recently published 'Geometria Practica'. Galileo's interest in Clavius' publications with respect to mathematical instrument design is corroborated by the content of his library that contained, beside Clavius' 'Astrolabium' and 'Geometria practica', also his 'Gnomonices libri octo' on sundials.[93]

Galileo's library has not received sufficient attention with respect to Galileo's interest in mathematical instruments. Such an enumeration of books should be treated with the utmost care, not taking it, for example, as exhaustive for Galileo's knowledge of mathematical instrument design. However, it is only used here as an indication of Galileo's interest. Leaving aside copies of several editions of Ptolemy's 'Geographia', for example Ercole Bottrigaro's Italian translation of the work, obviously important for perspective, about 11 books in Galileo's possession are directly and primarily related to mathematical instrument design. Four of these are about astrolabes, three about sundials, and four mostly about other instruments. On astrolabes, there is Gemma Frisius' 'De astrolabo catholico' (1556), Guidobaldo del Monte's 'Planisphaeriorum universalium theorica' (1579), Johannes Stöffler's 'Elucidatio fabricae ususque astrolabii' (1585) and Clavius' 'Astrolabium'. On sundials, there is Federico Commandino's 'Claudii Ptolemaei liber de analemmate' (1562), including the 'Liber de horologiorum descriptione' that Commandino added, Andreas Schöner's 'Gnomonice'[94] (1562), also containing a short section on astrolabes, and the already mentioned 'Gnomonices Libri Octo' (1581) of Clavius. (Figure 5.21)

The four remaining books were related to Galileo's own work on mathematical instruments. As will become evident, Niccolò Tartaglia's 'La nova scientia' (1562), on the 'squadra' and the 'squadra dei bombarieri', and Egnatio Danti's 'Trattato del radio latino' (1586), on a triangulation instrument with universal ambitions, primarily meant for use by military men, and similar to the 'archimetro' of Galileo's teacher Ricci, were influential in Galileo's design of his geometric and military compass. Galileo's copy of Tartaglia's 'Questi et inventioni diverse' (1554) was useful when making surveying compasses, as was Gemma Frisius' edition of Apian's 'Cosmographia' (1584), which dealt with the use of a surveying compass, among other things of interest to the designer of mathematical instruments.[95] Indeed, beside the well-known geometric and military compasses, Galileo also produced a number of surveying compasses.

As has been shown, mathematical instrument design was a primary concern of many mathematicians belonging to Pinelli's circle, for example Moleto and Contarini. Galileo

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[93] Favaro, *La Libreria di Galileo Galilei*, p. 257.

[94] Ibid, pp. 257-8. See Schöner, Andreas. *Gnomonice, hoc est de descriptionibus horologiorum sciotericorum omnis generis proiectionibus circulorum sphaericorum ad superficies cum planas tum convexas, concavasque, sphaericas, cylindricas, ac conicas*. Norimbergae: Apud Ioannem Montanum & Viricum Neuberum, 1562. Favaro has not noted that the copy of the Biblioteca Nazionale di Firenze (Magliabechiano 5. 2. 184) shows the 'W' as Viviani's mark on its frontispiece. Thus, this was Galileo's copy.

[95] Favaro, *La Libreria di Galileo Galilei*, pp. 260, 267-8. For Gemma Frisius' use of the surveying compass, see his 'De locorum describendorum ratione, & de eorum distantijs inveniendis, nunquàm antehac visus' in Apianus, Petrus. *Cosmographia Petri Apiani per Gemmam Frisium apud Lovanienses Medicum, & Mathematicum insignem*. Antverpiae: Ioannes Bellerus, 1578, ff. 52, 54-6.

corresponded with many of them, for example, Moleto and Contarini, about practical matters.[96] Although no correspondence about mathematical instruments seems to have been preserved, Galileo's interests in this period were related to the interest in mathematical instruments of the Pinelli circle. In 1599, Galileo hired a craftsman, Marcantonio Mazzoleni, to produce his recently developed geometric and military compass, therewith establishing a workshop. Some of these instruments Galileo used as a gift to members of the Pinelli circle, including Pinelli himself.[97] However, most of the instruments produced in Galileo's workshop were sold to elite students, aspiring a military career, who lived in Galileo's house and enjoyed his private education. An important aspect of this private education was the use of the geometric and military compass. Typically, he would sell to a student an instrument and a copy of the manual, which would develop into the 'Del compasso geometrico e militare', published in 1606, including some private education about its operations.

Drake, Schneider and Camerota have argued that Galileo's geometric and military compass was the culmination point of a Renaissance tradition of making instruments applicable to all areas of measuring.[98] During the second half of the sixteenth century a range of measuring instruments was available on the market. Some of these instruments were highly specialized, for example, Pifferi's 'monicometro', but often the same instrument was used for several practical mathematical activities.[99] Instruments better known for their astronomical purposes, for example, the astrolabe and the geometrical quadrant, were also used for purposes of terrestrial measurement.[100] To measure heights, depths and distances, the surveyor made use of the shadow square on the back of the astrolabe's mater and of the alidade in its centre.[101] (Figure 5.22) The 'umbra recta' is the shadow casted on the horizontal side of the shadow square of a vertically hold astrolabe, while the 'umbra versa' is the shadow casted on the vertical side of the shadow square. Both have scales divided into twelve parts, thus, running from zero to twelve at their intersection point.

[96] Three letters of Contarini and Galileo have been preserved, see Contarini to Galileo (22 December 1592); Galileo to Contarini (3 March 1593); Galileo to Contarini (22 March 1593), in Galileo, *Opere*, Vol. 10, pp. 52-60.

[97] Entry of 5 October 1599: 'uno strumento donato al S. Pinelli', in Galileo, *Opere*, Vol. 19, p. 147.

[98] Drake, Stillman. 'Tartaglia's Squadra and Galileo's Compasso.' *Annali dell' Istituto e Museo di Storia della Scienza* 2 (1977): 35-54; Camerota, *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*, pp. 126-30; Schneider, Ivo. *Der Proportionalzirkel: Ein Universelles Analogrecheninstrument der Vergangenheit*. Vol. 38, *Deutsches Museum Abhandlungen und Berichte*. München Düsseldorf: R. Oldenbourg Verlag VDI-Verlag Gmbh, 1970, pp. 5-64.

[99] Pifferi was a Carmelite monk and mathematician, who taught shortly at the university of Pisa before Galileo's arrival, and thereafter at the University of Siena. Pifferi's 'Monicometro instromento da misurar con la vista stando fermo' (1595) allowed to measure heights, depths and distances without the measurer needing to move to take more than one point of view. Pifferi, well known to Clavius, was present at Galileo's demonstration of the telescope in Rome in 1611. See Lattis, James M. *Between Copernicus and Galileo: Christoph Clavius and the Collapse of Ptolemaic Cosmology*. Chicago London: The University of Chicago Press, 1994, p. 188.

[100] Schechner Genuth, Sara, and Roderich and Marjorie Webster, eds. *Western Astrolabes*. Edited by Bruce Chandler. Vol. 1, *Historic Scientific Instruments of the Adler Planetarium & Astronomy Museum*. Chicago: Adler Planetarium & Astronomy Museum, 1998, pp. 18-24; Turner, Anthony. *Early Scientific Instruments Europe 1400-1800*. London: Sotheby's Publications, 1987, pp. 11-6. The astrolabe used for terrestrial measurement was widely discussed, for example, in Johann Stöffler, 'Elucidatio fabricae ususque astrolabii', 1524; Egnazio Danti, 'Trattato dell' uso et della fabbrica dell' astrolabio', 1569; Gemma Frisius, 'De astrolabo catholico liber', 1583.

[101] Camerota, *Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*, pp. 44-8.

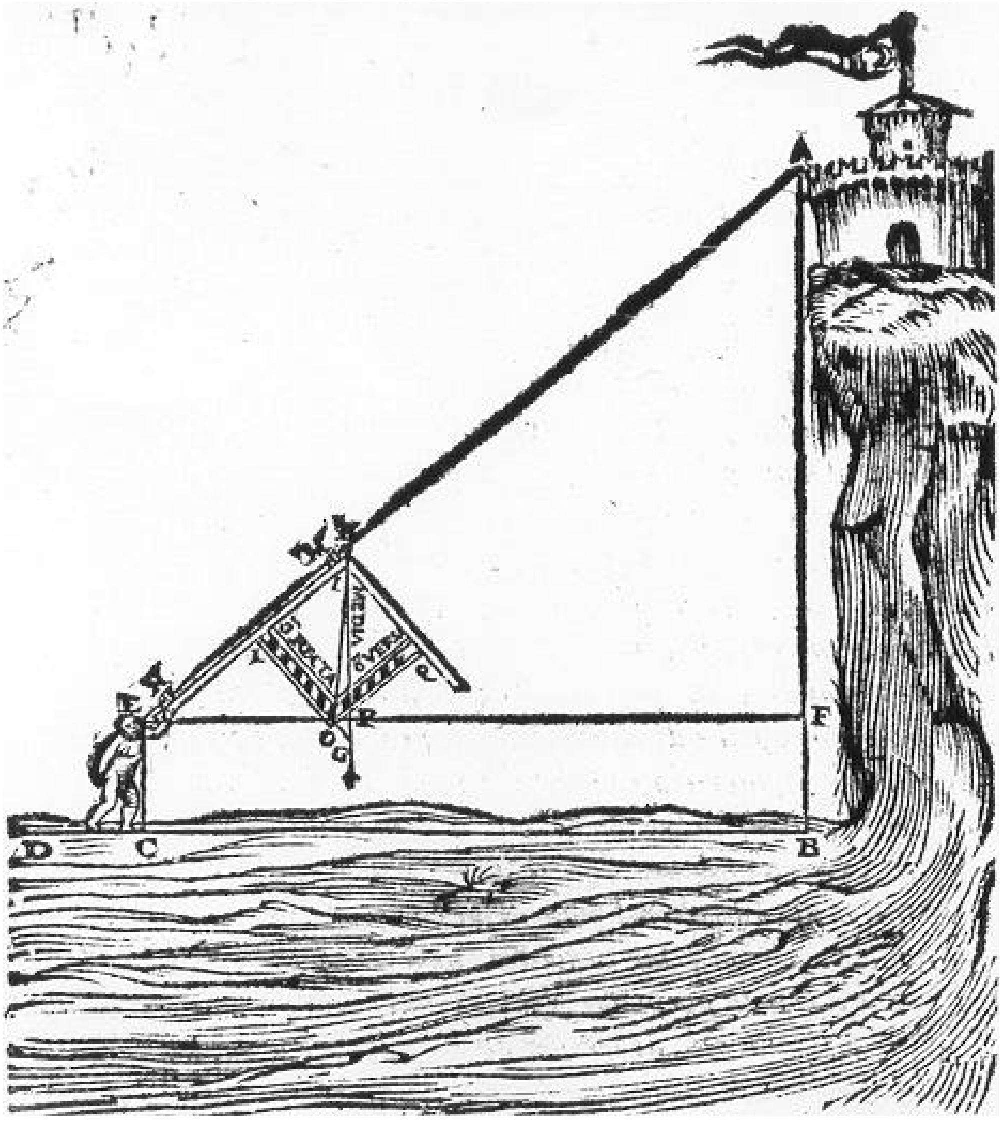

Figure 5.22

The ‘umbra versa’ is the tangent of the angle of inclination of the solar (or visual) ray, while the ‘umbra recta’ is its cotangent. For angles from 0° to 45° the solar or visual ray intersects the ‘umbra versa’, while for angles from 45° to 90° it intersects the ‘umbra recta’.

Measuring the height of the tower was based on triangulation, thus on constructing similar triangles by using the shadow square. When the distance from the sighting point to the tower was known, two similar triangles were formed by, on the one hand, the known distance to the tower, the alidade inclined to the top of the tower, and the unknown height of the tower, and, on the other hand, the alidade, one side of the shadow square and the segment of the ‘umbra versa’ or ‘umbra recta’ intersected by the alidade. The geometrical quadrant and the ‘quadrato geometrico’, were also used to measure depths, heights and distances.[102] To that end, both

[102] Ibid., pp. 140-1.

instruments had a shadow square that was used in the same way as one on an astrolabe, while loosing some of the astrolabe's features intended for astronomical purposes. The 'quadrato geometrico' was even nothing else but an enlarged shadow square, stripped of all other features for other purposes than terrestrial measurement, either in topography, cartography or surveying.

In general, at the end of the sixteenth century, the military concerns of the courtly patrons of the mathematicians provided the framework and the motivation for the boom in measuring instruments. It has often been observed that the perspective manuals of the end of the sixteenth century had an audience mainly interested in military fortifications, that the men, often known today as 'artists', also were assigned the function of supervising the military fortifications of their patron, and that instruments, often considered, helpful in a painter's workshop, like Alberti's veil or Dürer's 'sportello', were used for military topography and surveying.[103] Moreover, there was a natural link between surveying and military concerns, for example, range finding in gunnery.[104] The elevation of the cannon depended on the distance of the target, which was inaccessible at the risk of one's life. Surveying instruments, incorporating triangulation techniques, allowed measuring the distance to the target without approaching the enemy. To that end, Tartaglia invented the 'squadra dei bombardieri', presented in two versions. To measure the height and distance of a target, the 'squadra dei bombardieri' was equipped with two equal legs and a shadow square, that was operated in the same way as the shadow square of a quadrant, while to point a cannon to its target, one of the legs of the 'squadra dei bombardieri' was made longer so it could be inserted into the cannon's mouth. A plumb line suspended from the vertex showed the elevation of the cannon on the quadrant arc, divided into twelve equal parts, called 'points'.[105] (Figure 5.23)

Tartaglia's 'squadra dei bombardieri' had many rivals on the sixteenth century market, for example, the 'radio latino'. Danti's annotations to the 'Trattato del Radio Latino', an instrument invented by Orsini, explained all the uses such a measuring device could have for military men, like measuring heights, depths and distances, pointing a cannon at its target, making a ground plan of a fortification, mapping the environment or determining the position of the sun or the stars.[106] The 'archimetro' of Galileo's teacher Ricci was similar to the 'radio latino'.[107] (Figure 5.24) To measure by sight heights, depths and distances, similar triangles were not constructed by the use of a shadow square, as with the astrolabe and its variants, but by folding the legs of the instrument into a triangle similar to the one made by the visual rays of the measurer.

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[103] Hale, J.R. *Renaissance Fortification: Art or Engineering?* Norwich: Thames and Hudson, 1977. For the military application of Alberti's veil, used to draw the perspective image of a fortification, see Veltman, Kim H. 'Military Surveying and Topography: The Practical Dimension of Renaissance Linear Perspective.' *Revista da Universidade de Coimbra* 27 (1979): 329-68, pp. 339-44, 357-67. For the military audience of the treatises on perspective of, for example, Commandino, Benedetti and Guidobaldo, at the end of the sixteenth century, see Field, J.V. 'Perspective and the Mathematicians: Alberti to Desargues.' In *Mathematics from Manuscript to Print, 1300-1600*, edited by Cynthia Hay, 236-63. Oxford: Clarendon Press, 1988, in particular, pp. 256-62.

[104] Bennett, Jim, and Stephen Johnston. *The Geometry of War 1500-1750*. Oxford: University of Oxford Old Ashmolean Building Museum of the History of Science, 1996, p. 11.

[105] Tartaglia, Niccolò. *Quesiti et Inventioni Diverse*. Brescia: Ateneo di Brescia, 1959, pp. 5-35. See also Tartaglia, Niccolò. *Nova Scientia Inventa da Nicolo Tartalea*. In Vinegia: Per Stephano da Stabio, 1537, Book III.

[106] Danti, Egnatio. *Trattato del radio latino: Istrumento giustissimo & facile più d' ogni altro per prendere qual si voglia misura, & positione di luogo*. Roma: Apresso Marc' Antonio Moretti & Iacomo Brianzi, 1586.

[107] Vinci, *Ostilio Ricci da Fermo, maestro di Galileo Galilei*, pp. 23-9.

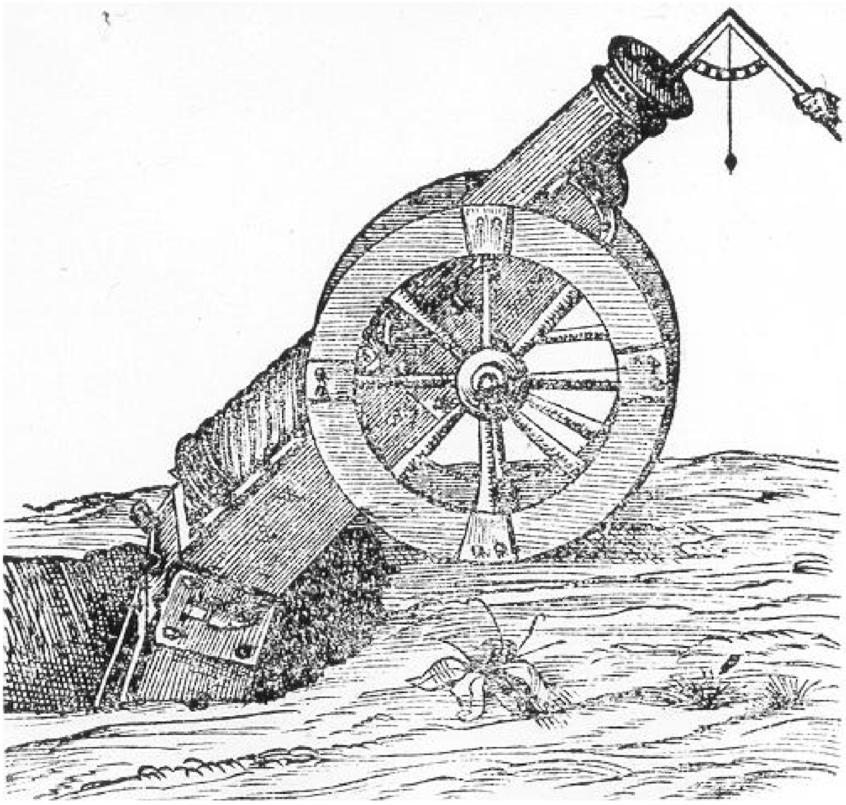

Figure 5.23

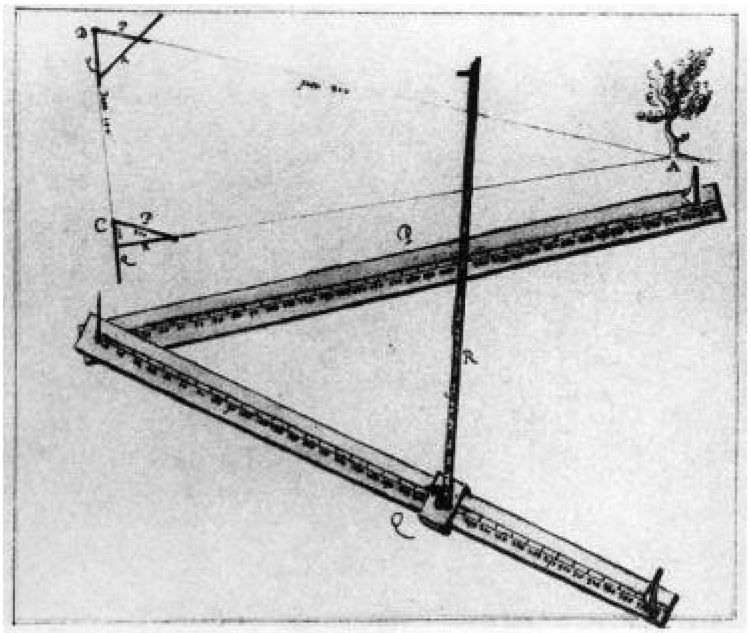

Figure 5.24

The archimetro consisted of three folding legs, scaled to avoid making calculations. To measure the height of a tower, a sighting point was chosen at a known distance from the tower. Two legs were folded making a right angle, one parallel to the horizon, the other parallel to the tower. The third leg, with a sighting device, was placed along the leg parallel to the horizon at the point of the scale corresponding to the distance to the tower. Then, the third leg was folded along the visual ray sighting the top of the tower, intersecting the leg parallel to the tower. The scale on the latter leg indicated the height of the tower, because of the similarity of the triangle made by the legs of the 'archimetro' and the triangle of the visual rays sigthing the top and the base of the tower.

In 1597, the first version of Galileo's geometric and military compass resembled Tartaglia's 'squadra dei bombardieri', but Galileo made it foldable for ease of use, like Ricci's 'archimetro', and he arranged that the gunner would no longer have to expose himself to danger by applying the 'squadra dei bombardieri' to the cannon's mouth.[108] However, Galileo's first version also had the functions of Guidobaldo's proportional compass to divide lines and circumferences into equal parts and to construct regular polygons.[109] After 1599, Guidobaldo's scales were removed for other scales that made Galileo's compass into a universal mechanical calculator. For surveying purposes, detailed arithmetical calculations of a triangulation were now no longer necessary, because Galileo's compass allowed doing this mechanically by using the arithmetic scale on the compass' legs. On earlier instruments, to measure the height of a tower, the shadow square with its umbra versa and its umbra recta, each divided into 100 parts, and a plumb line were used, in the same way as the shadow square on a geometrical quadrant. When the height of the tower exceeded the distance of the measurer to the tower, the plumb line cut the 'umbra retta'. In that case, to know the height of the tower, the 'umbra retta' had to be converted into 'umbra versa'.

This calculation was necessary with earlier instruments, but by using a scale divided into 100 parts, instead of 12, as was usual, the calculation was made easier. To find the value of the 'umbra versa' it sufficed to divide 10000 by the value of the 'umbra retta', because the 'umbra retta' divided by 100 equals 100 divided by the 'umbra versa'. However, for the military men using Galileo's compass it even was no longer necessary to make this simple calculation, because it could be done mechanically.[110] The aperture of the compass was made 100 points at the value of the 'umbra retta' set out along the scale. Then, without altering the aperture of the instrument, it was measured at the value of 100 along the scale. This latter aperture was the value of the 'umbra versa'. For example, when measuring the height of a tower, the plumb line cut the 'umbra

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[108] Galileo, Le Operazioni del Compasso Geometrico e Militare, in Galileo, *Opere*, Vol. 2, pp. 412-3, translation in Galilei, Galileo. *Operations of the Geometric and Military Compass 1606*. Translated by Stillman Drake. Vol. 1, *Dibner Library National Museum of History and Technology.* Washington, D.C.: Smithsonian Institution Press, pp. 79-80. For discussion, see Drake, 'Tartaglia's Squadra and Galileo's Compasso, pp. 40-3.

[109] The proportional compass is a further development of the reduction compass. The redution compass allowed to divide lines in equal parts or according to a given proportion and to divide circumferences in grades and minutes. The invention of the proportional compass is attributed to Guidobaldo del Monte, student of Federico Commandino, but the first complete elaboration is due to Michel Coignet. Beside the operations of the reduction compass, the proportional compass also allowed to find the length of the sides of regular polygons inscribed in a circle. See Rose, Paul L. 'The Origins of the Proportional Compass from Mordente to Galileo.' *Physis* 17 (1968): 53-69; Rosen, Edward. 'The Invention of the Reduction Compass.' *Physis* 10 (1968): 306-8; Camerota, *Il Compasso di Fabrizio Mordente: Per la Storia del Compasso di Proporzione*, pp. 9-128.

[110] Galileo, Le Operazioni del Compasso Geometrico e Militare, in Galileo, *Opere*, Vol. 2, pp. 414-24, translation in Galileo, *Operations of the Geometric and Military Compass*, p. 80-92. For discussion, see Camerota, *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*, pp. 128-30.

retta' at the value of 50. This value was mechanically converted into a 200 value of the 'umbra versa'. When the measurer was 100 units away from the base of the tower, this value of the 'umbra versa' also was the height of the tower, without any need for more calculations.

While it is well-known that Galileo's workshop produced his geometric and military compasses, and that, when the produced instruments were not meant as gifts to patrons, this activity was related to his private education, the production of surveying compasses or 'bussole' in Galileo's workshop seems, for some reason, to have escaped attention. Moreover, again, Galileo's workshop activity and his private education are interrelated. Galileo's 'Ricordi autografi' show that about 15 'bussole' were made between 1599 and 1609, a smaller number than the number of 98 geometric and military compasses (and related instruments) that were produced in the same period, but still a considerable number. Moreover, the records also show that most of these instruments were sold to students taking private classes with Galileo.[111]

It might have been possible to be taught the use of the surveying compass as a separate subject, but it is more likely that the teaching of its use was part of Galileo's larger course on military fortification. Indeed, one of Galileo's manuals on military fortification shows its use to contruct 'minie', a traditional subject in treatises on military architecture.[112] (Figure 5.25)

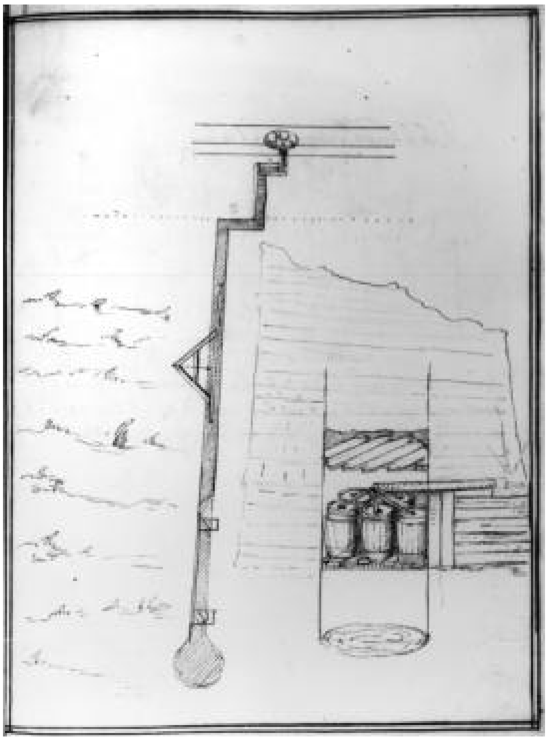

Figure 5.25

[111] Galileo, Ricordi autografi, in Galileo, *Opere*, Vol. 19, pp. 133-49. I would like to thank Matteo Valleriani, who is currently working on the 'Ricordi autografi', for counting the number of the geometric and military compasses.

[112] Galileo, Breve instruzione all' architettura militare, in Galileo, *Opere*, Vol. 2, pp. 46-7. Galileo's method is identical to the one described by Giulio Parigi, Bibliothèque Nationale de France (Paris), Italien 486, ff. 29v-30r.

Moreover, when ca. 1610 Pietro Duodo asked Galileo for advice on what to teach concerning mathematics at the newly founded Accademia Delia, intended to provide aspiring military men with an appropriate education, Galileo recommended, among other things, the teaching of the use of the 'bussola', and 'other instruments', to draw a plan.[113] This suggests that Galileo considered teaching the use of the surveying compass essential to the education of military men. In fact, there was a tradition, to which Galileo belonged, that considered the 'bussola' part of the science of military fortification. For example, in his 'Quesiti et inventioni diverse', Tartaglia taught the use of the surveying compass for topographical purposes along with the science of military fortification.

To the best of my knowledge, no drawing of Galileo's surveying compass has survived. Most likely, Galileo's surveying compass was not different from Tartaglia's in his 'Quesiti et inventioni diverse', in itself derivative of Alberti's 'orrizonte' of the 'Ludi matematici', the manual used as textbook by Galileo's teacher Ostilio Ricci. Both instruments were used to draw a map, that is for topographical purposes. Alberti's 'orrizonte' consisted of a circle divided into equal parts along its circumference.[114] A magnetic compass was used to orient the 'orrizonte' with its 0° towards magnetic north. By aligning the center of the instrument and the object, often a remarkable feature of the land, using a plumb line, the polar coordinate of the object was read from the scale on the circumference of the instrument. The position of an object was determined by taking its polar coordinates from two different sighting points, with the distance between the two sighting points known. A topographical map was made by transferring the polar coordinates of each object to a piece of paper. Tartaglia's 'bussola' was operated in the same way. (Figure 5.26) The most important difference was the scale used along the circumference of the disc of the instrument. In Tartaglia's case, it was divided using eight directions of the wind, with each part divived into 45°, making up for a total of 360°, allowing more precise measurements to be taken than with Alberti's 'orrizonte'. Also, the magnetic compass was now inset, in order to align the appropriate direction of the wind, that is 'tramontana', with magnetic north. Finally, one version of Tartaglia's surveying compass was equipped with sighting vanes in order to align more easily the sighted object and the center of the instrument.[115]

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[113] 'Cognizione della bussola e di altri strumenti, per torre in disegno ogni sorta di pianta, cosi da vicino come da lontano' in Galileo, 'Raccolta di quelle cognizioni che a perfetto cavaliero e soldato si richieggono, le quali hanno dependenza dalle scienze matematiche'. Galileo, *Opere*, Vol. 2, p. 607. Beside the use of the surveying compass, Galileo also recommended the teaching of (1) arithmetics 'per l' uso delle ordinanze degli eserciti e di molte altre occorrenze', (2) geometry 'per misurare ogni pianta superficiale, tanto regolare quanto irregolare, e per misurare tutte le figure e corpi solidi', (3) mechanics, (4) artillery, (5) the use of instruments to measure by sight heights, distances, and depths, and to level sites, (5) how to draw in perspective 'per la quale le fortezze e tutte le loro parti, come anche ogni machina e strumento bellico, si possa rappresentare e porre avanti gli occhi', and (6) military architecture. Together, this course proposal of Galileo must have made a course very similar to Giulio Parigi's. On Galileo and the Accademia Delia, see Quaranta, Mario. 'Galileo Galilei e l' Accademia Delia di Padova.' In *Galileo e la Cultura Padovana*, edited by Giovanni Santinello, 203-31. Padova: CEDAM, 1992.

[114] Alberti, 'Ludi Rerum Mathematicarum', pp. 163-9. See also Camerota, *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*, p. 144.

[115] Tartaglia, *Quesiti et Inventioni Diverse*, pp. 54-63. See also Filippo Camerota, *Giorgio Vasari il Giovane: Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*, pp. 145-6.

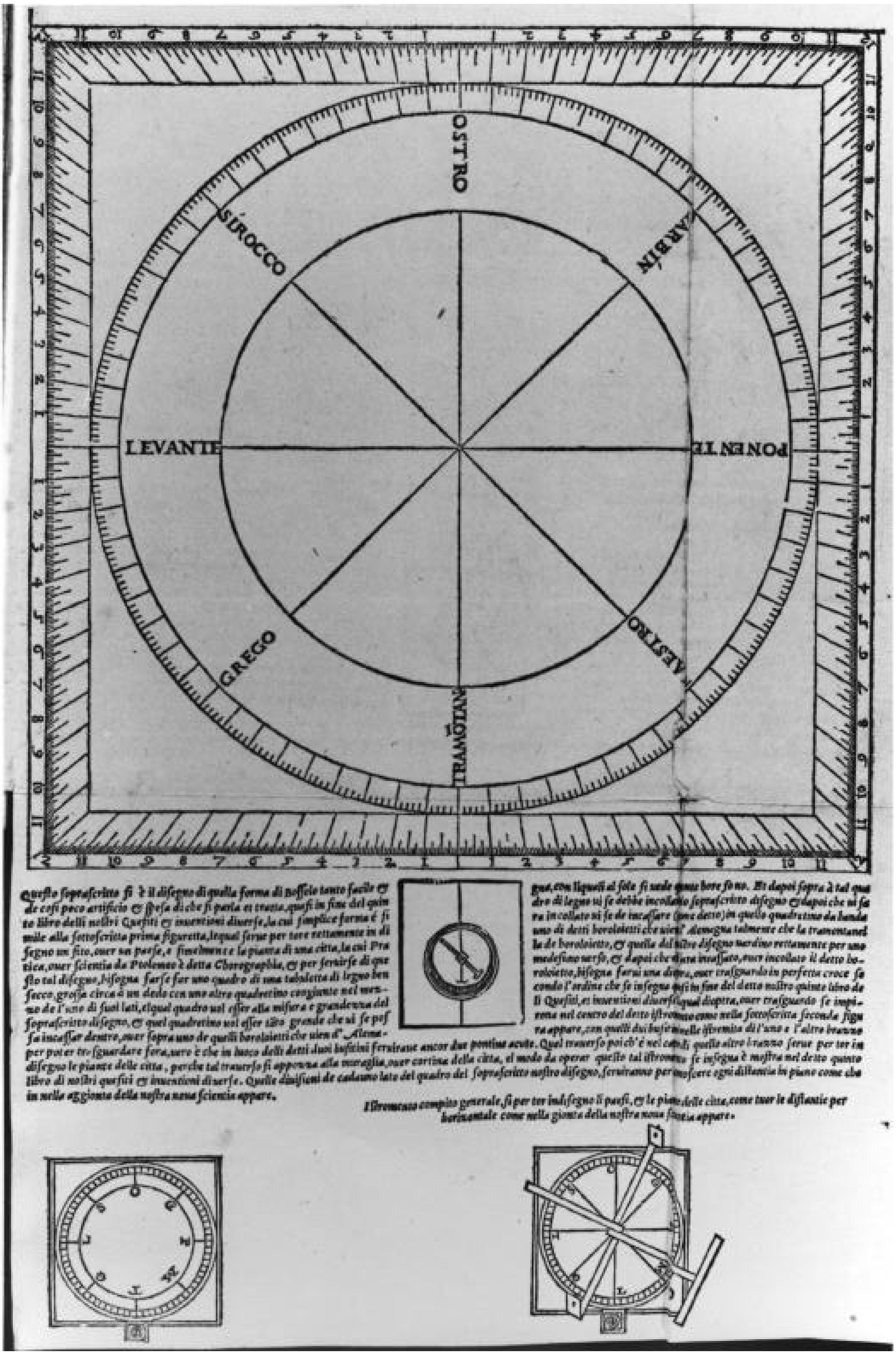
OSTRO
SIROCCO
GARBIN
LEVANTE
PONENTE
GREGO
MAESTRO
TRAMONTAN

Figure 5.26

Galileo's 'Ricordi autografi' also show that to some of his students he was teaching a course on perspective.[116] Unlike his course on military fortification, none of Galileo's course notes on perspective have been preserved. However, Viviani, Galileo's last student and first biographer, gave some indication of what this course must have been about. In his biography of Galileo, Viviani wrote that 'for the benefit of his students, at that time, Galileo wrote several treatises, among others, one about fortification, according the the use of those days, one about gnomonics or practical perspective, a compendium about the sphere, and a treatise on mechanics'.[117] Thus, if Viviani is reliable, Galileo's course on perspective was about gnomonics. From the perspective of the disciplinary categories of those days, this does not need to be a surprise. In the Vitruvian tradition, confirmed in Barbaro's sixteenth century commentary, gnomonics was a part of architecture, as was mechanics. Although there was a tendency for architecture, on the one hand, and mechanics, and military fortification that was associated with it, on the other hand, to move farther apart during the sixteenth century, they were still sufficiently associated in the mind of a mathematician like Galileo, to be both part of the education offered to military men in training.[118]

However, unlike for the surveying compass, the 'Ricordi autografi' do not show a larger scale production of sundials or other mathematical instruments. An entry, dated 4 July 1602, mentions an 'orologio', without further specifications, which Galileo sold to his assistant Mazzoleni.[119] In fact, the only clock Galileo is known to have most likely made was a magnetic water-clock that, near the end of his life, he discussed with Peiresc as made 'many years ago'.[120] Also, an entry for June 1602, shows that Galileo asked Mazzoleni for a large plate of messing, 'like the one left to me to make the aranea of an astrolabe'.[121] An entry like this suggests that mathematical instruments like astrolabes were made by Galileo and Mazzoleni, but most likely, for personal use. Different from the surveying and the geometric and military compass, there is no evidence for a systematic relation between Galileo's course on perspective and any instruments made in his workshop. Consequently, notwithstanding the lack of material evidence for this particular

[116] The entries for 1601 mention three German students, 'S. Alberto et suo compagno' and 'S. Consigliero' who took classes on perspective taught by Galileo. Galileo, Ricordi Autografi, in Galileo, *Opere*, Vol. 19, p. 150.

[117] 'et a contemplazione de' suoi scolari scrisse allora varii trattati, tra' quali uno di fortificazione, secondo l'uso di quei tempi, uno di gnomonica e prospettiva pratica, un compendio di sfera, et un trattato di mecchaniche'. Viviani, Racconto istorico della vita di Galileo, in Galileo, *Opere*, Vol. 19, p. 606.

[118] According to Vitruvius, there are three areas of activity for the architect: 'l' edificatione, la lineatione regolata per l' ombre de' stili,e l' arte di far le machine, ossia l' Edificatione o fabrica, la Gnomonica o l' arte di far gli horologi, e la Machinatione o l' arte di far le machine'. Barbaro, *I Dieci Libri dell' Architettura*, p. 27. For discussion, see Wilkinson, Catherine. 'Renaissance Treatises on Military Architecture and the Science of Mechanics.' In *Les Traités d' Architecture de la Renaissance: Actes du Colloque Tenu à Tours du 1er au 11 Juillet 1981*, edited by Jean Guillaume, 467-76. Paris: Picard, 1988. See also De La Croix, Horst. 'The Literature on Fortification in Renaissance Italy.' *Technology and Culture* 4 (1963): 30-50.

[119] Galileo, Ricordi autografi, in Galileo, *Opere*, Vol. 19, p. 136.

[120] 'The water-clock will truly be a thing of extreme marvel if it is true that the globe suspended in the middle of the water goes naturally turning by an occult magnetic force. Many years ago I made a similar invention, but with the aid of a deceptive artifice'. Galileo to Peiresc, 12 May 1635, in Dibner, Bern, and Stillman Drake. *A Letter from Galileo*. Norwalk, Connecticut: Burndy Library, 1967, p. 52. For this bogus device and how it relates to Kircher's sunflower clock, see Hankins, Thomas L., and Robert J. Silverman. *Instruments and the Imagination*. Princeton, New Jersey: Princeton University Press, 1995, pp. 14-36.

[121] 'E più deve darmi una piastra tonda di ottone todesco grande, come quella che mi è restata per far l' aranea d' un astrolabio'. Galileo, Ricordi autografi, in Galileo, *Opere*, Vol. 19, p. 135.

aspect of instrument-making, Galileo's training, his library and the courses offered privately to his students show that he must have been familiar with the mathematical aspects, in particular perspective, of mathematical instrument design. Moreover, Galileo applied his knowledge of perspective, as embodied in mathematical instruments, in the controversy on sunspots.

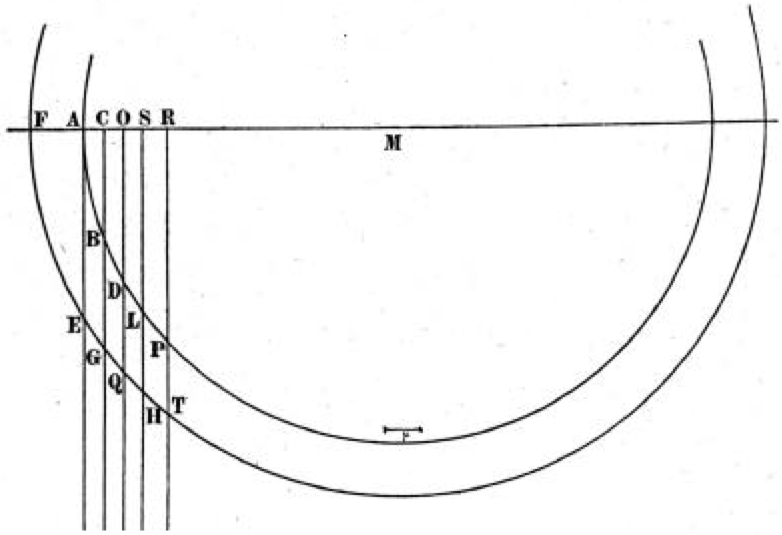

Figure 5.27

In his second and, again, in his third letter to Welser, Galileo refuted Scheiner's interpretation of the sunspots, using a diagram to compare the appearance of the spots when satellites with when they are considered to be in the immediate vicinity of the sun's surface.[122] (Figure 5.27) ABD is the circle of the circumference of the sun on which the spot μ appeared to be moving. The eye of the observer on Earth was considered to be in the same plane. Moreover, different from Scheiner, Galileo realized that the eye was 'at such a large distance, that the rays produced from the eye to the diameter of the sun are like parallel lines'.[123] When the spot was on the surface of the sun, occupying successively the intervals PL, LD, DB, BA, its size would diminish according to the ratios RS, SO, OC, CA, obtained by projecting orthographically these intervals onto the plane through M, the center of the circle described by the moving spot μ. When the spot was a satellite and moved in an orbit around the sun through the points T, H, Q, G, E, as Scheiner claimed, the ratios of diminution would be different. Consequently, Galileo showed that the ratios of diminution obtained when the spots are on the surface of the sun were in agreement with his measurements of the size of the spots, obtained from very detailed drawings that were the result of recording the projection of the sun's image through a telescope directly on paper.

[122] Galileo, Istoria e dimostrazioni intorno alle macchie solari, in Galileo, *Opere*, Vol. 5, pp. 121-2, 213-4.

[123] 'in lontananza immensa, tal che i raggi di quello prodotti al diametro di esso sieno come linee paralelle'. Galileo, Istoria e dimostrazioni intorno alle macchie solari, in Galileo, *Opere*, Vol. 5, p. 213.

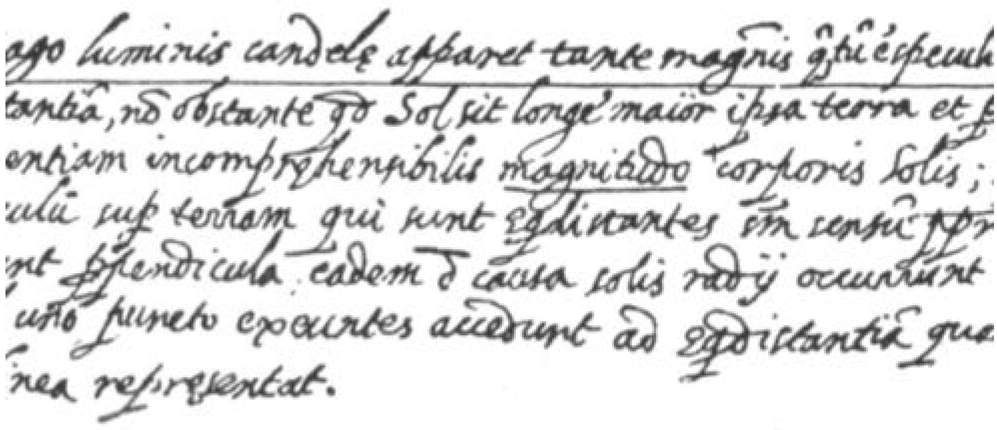
ago luminis candele apparet tante magnis q̄ tu è specul
tantia, nō obstante q̄d Sol sit longe maior ipsa terra et
entiam incomprehensibilis magnitudo corporis solis;
ulū sup terram qui sunt equidistantes im sensu
nt perpendicula eadem d causa solis radij occurrunt
uno puncto concurrentes accedunt ad equidistantia que
nea representat.

Figure 5.28

The application of an orthographic projection in this context was not self-evident. At the end of the sixteenth century, it was well known from medieval perspectiva that the rays of the sun had to be considered parallel, although, at the end of the sixteenth century, there still was some confusion about the reason why. For example, in a letter of 1584, Moleto explained that this was the case, not because of the size of the sun, but because of its distance from the Earth, referring to and repeating the appropriate theorem from Witelo.[124] As has been shown, Galileo was well acquainted with this proposition, since he underlined a word of the reference to Witelo's theorem in his copy of Ausonio's 'Theorica'.[125] (Figure 5.28) However, although the parallel rays of the sun were a well-known proposition of optics, the reference to the orthographic projection as a projection by the parallel rays of the sun was absent from contemporary literature on the analemma or the astrolabe. The equivalence of the orthographic projection with a projection along the parallel rays of the sun had to wait until the publication of the 'Opticorum libri sex' of Aguilon, who also coined the terms 'stereographic' and 'orthographic'. Aguilon took the itch out of Guidobaldo's remark that the orthographic projection could not be considered perspective, because the eye was at infinity, by interpreting infinity as exceeding any reasonable distance. However, Galileo's diagram in his letter on sunspots preceded Aguilon in considering the orthographic projection as a projection along the parallel rays of the sun.[126]

Moreover, any exclusively painterly influence on Galileo for this particular problem can be excluded. Again, it was well known, also to painters, that the rays of the sun should be considered parallel.

[124] Moleto to Pinelli, 5 February 1566, B. A. M., A 71 Inf., ff. 1r-1v, in reply to a question concerning the shadows produced by gnomons, using proposition 35 of book 2 of Witelo's 'Perspectiva'.

[125] The word 'magnitudo' is underlined. *Theorica speculi concavi sphaerici*. See Appendix II.

[126] 'haud erit difficile proiectionis rationem cum infinita oculi distantia conciliare, cum non absolute infinita intelligatur; sed admodum magna, quae iusti intervalli fines excedat'. Aguilonius, Franciscus. *Opticorum libri VI*. Antverpiae: Plantin, 1613, pp. 521, 503-4. Thus, in this context, Aguilon referred to the shadows of gnomons caused by the parallel rays of the sun. Galileo was independent of Aguilon, because, only in 1614, he became acquainted with Aguilon's 'Opticorum libri sex'. See Sagredo to Galileo, 10 April 1614, in Galileo, *Opere*, Vol. 12, p. 51.

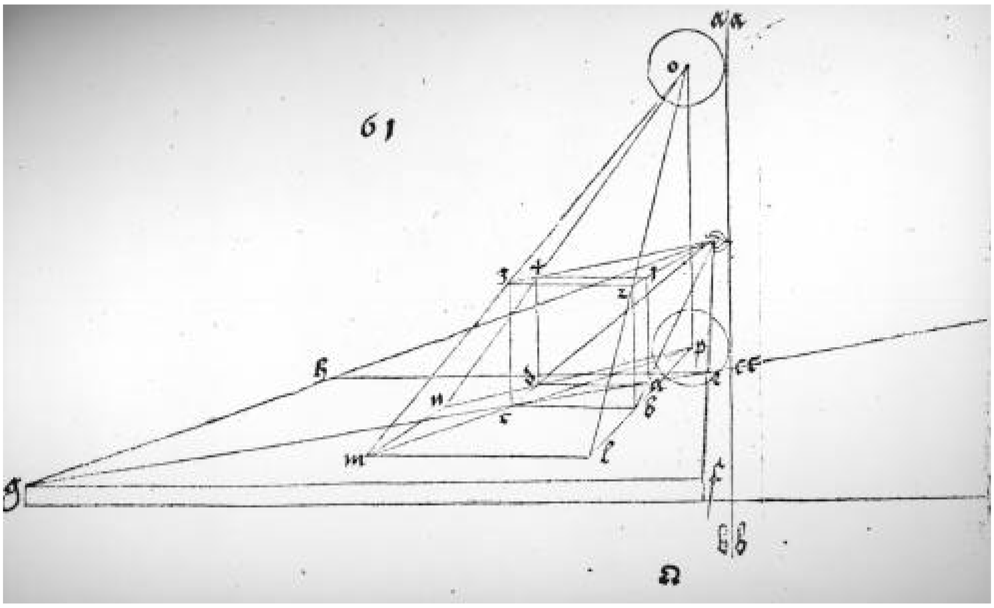

Figure 5.29

However, when discussed in the context where it was relevant for painters, that is the drawing of cast shadows, as developed by Dürer in his 'Underweysung der Messung', only sources of light at a finite distance were used. Moreover, by showing in his final perspective drawing a light source that could easily be interpreted as the sun, while having used a source of light at a finite distance in the plan and elevation drawing, this could have been understood as if Dürer was confusing solar light and candlelight.[127] (Figure 5.29 – Figure 5.30) This was how it was understood by Aguilon, when he reminded painters of the fact that 'those things which are exposed to the direct rays of the sun are to be represented orthographice'.[128] Thus, the concept of a projection along the parallel rays of the sun did not arise in this painterly context.

[127] For Dürer's discussion of shadow in his 'Underweysung der Messung', see Kaufmann, Thomas DaCosta. *The Mastery of Nature: Aspects of Art, Science, and Humanism in the Renaissance*. Princeton, New Jersey: Princeton University Press, 1993, pp. 49-78. Hallyn has argued that Galileo, when discussing shadows cast by lunar mountains in a letter to Grienberger of 1 September 1611, implied the sun to be at a finite distance, as if he was interpreting Dürer's geometrical method of shadow projection as applying to solar shadow projection. However, since in Galileo's accompanying diagram the rays issuing from the sun and the visual rays were drawn as a bundle of rays, while strictly speaking both should be drawn parallel, this diagram should rather be interpreted as a conceptual-visual aid than as explaining the geometry of solar shadow projection. See Hallyn, 'Le Regard Picturale de Galilée sur la Lune', pp. 32-3. For the diagram, see Galileo to Grienberger, 1 September 1611 in Galileo, *Opere*, Vol. 11, p. 185.

[128] 'quae autem directos solis radios admittunt, orthographice sunt designanda'. Aguilonius, *Opticorum libri VI*, p. 683, translation in Kaufmann, *The Mastery of Nature*, p. 73. Of course, drawing a shadow casted by the sun in perspective was not done using an orthographical projection. A vanishing point needs to be determined. Absent from Kaufmann's discussion of the theory of shadow projection is the work of Galileo's friend Ludovico Cigoli. Cigoli was the first to succeed in drawing the shadow cast by the rays of the sun by determining the position of the vanishing point specific to the direction of the solar rays, considered parallel among them. This could be done due to the availibility of Guidobaldo's general theorem of the vanishing point. See Profumo, Rodolfo, ed. *Trattato Pratico di Prospettiva di Ludovico Cardi Detto Il Cigoli*. Roma: Bonsignori Editore, 1992, pp. 143-7.

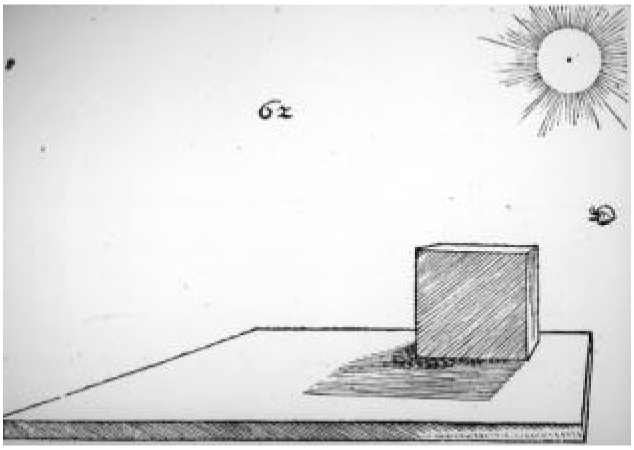

Figure 5.30

Consequently, while Galileo's drawings and interpretation of his astronomical observations show his mastery of disegno, the knowledge of perspective and projection techniques embodied in mathematical instruments, with which Galileo as a mathematical practitioner was acquainted, also accounts for Galileo's knowledge of perspective. In chapter 2, it has been argued that when mathematical practitioners appropriated optics in the sixteenth century, the aims of optics changed from vision, perception and cognition to solving more limited problems of measurement within a Euclidean framework and instrument design. Since Galileo was such a sixteenth century mathematical practitioner, well acquainted with instrument design and perpspective embodied in these instruments, he must have shared the aims of optics as formulated in the tradition of mathematical practitioners. This is confirmed by the kind of teaching of optics in Ricci's class. Moreover, as will be shown, Ricci's teaching of optics was not limited to perspective, as is usually assumed. It also included the study of catoptrics and dioptrics.

4. The Catoptrics of a Mathematical Practitioner

In the MS Riccardiana 2110, a copy of Alberti's 'Ludi matematici', as has been discussed, a textbook used by Ricci, is bound together with a manual on perspectiva, known as 'Della prospettiva'. While Settle thought this manuscript copy was originally made for Giovanni de' Medici, one of Galileo's fellow students in Ricci's class on perspective, and the treatise on perspective possibly was, like the 'Ludi matematici', also written by Alberti, Parronchi, Battisti and Battisti, and De Nil have since revised that claim.[129] None of the latter scholars noted that this manuscript belonged to Giovanni de 'Medici at the end of the sixteenth century. Settle, on the other hand, was wrong in thinking that it was originally made for Giovanni de' Medici, although the fact that it at one time belonged to Giovanni de' Medici, and Settle's conclusion that, therefore, it was used as a textbook on perspective in Ricci's class still stands.

[129] Settle, 'Ostilio Ricci, a Bridge between Alberti and Galileo', p. 125; Parronchi, Alessandro. *Studi su la Dolce Prospettiva*. Milano: Aldo Martello Editore, 1964, pp. 583-641; Battisti and Battisti, *Le Macchine Cifrate di Giovanni Fontana*, p. 24; De Nil, Erwin. 'Della Prospettiva, een Revelerend Handschrift.' In *Perspectiva tussen Aristoteles en Zeki*, edited by Marc De Mey and Erwin De Nil, 87-192. Gent: Communicatie & Cognitie.

In fact, Parronchi noted that, before it came into the hands of Giovanni de' Medici, it belonged to Benedetto Varchi, whose name was later scratched out on the first page of the manuscript.[130] However, while Parronchi thought the authorship of the 'Della prospettiva' could be attributed to Toscanelli, De Nil has argued that the evidence for this is inconclusive.[131] In fact, Battisti and Battisti have attributed the 'Della prospettiva' to Giovanni Fontana, based on the similarity of the unknown addressee of the 'Della Prospettiva', Poliseo or Polixeo, to the adressed Polixcus or Poliscus in other work of Fontana, the 'Horologium aqueum'.[132] The 'Della prospettiva' presumably is the lost treatise on perspective, dedicated to Jacopo Bellini, of Giovanni Fontana. De Nil's argument that, because of the similarities of content between the 'Della prospettiva' and the optical work of Biagio Pelacani, the 'Della prospettiva' should be attributed to someone of the circle of Biagio Pelacani, support the claim of Battisti and Battisti that Giovanni Fontana, a student of Pelacani, is most likely the author of this treatise on perspective.

Fontana's 'Della prospettiva' can be characterized as an introductory course on optics. There is nothing exceptional about this treatise, but, besides showing that Galileo must have been acquainted with not only perspective, but also catoptrics and dioptrics, at an early age, it gives some indications about Galileo's early knowledge of optics. Fontana's 'Della prospettiva' was built around the traditional tripartite division of optics into direct vision, reflection and refraction, However, most attention was devoted to catoptrics, and, to a lesser extent, dioptrics. Fontana's discussion of direct vision was limited to the mathematics of the visual pyramid. Fontana introduced the medium, and the species in the medium, as a prerequisite of vision.

> Secondly, it needs to be noted with the utmost care, dear Polixeo, that every visible thing, of whatever finite quantity or figure, has from its own nature, according to what some perspectivists and philosophers say, the possibility to produce by celestial influences some invisible and occult qualities in the medium, which are multiplicated all around over a certain distance in the form of rays.[133]

Thus, in the footsteps of his teacher Pelacani, Fontana considered the multiplication of the species in the medium to be equivalent with the rays of the visual pyramid.[134] Consequently, Fontana's discussion of direct vision is limited to the mathematics of the visual pyramid.

> But note, dear Polixeo, that the thing is seen by a visual pyramid. Whence you need to know that the pyramid is in the form of a triangle. When a thing is seen, a ray proceeds from every point of the thing which ends in the eye of the observer, from the point in the middle, whichever it is, as from the extremes. Therefore, by necessity a triangle is formed, and this triangle has two extremities and a basis, which in this

[130] Parronchi, *Studi su la Dolce Prospettiva*, pp. 594-8.

[131] De Nil, 'Della Prospettiva, een Revelerend Handschrift', pp. 93-7.

[132] Battisti and Battisti, *Le Macchine Cifrate di Giovanni Fontana*, p. 24

[133] 'E da notare secondariamente con diligentia, Polixeo carissimo, che ogni cosa visibile in qualunque quantità finita, o vero figura si fusse, ha da sua propria natura, secondo el detto d' alcuni perspettivi, o vero secondo il detto d' alcuni filosofi, dalle influentie celestiali poter producere in el spatio mezzo alcune qualità invisibili et occulte le quali sonno multiplicate per certa distantia in circuito in forma di razi.' Parronchi, *Studi*, p. 600.

[134] For discussion, see Frangenberg, Thomas. 'Perspectivist Aristotelianism: Three Case-Studies of Cinquecento Visual Theory.' *Journal of the Warburg and Courtauld Institutes* 54 (1991): 137-58, pp. 142-4.

> case is nothing else but the object seen. And this complete triangle from the object seen to the eye is called the pyramid of rays, by which this eye sees the already mentioned object.[135]

After a short introduction to the mathematics of angles, Fontana stated that the Euclidean principle that the size of an observed object is dependent upon the size of the angle formed by the visual pyramid.[136] Fontana did not discuss ocular physiology, psychology or epistemology.
After the introduction of the familiar mechanical analogy of projectile motion and the reflection of visual rays and the law of equal angles, Fontana discussed the Euclidean proposition, present in many contemporaneous manuals on measuring by sight, to measure the height of the mirror by the similar triangles formed by the tower's reflection in a mirror located horizontally between the measurer and the tower.[137] (Figure 5.31)

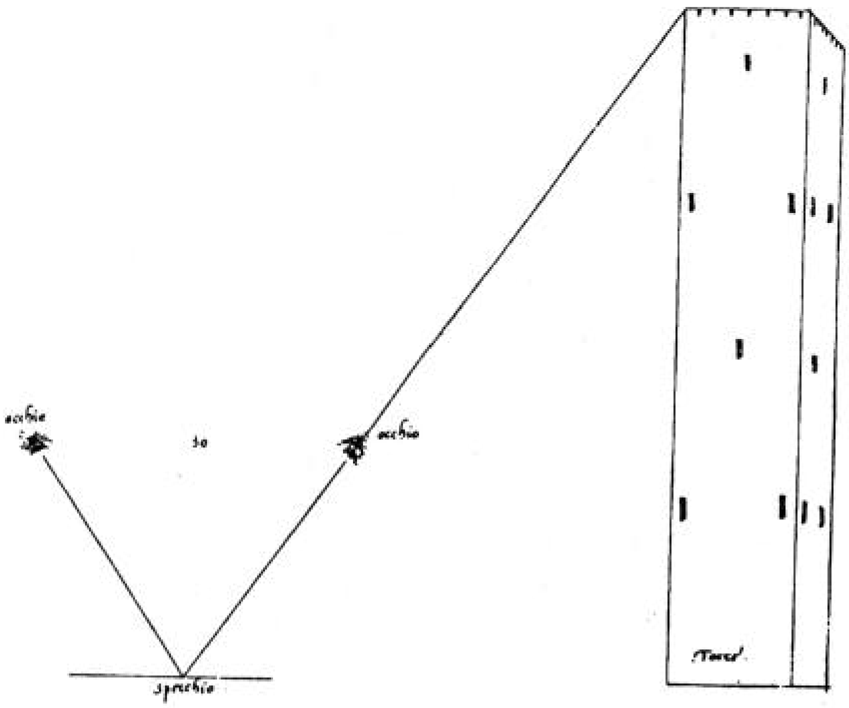

Figure 5.31

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[135] 'Ma nota, Polixeo carissimo, che la cosa ch' è si vede per una piramide. Onde tu de' sapere che la piramide è in forma de triangolo. Quando la cosa si vede, da ciascun punto della cosa procede uno razo terminando nell' occhio di colui che vede cosi dal punto di mezo qualunque si sia come dalli estremi, per la qual cosa è di necessitade che si formi uno triangolo, et queto triangolo ha due extremità e la basis, la qual non è altro al presente se non la cosa veduta. Et tutto questo triangolo dalla cosa veduto fino all' occhio si chiama la piramide radiosa, per la qualequesto occhio vede la predetta cosa'. Parronchi, *Studi su la Dolce Prospettiva*, p. 601.

[136] Ibid., pp. 601-4.

[137] Ibid., pp. 610-11.

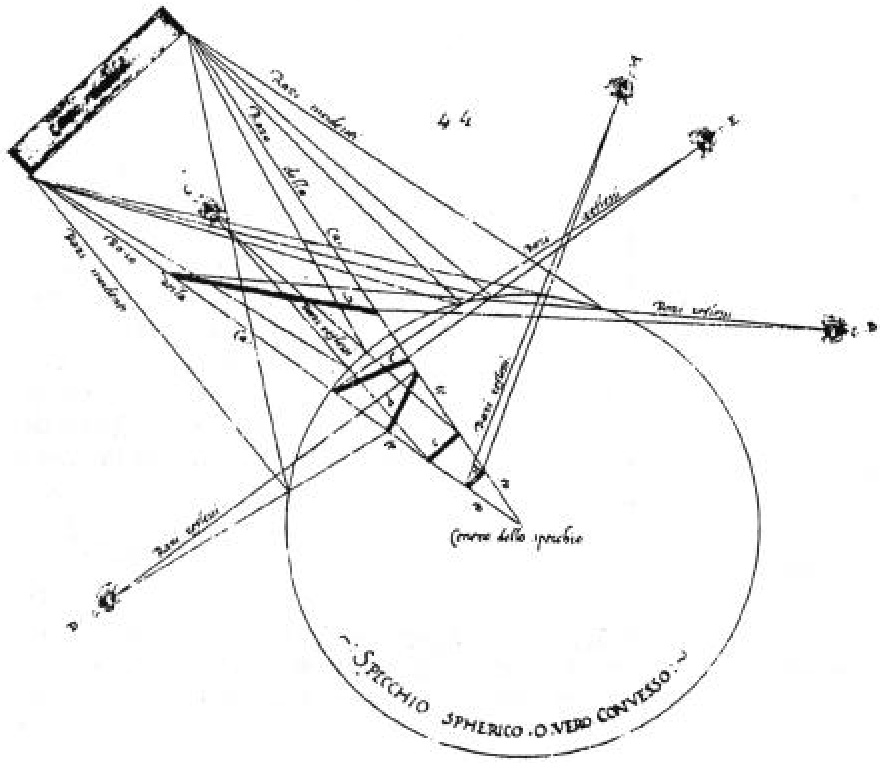

Figure 5.32

The remainder of Fontana's section on catoptrics was completely devoted to the application of the cathetus rule to plane, convex and concave spherical mirrors to determine the loci of images.[138] Fontana's discussion is competent, with the exception of the location of an image outside a convex mirror, reminiscent of Dee's mistake, as discussed in chapter 2, and it has nothing to offer beyond medieval optics.[139] (Figure 5.32) However, he also concluded, however, without giving arguments, that 'it will, consequently, not appear to you something magical when for some time someone sees an appearance in the air as if it was a city or people that move in the air'.[140] As has been shown, Ausonio's 'Theorica' clearified the issue.

[138] Ibid., p. 614-31. For contemporaneous references to catoptrical altimetry, see Alberti, 'Ludi Rerum Mathematicarum', pp. 138-9; Bartoli, Cosimo. *Del Modo di Misurare le Distantie, le Superficie, i Corpi, le Piante, le Provincie, le Prospettive, & Tutte le Altre Cose Terrene, che Possono Occorrere a gli Huomini: Secondo le Vere Regole d' Euclide, & de gli Altri Più Lodati Scrittori*. Edited by Theodore Besterman. Vol. 24, *The Printed Sources of Western Art*. Portland, Oregon: Collegium Graphicum, 1972, ff. 25v-26v; Fine, Oronce. *Opere di Orontio Fineo del Delfinato Divise in Cinque Parti, Arimetica, Geometria, Cosmografia, e Orivoli, Tradotte da Cosimo Bartoli, Gentilhuomo, & Academico Fiorentino, et gli Specchi, Tradotti dal Cavalier Ercole Bottrigaro, Gentilhuomo Bolognese*. Translated by Cosimo Bartoli. Venetia: Presso F. Franceschi, 1587, ff. 40v-41r; Magini, Giovanni Antonio. *De dimitiendi ratione*. Venetia: Roberto Meietti, 1592, ff. 87r-92v; Camerota, *Raccolto Fatto dal Cav:Re Giorgio Vasari di Varii Instrumenti per Misurare con la Vista*,, pp. 137, 176-7, 200-2.

[139] Parronchi, *Studi su la Dolce Prospettiva*, p. 620.

[140] 'Non ti parrà adunque cosa magica se per tempo alcuno vedessi alcuna apparitione nell' aere, come sarebbe città, o vero huomini che si movessino per l' aere'. Ibid., p. 631.

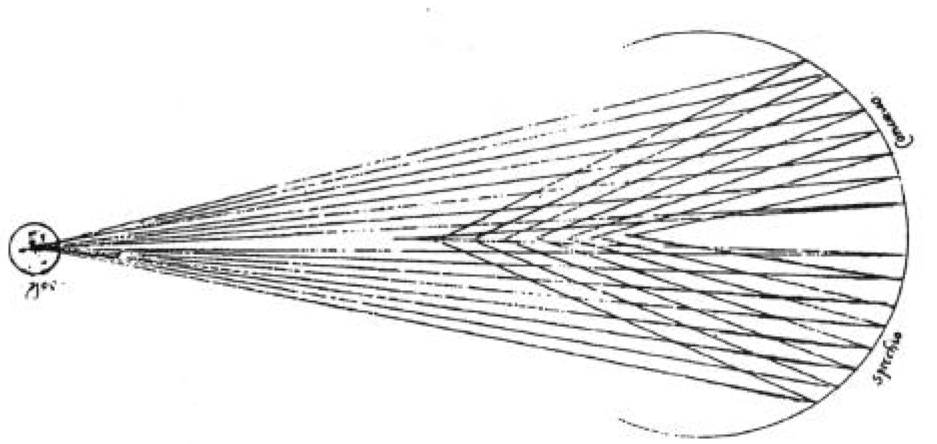

Figure 5.33

As has been discussed in chapter 2, Fontana was interested in the design of the parabolic burning mirror, as he left marginal notes to Alhazen's 'De speculis comburentibus'. In the 'Della prospettiva', he mentioned the parabolic burning mirror that makes the reflected rays converge in one point.[141] However, Fontana also noted that a concave spherical mirror did not make the solar rays converge in one point.[142] There was no attempt at determination of the focal point of a concave spherical mirror. Moreover, Fontana's drawing shows the solar rays diverging from the sun. (Figure 5.33) Again, as has been shown, Ausonio's 'Theorica speculi concavi sphaerici' clearified the issue. It also exemplifies a point already made, that is, that solar rays are to be considered parallel was not self-evident in the fifteenth and sixteenth centuries. If Fontana's 'Della prospettiva' was Galileo's textbook of optics, and one of his very few sources, as it most likely was, then Galileo's interest in the proposition that solar rays are to be considered parallel, to the extent of having made the already discussed underlinement of 'magnitudo' in the corresponding section of Ausonio's 'Theorica', is not particularly surprising.

Finally, Fontana's section on dioptrics discussed the application of the cathetus rule to refraction.[143] (Figure 5.34) He also considered atmospheric refraction. He attributed the cause of the apparent enlargement of the sun, the moon and the stars near the horizon with respect to when they are located near zenith, to the refraction of their rays in the denser vapors near the earth.[144] Next, Fontana discussed eyeglasses. He noted that 'the elderly and those who have weak eyesight use eyeglasses, because the likenesses which pass through the glass of the eyeglasses, which is denser than the air, are broken and show the letter larger than it is'.[145] Consequently, Fontana attributed the use of convex lenses to their magnificying power.

[141] Ibid., p. 632.

[142] Ibid., p. 632.

[143] Ibid., p. 637.

[144] Ibid., p. 638.

[145] 'li vecchi et quelli che hanno debile visione usano gli occhiali, perché le similitudini che passano per lo vetro delli occhiali, che è denso più che l' aere, si spezzano et dimostrano la lettera più grossa che non è'. Ibid., p. 639.

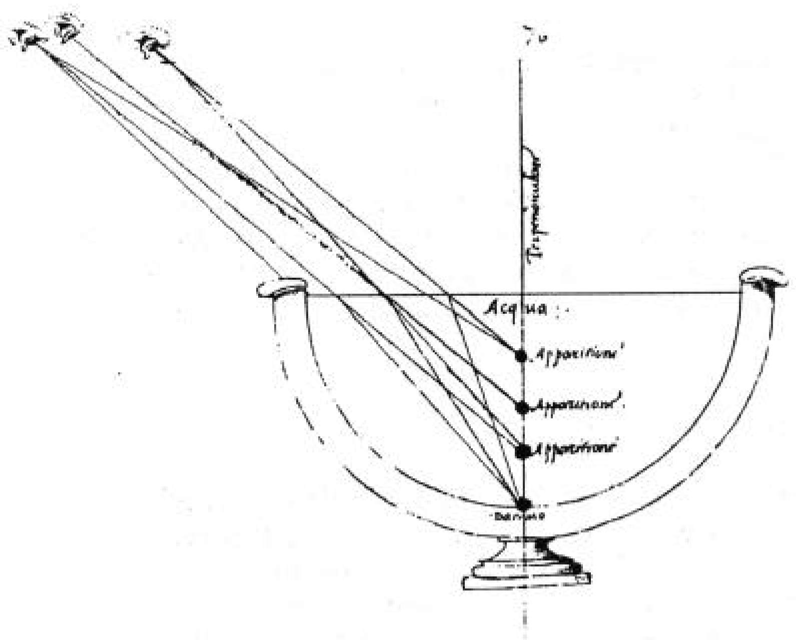

Figure 5.34

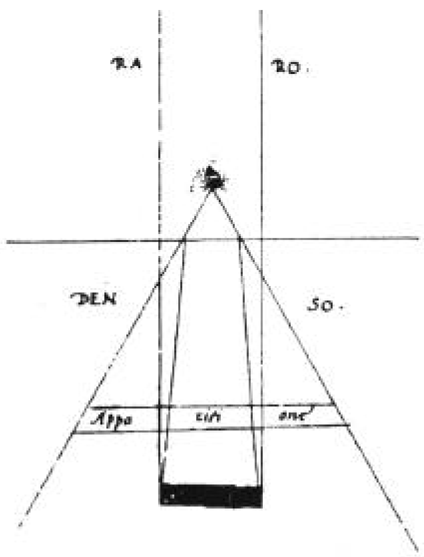

Figure 5.35

However, Fontana's corresponding figure is very confused. Fontana located, apparently arbitrarily, in obvious rejection of the cathetus rule, a magnified image of an object seen through a plane interface between a rarer and a denser medium. (Figure 5.35)

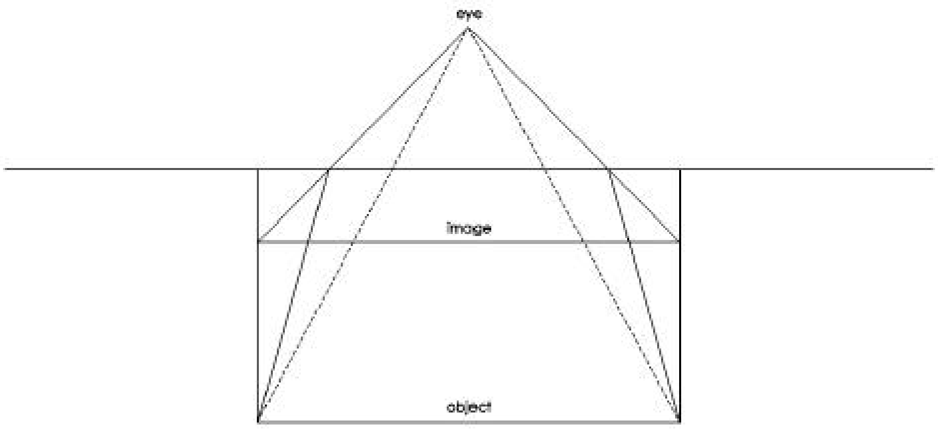

Figure 5.36

What Fontana most likely meant to represent, was the magnified image of an object under water. (Figure 5.36) However, Fontana did not discuss magnification of curved surfaces as would have been appropriate for convex lenses.

Finally, Fontana discussed the enhancement of candlelight by its refraction. He noted that 'if you place a candle in a body of glass, which is in another larger body, in such a way that the space in between those two glasses is full of cold water, then this small candle will make a great splendor and it will highly and wonderfully illuminate the room'.[146] Fontana illustrated the same principle in his 'Bellicorum instrumentorum liber', in which he placed candles inside small in-set spheres of glass of a mitre and a crozier.[147] (Figure 5.37) In his 'Della Prospettiva', he also illustrated how to light a candle with solar rays, which are again diverging, refracted in sphere filled with water.[148] (Figure 5.38) Again, as discussed below, Ausonio's 'Theorica' clearified the issues by placing a candlelight in the focal point of the concave mirror to enhance the light.

Thus, Fontana's 'Della Prospettiva' introduced Galileo to the basics of optics, catoptrics and dioptrics as they were practiced by a mathematical practitioner. The exclusive focus of the 'Della prospettiva' on the Euclidean mathematics of the visual pyramid, the discussion of the cathetus rule for image formation in mirror and burning mirrors, the discussion of lenses and the enhancement of light with candles and optics, is characteristic of the practical optics of mathematical practitioners as it has so far been discussed.

[146] 'Se tu metti una candela in uno corpo di vetro, il qual corpo sia in un altro corpo maggiore, per modo che tutti il spatio per mezzo di quelli dui vetri sia pieno d' acqua fredda, dico che quella piccola candela farà un grande splendore et illuminerà molto la camera et maravigliosamente'. Ibid., p. 640.

[147] Giovanni Fontana, Bellicorum Instrumentorum Liber, Bayerische Staatsbibliothek (München), Cod. Icon. 242, f. 19v. Reproduced in Batisti and Battisti, *Le Macchine Cifrate di Giovanni Fontana*, pp. 67, 115.

[148] Parronchi, *Studi su la Dolce Prospettiva*, p. 641.

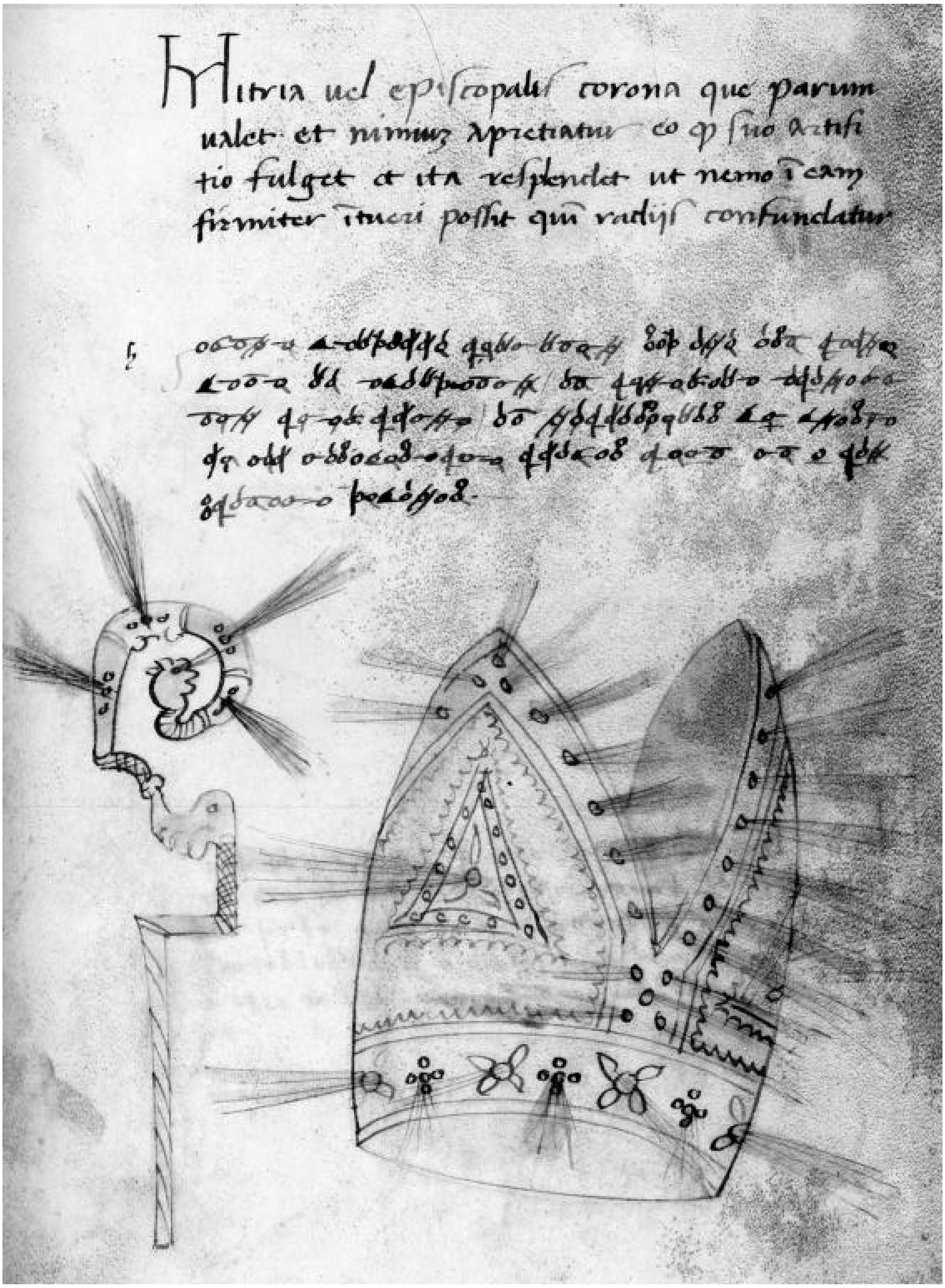
Mitria uel episcopalis corona que parum
ualet et nimium apretiatur eo quod suo artifi
tio fulget et ita resplendet ut nemo in eam
firmiter intueri possit quin radijs confundatur

Figure 5.37

Figure 5.38

Fontana's 'Della prospettiva' might have been useful to Galileo, when teaching mathematics, most likely, including perspectiva, at the Vallombrosan convent San Michele Arcangelo at Passignano, on the road between Florence and Siena, to a certain Don Epifanio, already in 1588.[149] However, it has also become clear how Galileo might have regarded Ausonio's 'Theorica' as a certainly intellectually more demanding solution to problems raised in the 'Della Prospettiva'. Consequently, when Galileo copied Ausonio's 'Theorica' between 1592 and 1601, it must have hit fertile ground.

Was there a more specific purpose behind Galileo's copy of Ausonio's 'Theorica'? In his 'Two New Sciences' of 1638, Galileo noted that, he had 'seen lead instantly liquefied by a concave mirror three spans in diameter', and that he was of the opinion that if the mirror were very large, smooth, and of parabolic shape, it would liquefy any metal in short time. For we see that a spherically concave mirror, neither very large nor well polished, liquefies lead with great power and burns every combustible material – effects that give credibility to the wonders of the mirrors of Archimedes'.[150] Consequently, Galileo was familiar with the legendary story of the burning mirror of Archimedes. Moreover, he was well aware of the latest publications on the subject and even lent credibility to the legend. In the 'Two New Sciences', after seeing the recently finished book on the burning mirror of Buonaventura Cavalieri, a student of Galileo's pupil Castelli, he remarked that he believed that Archimedes' burning mirror had actually existed at the time.

[149] Archivio di Stato (Firenze), Conv. 179, n. 16, , f. 157t: 'A dì 3 settembre [1588] ... A Galileo Galilei lire trentacinque piccoli portò detto contanti. A buon conto di haver insegnato mathematica a D. Epifanio'; f. 159: 'A dì 15 d.o [novembre 1588]. A Galileo Galilei lire ventuna piccoli porto detto contanti di Don Epifanio nostro monaco per havergli insegnato tre mesi mathematica'; f. 159t: 'A dì 28 d.o [novembre 1588]. A Galileo Galilei sette piccoli portò contanti da D. Epifanio'. See Schiavo, Armando. 'Notizie Riguardanti La Badia di Passignano Estratte dai Fonti dell' Archivio di Stato di Firenze.' *Benedictina* 9 (1955): 31-92, p. 44. See also Schiavo, Armando. 'La Badia di San Michele Arcangelo a Passignano in Val di Pesa.' *Benedictina* 8 (1954): 257-87. I would like to thank Tom Settle for drawing my attention to Galileo's teaching at the Vallombrosan covent and these articles.

[150] 'veduto, dico, con uno specchio concavo di tre palmi di diametri, liquefare il piombo in un instante: onde io son venuto in opinione, che quando lo specchio fusse grandissimo e ben terso e di figura parabolica, liquefarebbe non meno ogni altro metallo in brevissimo tempo, vedendo che quello, nè molto grande nè ben lustro e di cavità sferica, con tanta forza liquefaceva il piombo ed abbruciava ogni materia combustibile; effetti che mi rendon credibili le maraviglie de gli specchi d' Archimede'. Galileo, Discorsi e dimostrazioni intorno a due nuove scienze, in Galileo, *Opere*, Vol. 8, p. 86, translation in Galilei, Galileo. *Two New Sciences*. Translated by Stillman Drake. Madison, Wisconsin: The University of Wisconsin Press, 1974, p. 48.

> As to Archimedes and the effects of his mirrors, all the miracles that are read in other authors are rendered credible to me by reading the books of Archimedes himself, long ago studied by me with infinite astonishment. And if any doubt lingered, the book lately published about the burning glass by Father Buonaventura Cavalieri, which I read with admiration, is enough to put a stop to all difficulties for me.[151]

Thus, Galileo's study of the work of Archimedes as a young student with Ostilio Ricci, and his deep appreciation of Archimedes' work throughout his career made Galileo a likely candidate to become involved in one of the many projects that aimed at the reconstruction of the Archimedean burning mirrors. Galileo's copy of the 'Theorica', whic9h he in the quoted passage of his 'Two New Sciences' seemed to link with the Archimedean legend, might have been a part of such a project. I would like to suggest that the Guidobaldo-Pigafetti project is a most likely candidate.
Guidobaldo del Monte was not only involved with the design of refractive dials, but also with mirrors. In a manuscript of Guidobaldo's notes, there is a theorem, of unknown date and unrelated to the rest of the notes, which shows that an object multiplies its species to every point of a mirror. The source of this theorem appears to be Alhazen.[152] (Figure 5.39)

> Let the mirror be ab and the image cd. I say that the species of the image cd is received in the mirror ab in a point. Let the eye be e, which sees the extremity c in the mirror in f. Next, df is connected so that the angle afg is equal to the angle bfd. If the eye is placed in g, it is manifest that g sees the point d in the mirror in point f. Next, let accept whatever point h of the image and fh is connected so that the angle afk is equal to the angle bfh. For the same reason, if the eye is in k, it is evident that the eye k sees the point h in the mirror in the same point f. And if whatever other point of the same image cd is accepted, it is shown that it will likewise be received in the mirror in the same point f. Thus, all the species of the image cd are received in the mirror ab in the point f. What needed to be demonstrated. … Moreover, if whatever point l of the mirror is accepted, then it is shown in the same way that the species of the image cd are received in point l. Henceforward, all the species of an image are received in the mirror in every point of the mirror.[153]

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[151] 'Intorno a gli effetti de gli specchi d' Archimede mi rese credibile ogni miracolo, che si leggi in più scrittori, la lettura de i libri dell' istesso Archimede, già da me con infinito stupore letti e studiati; e se nulla di dubbio mi fusse restato, quello che ultimamente ha dato in luce intorno allo specchio ustorio il P. Buonaventura Cavalieri, e che io con ammirazione ho letto, è bastato a cessarmi ogni difficoltà'. Galileo, Discorsi e dimostrazioni intorno a due nuove scienze, in Galileo, *Opere*, Vol. 8, pp. 86-7, translation in Galileo, *Two New Sciences*, pp. 48-9. The work of Cavalieri is Cavalieri, Bonaventura. *Lo specchio ustorio, overo, Trattato delle settioni coniche: et alcuni loro mirabili effetti entorno al lume, caldo, freddo, suono, e moto ancora*, Bologna: Presso Clemente Ferroni, 1632.

[152] Alhazen, Opticae Thesaurus, book IV, proposition 21, in Lindberg, *Opticae Thesaurus*, p. 114.

[153] 'Sit speculu ab imago cd. Dico speciem imaginis cd in speculo ab, in puncto recipi. Sit oculus e qui videat extremitate c in speculo in puncto f deinde connectat df fiatq angulus afg angulo bfd aequalis si in g ponat' oculus manifestu est quod g videt punctu di in speculo in puncto f. Accipiat denique in imagine cd quod vis punctu h atq iungat fh fiatq angulus afh angul bfh aequalis eadem ratione, si in k sit oculus patet oculu k videre in speculo punctu h in eode puncto f et si ipsius imaginis cd aliud quodvis punctu aliud accipiat similiter ostendet ipsu recipi in speculo in puncto f. Tota igitur species imaginis cd in speculo ab in puncto f recipit, quod demonstrare oportebat. Praeterea si in speculo aliud quodvis accipiat punctu l eodem modo ondetu imaginis speciem cd in puncto l recipi, ex quib sequitur imaginis speciem in speculo in omnib speculi punctis totam recipi'. See the copy in Baldi, 'Theorema Guidi Ubaldi e Marchionibus Montis de speculo', Bibliothèque Nationale de France (Paris), Lat. 10280, f. 208r. The second part of theorem in Guidobaldo's original is slightly more long-winded. 'Ponat' qdem oculum e videre punctum c in speculo in puncto f alteram verò extremitatem d in puncto L proculdubbio cd in speculo recipitur no in puncto, sed in quantitate fl. Cui respondendum est, qd tota spes imaginis cd, no solu recipit in in puncto f, ut demonstratu est, verum etiam in puncto L, qd eodem modo ondetur. Preterea no solu in punctis fl sed et in quodlibet puncto speculj. Idcirco qvis cd ab oculo e videat' sub qtitate fl, hoc no puerit, qa cd in speculo no recipiat in puncto;

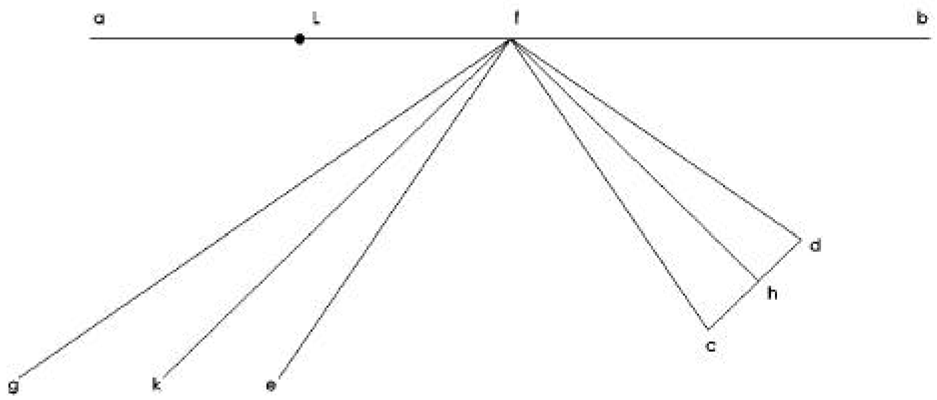

Figure 5.39

Consequently, every eye at a different location sees the image reflected from a different point, but for every eye and every point-object, there is only one such point. Guidobaldo added a short discussion of the cathetus rule, as applied in plane mirrors, to this theorem.[154] It has not been possible to find out why Guidobaldo wrote this theorem. However, there might have been a connection to the contemporaneous catoptrical culture at the Del Monte court.[155] It has even been suggested that the use of mirrors to project and draw pictures by Caravaggio, who became a client of the important patron Cardinal Francesco del Monte, Guidobaldo's brother, around 1594 or 1595, was related to the contemporaneous interest in optics of the del Monte court.[156]

The young Galileo came in contact with Guidobaldo around 1588, when he sent him a theorem on the center of gravity for approval.[157] Over the next twenty years, the correspondence between Guidobaldo and Galileo continued, as they appear to have discussed every major theme of mutual interest. They presumably first met personally in 1592, when Galileo was en route to Padua to take the chair of mathematics, for which Guidobaldo and his contact in Padua, Gian Vincenzo

sed qa unus tiu oculus i uno, et eodem situ, no potest eisdem lineis visualibus nisi unu tn punctu videre. Ut lineis cfe videt th punctu c; ppea his lineis nulla pt aliam partem videre ipsius imaginis cd, qvis tota recipat' in puncto f ut dictu est. Hoc itat evenit ratione situs ipsius oculj, et no qa spes no recipiat' in puncto. Cer: & his patet. Imaginis spem in speculo in orbus speculi punctis totam recipi'. Bibliothèque Nationale de France (Paris), Lat. 10246, ff. 45-8.

[154] Ibid., ff. 49-50.

[155] For the scientific and optical culture at the del Monte court, see Lapucci, Roberta. 'Caravaggio e i 'Quadretti nello Specchio Ritratti'.' *Paragone* 45 (1994): 160-70, pp. 160-1. See also Spezzafore, Luigi. 'La Cultura del Cardinal del Monte e il Primo Tempo del Caravaggio.' *Storia dell' Arte* 9-10 (1971): 57-92.

[156] Lapucci, 'Caravaggio e i 'Quadretti nello Specchio Ritratti'', pp. 164-5; Bassani, Riccardo, and Fiora Bellini. 'La Casa, le 'Robbe', lo Studio del Caravaggio a Roma.' *Prospettiva* 71 (1993): 68-76, pp. 74-5; Bassani, Riccardo, and Fiora Bellini. *Caravaggio Assassino: La Carriera di un 'Valenthuomo' Fazioso nella Roma della Controriforma*. Roma: Donzelli Editore, 1994, pp. 100, 204-5; Hockney, David. *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*. London: Thames & Hudson, 2001, pp. 113-23.

[157] Galileo, *Opere*, Vol. 10, pp. 25-6, 31, 33-39.

Pinelli, had been highly instrumental.[158] Renn, Damerow and Rieger have recently argued that during this visit Guidobaldo and Galileo talked about an experiment to show the parabolic trajectory of a projectile motion.[159] In 1597, Guidobaldo asked Galileo to tutor his son.[160] Around this same year, Guidobaldo's circle was involved in another project that dealt, like the work on projectile motion, with parabolas. Around 1597, Filippo Pigafetta, well known to Pinelli and translator of Guidobaldo's book on machines, discussed the idea of a museum of military architecture, to which the Medici collection of drawing and measuring instruments would be added, with the Grand Duke of Tuscany Ferdinando I.[161] The project never materialized, but the 'Stanza delle Matematiche' of the Uffizi, housing a collection of mathematical instruments, was decorated in 1600 by Giulio Parigi with ancient war instruments, which were in the original proposal meant to be actually present in the collection.[162] One of these instruments was Archimedes' burning mirror. (Figure 5.40)

Figure 5.40

[158] See Guidobaldo's invitation, 'Et s' ella vorrà andar a Venetia questa state, io l' invito a passar di qua, che non mancarò dal canto mio di far ogni opera per aiutarla e servirla'. 21 February 1592, in Ibid., p. 47.

[159] Renn, Jürgen, Peter Damerow, and Simone Rieger. 'Hunting the White Elephant: When and How Did Galileo Discover the Law of Fall?' *Science in Context* 13 (2000): 299-422, pp. 324-36.

[160] Guidobaldo to Galileo, 17 December 1597, in Galileo, *Opere*, Vol. 10, pp. 71-2.

[161] Prinz, Wolfram. '"Informazione di Filippo Pigafetta al Serenissimo di Toscana per una Stanza da Piantare lo Studio di Architettura Militare".' In *Gli Uffizi: Quattro Secoli di una Galleria*, edited by Paola Barocchi and Giovanna Ragionieri, 343-54. Firenze: Leo S. Olschki Editore, 1983.

[162] Prinz, Wolfram. 'Dal Modello al Dipinto: Macchine da Guerra di Archimede alla Fine del Cinquecento.' In *Atti del Convegno di Studi: Architettura Militare nell' Europa del XVI Secolo*, edited by Carlo Cresti, Amelio Fara and Daniela Lamberini, 409-16. Siena: Periccioli, 1988.

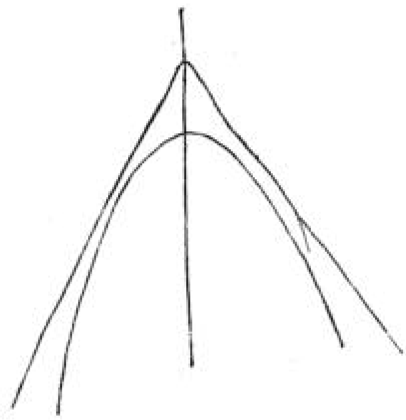

Figure 5.41

It is most likely that Galileo and Guidobaldo discussed the project, since they were very well informed about their work, and, in particular, the Archimedean burning mirror, because of their specific interest in parabolas at the time. Most of this work was kept a secret until the publication of the 'Two New Sciences', but, among friends, Galileo suggested the importance of the parabola. In 1599, when staying in Murano, he drew a parabola in the 'Album Amicorum' of Thomas Seggett.[163] (Figure 5.41) Consequently, the situation of the 'Two New Sciences', in which the parabolic burning mirror arose in a discussion of the parabolic trajectory of a projectile motion might reflect a situation of the 1590s, in which Guidobaldo and Galileo alreday saw a similar connection. Conseuqently, in such a context, Galileo's copy of Ausonio's 'Theorica' fitted his contemporaneous interests.

It is difficult to determine to what extent Galileo was involved in the design of mirrors in the 1590s, but it can be established that Galileo was in contact with mirror-makers of Murano, foremost with Girolamo Magagnati, a poet active in the Venetian glass and mirror industry.[164] Such contacts with mirror makers in Murano proved useful to Galileo when he began working on the telescope. A letter of 1 October 1610 to Giuliano de' Medici shows that Galileo had established his own lens making workshop, and, in the years to come, he would hire several crafstmen to operate the workshop.[165] However, to make a lens, Galileo did not have to start from scratch by making a plane glass blank. As discussed in chapter 1, Galileo's lenses were plano-convex and plano-concave. Willach has recently argued that the plane surface of telescope lenses

[163] See Galileo, *Opere*, Vol. 19, p. 204. For Seggett, see Favaro, Antonio. 'Tommasso Segeth.' In Favaro, Antonio. *Amici e Correspondenti di Galileo*. Edited by Paolo Galluzzi. Vol. 3. Firenze: Libreria Editrice Salimbeni, 1983.

[164] Magagnati is credited with the invention of a certain colored glass to imitate precious stones and a certain procedure that allowed to make mirrors larger and faster than before. For Magagnati, see Favaro, Antonio, 'Girolamo Magagnati', in Ibid., pp. 65-92; Drake, Stillman. 'Galileo Gleanings XIV: Galileo and Girolamo Magagnati.' *Physis* 6 (1957): 269-86; Zecchin, Luigi. *Vetro e Vetrai di Murano*. Venezia: Arsenale Editrice, 1990, pp. 356-64.

[165] Galileo to Giuliano de' Medici, 1 October 1610, in Galileo, *Opere*, Vol. 10, pp. 440-1. For the crafstmen that Galileo hired to grind and polish lenses, see Bedini, Silvio A. 'The Makers of Galileo's Scientific Instruments.' In *Atti del Simposio Internazionale di Storia, Metodologia, Logica e Filosofia della Scienza: Galileo Galilei nella Storia e nella Filosofia della Scienza*, 89-115. Florence: Gruppo italiano di storia della scienza, 1967, reprinted in Bedini, Silvio A. *Science and instruments in seventeenth-century Italy*. Aldershot: Variorum, 1994.

of the beginning of the seventeenth century shows the same surface texture as contemporaneous Venetian mirror plates.[166] This suggests that Galileo purchased pieces of flat mirror glass from the mirror makers of Murano. Consequently, it was only left to his own workshop to grind and polish one surface of the lens to the desired curvature. Afterwards, the lenses were cut from the larger pieces of mirror glass to the desired diameter, which resulted in the typically sharp edges of the lens.[167]

Moreover, for grinding and polishing a lens, techniques were used which derived not from the spectacle-makers, but from mirror-makers who were far better up to the job.[168] The lens was grinded in a copper mould, filed to desired curvature, and rotated on a lens-grinding machine. The glass was pasted to a pistil and grounded with successive finer grains of ‘sand’, a mixture of feldspar and quartz. After the grinding process, the lens was polished on a rotating plane of wood covered with a piece of fine leather. The polishing material was tripolite, which consists of the shells of diatoms. That Galileo bought pieces of glass from mirror-makers and used similar techniques to grind and polish one surface of such a piece of mirror glass in to a lens, is confirmed by a shopping list on the back of a letter of November 1609. Several items on this shopping list were to be bought at a mirror-maker shop, such as plane pieces of German glass, plane mountain crystal, pieces of mirror, tripolite, and a hand-held mirror.[169] Consequently, while it is unlikely that Galileo’s copy of Ausonio’s ‘Theorica’ between 1592 and 1601 was connected with a project to make a telescope, as it is more likely to have been connected with a project to make an Archimedean burning mirror, Galileo’s involvement with mirror-makers prior to 1610 gave him access to the knowledge embodied in this craft of mirror-making which would prove very useful when he started making telescope lenses. Moreover, as will be shown in the next chapter, the knowledge of image formation in a concave spherical mirror, in particular, the notion of the point of inversion, would prove influential when Galileo started making telescopes.

[166] Willach, Rolf. ‘The Development of Lens-grinding and Polishing Techniques in the First Half of the 17th Century.’ *Bulletin of the Scientific Instrument Society* 68 (2001): 10-15, p. 14

[167] Ibid., pp. 14-5.

[168] Ibid., pp. 10-2. For grinding and polishing techniques, see also Bedini, Silvio A. ‘A Treatise on Optics’ by Giovanni Christoforo Bolantio.’ *Annals of Science* 52 (1995), 103-126, reprinted in Bedini, Silvio A. *Patrons, Artisans and Instruments of Science, 1600-1750*. Aldershot: Ashgate Variorum, 1999.

[169] ‘vetri todeschi spianati, spianar cristallo di monte, pezzi di specchio, tripoli, lo specchiaro all ‘insegna del Re, specchio per fregare’. On the back of a letter of 23 November 1609, in Galileo, *Opere*, Vol. 10, p. 270.

VI. 'As Distant Cities in Flemish Paintings': The Point of Inversion and the Invention of the Telescope

1. The Telescopic Dream and the Delay of the Invention of the Telescope

In his 'The Invention of the Telescope', Van Helden came to the surprising conclusion that 'telescopes existed before anyone, including the men who made them, were aware of them'.[1] Van Helden's argument is only apparently contradictory, because the telescope was an invention that had long been foreseen.[2] What was new around 1600 was not the idea of telescopic magnification, but the construction of a working telescope. Moreover, the telescopic dream had been written in such hyperbolic terms that when actual instruments were finally produced, feeble in magnifying power as those first devices were, their makers prejudiced by the optical marvels of Roger Bacon and King Ptolemy, might not have recognized them as telescopes. As discussed in chapter 2, Bacon dreamt of a telescopic device useful for objects at 'incredible distances'.

> For we can so shape transparent bodies, and arrange them in such a way with respect to our sight and objects of vision, that the rays will be refracted and bent in any direction we desire, and under any angle we wish we shall see the object near or at a distance. Thus from an incredible distance we might read the smallest letters and numbers grains of dust and sand owing to the magnitude of the angle under which we viewed them. … In this way a child might appear a giant, and a man a mountain. He might appear of any size whatever, as we might see a man under as large an angle as we see a mountain, and close as we wish. Thus a small army might appear very large, and situated at a distance might appear close at hand, and the reverse.[3]

The military application of a telescopic device was a common element of the dream, for example, in the legend of the telescopic mirror in Pharos. In his 'Natural Magick', Della Porta noted not only that 'Archimedes at Syracuse with burning glasses defeated the forces of the Romans', but also that 'King Ptolemy built a tower in Pharos, where he set a glass, that he could for six hundred miles, see by it the enemies ships, that invaded his country, and plundered it'.[4] There is an echo of this motive of military application of a telescopic mirror on the top of a tower, when Galileo took the members of Venetian Senate to the top of the campanile to

[1] Van Helden, Albert. *The Invention of the Telescope*. Vol. 67, *Transactions of the American Philosophical Society*. Philadelphia: The American Philosophical Society, 1977, p. 19.

[2] For the telescopic dream, see also Lüthy, C.H. 'Atomism, Lynceus, and the Fate of Seventeenth-Century Microscopy.' *Early Science and Medicine* 1 (1996): 1-27, pp. 1-6.

[3] 'Nam possumus sic figurare perspicua, et taliter ea ordinare respectu nostri visus et rerum, quod frangentur radii et flectentur quorsumcumque voluerimus, ut sub quocunque angulo voluerimus videbimus rem prope vel longe. Et sic ex incredibili distantia legeremus literas minutissimas et pulveres ac arenas numeraremus propter magnitudinem anguli sub quo videremus … Et sic posset puer apparere gigas, et unus homo videri mons, et in quacunque quantitate, secundum quod possemus hominem videre sub angulo tanto sicut montem, et prope ut volumus. et sIc parvus exercitus videretur maximus, et longe positus apparet prope, et e contra'. Roger Bacon, Opus Majus. Quoted and translated in Van Helden, *The Invention of the Telescope*, p. 28.

[4] Porta, John Baptista. *Natural Magick*. New York: Basic Books, Inc., 1957, p. 355.

convince them of the military value of his telescope, not consisting of a mirror, of course, but of a convex and a concave lens, by having them look at distant ships approaching the harbor.[5]

The legend of the telescopic mirror of Pharos had echoes in other contemporaneous literary motives. In 1557, Tramezzino published the first translation in a European language of an originally Persian tale, the 'Peregrinaggio di tre giovani, figluoli del Re di Serendippo'.[6] This tale contained a reference to a 'mirror of justice', of which the shape would have been discovered by philosophers of antiquity.[7] This mirror was not aimed at protecting a state against foreign enemies approaching the harbor, but against those who intended to harm the state from inside. It was claimed that this mirror revealed the evildoer, and, as such, it was meant as a guarantee for civil obedience. As shown in chapter 3, Ausonio was connected to the workshop of Tramezzino, presumably in the 1550s, and, beside in Tramezzino's cartographic production, Ausonio also might have had a hand in the publication of the 'Peregrinaggio'. A letter sent to Ausonio at the workshop of Tramezzino, of an unknown date and an unknown correspondent, discussed a procedure 'to see within a room everything that is done far away in a square or other place by means of a mirror and perspective'.[8] The procedure is obscure (to the present author), but it involved the placement of a plane mirror, or several plane mirrors or a convex mirror to obtain a larger field of view, on top of a tower. The mirror needed to be oriented so that it allowed viewing the festivities on the market square below the tower. It seems that this mirror was intended to reflect the image to a room, presumably darkened, on top of a second tower. (Figure 6.1)

> First of all it is necessary to choose a high tower or campanile that is near enough to the square that one must see that if someone were standing on its summit he could see another distant tower and the room where one wishes finally to make the observation. And the same observer should be able to see the whole square where this festivity is taking place. It is necessary to put a large, flat, and very clear mirror here.[9]

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[5] 'e sono stato moltissimi i gentil'huomini e senatori, li quali, benchè vecchi, hanno più d' una volta fatte le scale de' più alti campanili di Venetia per scoprire in mare vele e vasselli tanto lontani, che venendo a tutte vele verso il porto, passavano 2 hore e più di tempo avanti che, senza il mio occhiale, potessero essere veduti: perchè in somma l' effetto di questo strumento è il rappresentare quell' oggetto che è, verbi gratia, lontano 50 miglia, così grande e vicino come se fussi lontano miglia 5'. Galileo to Benedetto Landucci, 29 August 1609, in Galileo, *Opere*, Vol. 10, p. 253.

[6] Armeno, Christoforo. *Peregrinaggio di Tre Giovani, Figluoli del Re di Serendippo*. Edited by Heinrich Gassner and Hermann Varnhagen, *Erlanger Beiträge zur Englischen Philologie*. Erlangen: Verlag von Fr. Junge, 1891. On Armeno, see Melfi, E. 'Cristoforo Armeno.' *Dizionario Biografico degli Italiani*, Vol. 31, 71-72. I am grateful to Eileen Reeves for pointing out to me the existence of this work, and information on its reference to mirrors.

[7] 'Fu dagli antichi filosofi di questo imperio, i quali i predecessori miei hanno in ogni tempo assai stimato, ritouata una forma di specchio, il quale essi chiamauano specchio di giustitia, percio che hauea questa uirtu, che, oue due insieme piatiuano, facendo il giudice quelli in esso guardare, à colui, che ingiusta dimanda facea, la faccia incontanente nera diuenia, et quello, che dirittamente si difendea, nel primo suo color rimanendo, dal giudice uittorioso se ne giua'. Armeno, *Peregrinaggio di Tre Giovani, Figluoli del Re di Serendippo*, p. 22.

[8] 'per vedere in una camera tutto quello che si fà da lontano in una piazza o altro luogo per forza di specchio et prospetiva'. B. A. M., G120 Inf., f. 74v. For a transcription of this letter to Ausonio, see Appendix I. I would like to thank Eileen Reeves for her most generous help with the transcription of this letter.

[9] 'In prima bisogna ellegere una torre o campanile alto che sia tanto appresso la piazza che si hà da vedere che stando uno alla sommità di quella possi vedere una torre lontana da quella e che sia al verso della camera dove alla fine si vole vedere. Anchora l'istesso possi vedere tutta la piazza dove si ha da fare detta fusta o bagordo. In quel luogo si deve mettere uno speglio piano et grande molto bene visibile'. Ibid, f. 74v. See Appendix I.

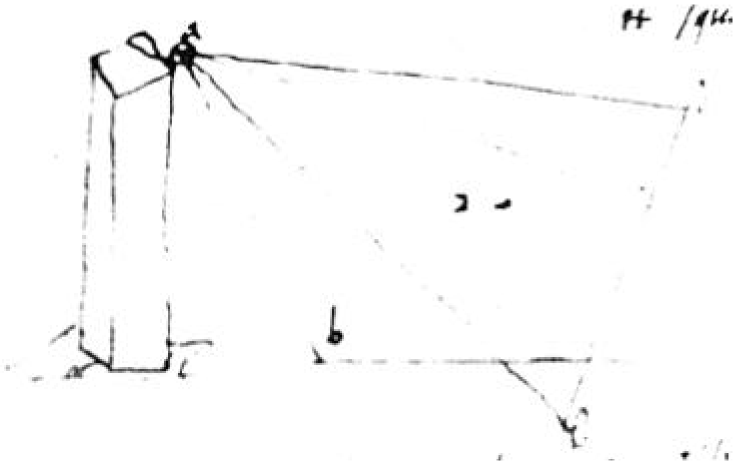

Figure 6.1

Whether such a procedure might have actually worked is less important for our purposes. The point is that these legends show that there was a dream of telescopic magnification, which was much older than the invention of a working telescope. The dream of telescopic magnification might take different shapes, either involving some kind of telescopic mirror, as in the legend of Pharos, or some kind of refracting device, as Bacon seemed to suggest. Moreover, as the letter to Ausonio and many other documents, some of which have been quoted in chapter 2, suggest, there was an active search for a technological procedure that might obtain such a telescopic effect.

However, there is a difference between dream and reality. When did it become technologically feasible to make a telescope? Lens-shaped objects and mirrors had been around since antiquity.[10] However, while some of these lens-shaped objects might have been intended as part of art works, like the lens making the eye of a bull's head rhyton of ca. 1550-1500 BC, to obtain a magnified virtual image for the observer of the art work when he looked into the 'eye', with the exception of Seneca, they were not mentioned as being used as a visual aid.[11] The invention of eyeglasses, with convex lenses to correct presbyopia, has been dated to the second half of the thirteenth century. Neaman has discovered a Ghent psalter from between 1250 and 1270 that shows a fable figure wearing spectacles, while Rosen has argued that such spectacles were first made in Italy, in the region of Pisa, between 1280 and 1285.[12] Concave lenses to correct myopia presumably came

[10] Riekher, Rolf. *Fernrohre und Ihre Meister: Eine Entwicklungsgeschichte der Fernrohrtechnik*. Berlin: Veb Verlag Technik, 1957, pp. 12-5; King, Henry C. *The History of the Telescope*. London: Charles Griffin & Company Limited, 1955, p. 25; Turner, G. L'E. 'Animadversions on the Origins of the Microscope.' In *The Light of Nature: Essays in the History and Philosophy of Science Presented to A. C. Crombie*, edited by J.D. North and J. J. Roche, 193-207. Dordrecht Boston Lancaster: Martinus Nijhoff Publishers, 1985, pp. 193-4.

[11] Enoch, Jay M. 'The Enigma of Early Lens Use.' *Technology and Culture* 39 (1998): 273-91.

[12] Neaman, Judith S. 'The Mystery of the Ghent Bird and the Invention of Spectacles.' *Viator* 24 (1993): 189-214; Rosen, Edward. 'The Invention of Eyeglasses.' *Journal of the History of Medicine and Allied Sciences* 11 (1956): 13-46. For the history of spectacles, see also Von Rohr, Moritz. 'Additions to Our Knowledge of Old Spectacles: A Summary of Papers Published in 1923-24 Relating to the Subject of the Thomas Young Oration of 1923.' *Transactions of the Optical Society* 26 (1924-25): 175-87; Von Rohr, Moritz. 'Aus der Geschichte der Brille mit Besonderer Berücksichtigung der auf der Greeffschen Beruhenden Jenaischen Sammlung.' *Beitrage zur Geschichte der Technik und Industrie* 17 (1927): 3-50; Greeff, R. 'Die Veglia des Carlo Dati über die Erfindung der Brillen.' *Zeitschrift für ophthalmologische Optik* 5 (1917): 65-77.

slightly later. Ilardi has discovered documents that show that spectacles with concave lenses were produced in Florence in 1451.[13] However, the spectacles shown on Van Eyck's 'Canon Van der Paele' of the beginning of the fifteenth century, are most likely composed of concave lenses. (Figure 6.2)

Figure 6.2

Although Canon Van der Paele is reading a book, which would suggest the use of convex lenses, the optical effect of the spectacle lens on the text seen through the lens shows that Van der Paele's spectacles are made of concave lenses.[14] When a concave lens is oriented to the text, as in Van Eyck's painting, the text will indeed appear diminished. As Van Eyck shows, there is virtually no effect where the lens touches the book, while the text will look progressively smaller, squeezed and moved upwards, that is the text will appear to move above its line from the point of view of the observer of the painting, when the distance between lens and text grows. Seen through a convex lens the text will also seem to move upward, but it will now be progressively enlarged and spread out. Consequently, although presumably rare and in this case most likely symbolically used, Van Eyck's 'Canon Van der Paele' shows that concave spectacle lenses were already used in the first half of the fifteenth century, around 1436, in Flanders.

If concave mirrors, convex and concave lenses were available from the mid-fifteenth century and the dream of telescopic magnification was culturally omnipresent, then why did it take so long before the telescope was invented? Ronchi has called attention to the scholarly optical tradition's lack of attention to lenses.[15] He has attributed this lack of attention to a 'conspirancy of silence'. The general distrust of vision caused scholars to keep silent about lenses, because they were

[13] Ilardi, Vincent. 'Eyeglasses and Concave Lenses in Fifteenth-Century Florence and Milan: New Documents.' *Renaissance Quarterly* 29 (1976): 341-60. See also Ilardi, Vincent. 'Renaissance Florence: The Optical Capital of the World.' *Journal of European Economic History* 22 (1993): 507-41; Ilardi, Vincent. *Occhiali alla Corte di Francesco e Galeazzo Maria Sforza*. Milano: Metal Lux, 1976.

[14] Desneux, Jules. *Rigueur de Jean Van Eyck: A Propos d' un Diagnostic Médical sur un Tableau de 1436*. Bruxelles: Editions des Artistes, 1951, pp. 27-30.

[15] Ronchi, *The Nature of Light: An Historical Survey,* pp. 69-76.

thought to distort reality. Lindberg and Steneck have shown that Ronchi's argument for a general distrust of vision is not backed up by the sources.[16] Their argument was primarily intended at refuting Ronchi's claim, but they were not able to give an alternative explanation for what has been regarded as the lack of attention to lenses from the scholarly optical tradition. However, even this lack of attention to lenses was not as absolute as suggested by Ronchi. As shown in the last chapter, a mathematical practitioner of the beginning of the fifteenth century, like Fontana, discussed convex eyeglasses in his 'Della Prospettiva'. By the middle of the sixteenth century, as will become evident, mathematical practitioners abundantly discussed lenses. Moreover, De Mey has shown that around the middle of the sixteenth century the ocular drawings of Ryff and Vesalius introduced, for the first time, the crystalline body as a lenticular shape.[17] (Figure 6.3)

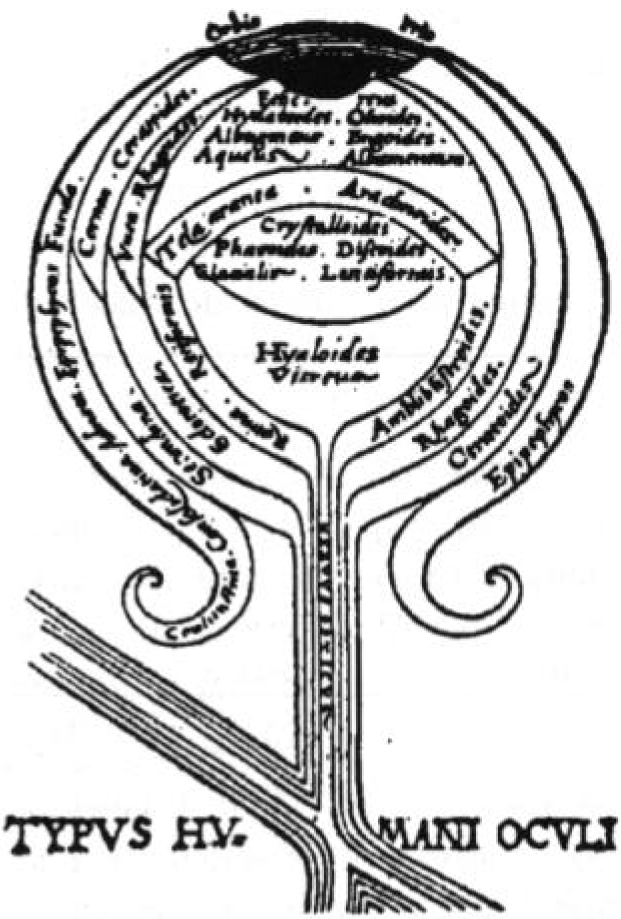

Figure 6.3

[16] Lindberg, David C., and Nicholas Stenec k. 'The Sense of Vision and the Origins of Modern Science.' In *Science, Medicine and Society in the Renaissance: Essays to Honor Walter Pagel*, edited by Allen G. Debus, 29-45. New York: Science History Publications, 1972. Reprinted in Lindberg, David C. *Studies in the History of Medieval Optics*. London: Variorum Reprint, 1983, pp. 29-45.

[17] De Mey, Marc. 'Artistic and Intellectual Factors in Scientific Discovery: Ocular Anatomical Drawings.' *Scientiarum Historia* 23 (1997): 73-108, in particular, pp. 94-9.

Consequently, the appropriation of lenses by a mathematical practitioner like Ryff gave way to the introduction of the lens in to the eye and the process of vision itself in the medical tradition.

If Ronchi's claim, even in its milder versions, is untenable, then the delay of the invention of the telescope presents a puzzle to the history of optics. The puzzle becomes even more complex, when Van Helden's claim is taken into account that the first refracting telescopes, made in Holland, were the result of the trial and error procedures of a simple craftsman with a convex and a concave lens readily available in his own workshop.[18] Then, why wasn't any craftsman so lucky to stumble in to the invention of a telescope, let's say, around 1500? The delay of the invention of the telescope becomes a still more complex puzzle, when it is assumed, with Lindberg, that Kepler's 'Paralipomena' is an outgrowth from and a simple continuation of the perspectivistic geometrical techniques of medieval optics, which were, pace sixteenth century optics, fully capable of dealing with lenses, and Kepler's 'Dioptrice' was the application of the optical principles of the 'Paralipomena' to image formation in a telescope.[19] Then, if Ronchi's claim of a general distrust of vision is untenable, why did no single perspectivist before Kepler applied his mathematical means to lenses in order to invent a so desired telescopic device? Consequently, this puzzle shows that the date of the invention of the telescope does not fit the history of optics.

In this chapter, it will be argued that the puzzle of the delay of the invention of the telescope is resolved when the sixteenth century prehistory of the telescope is taken into account.[20] The prehistory of the telescope allows the invention of the telescope to become an integral part of the history of optics. It needs to be pointed out that the telescope cannot be the result of medieval optics as such. The medieval geometric techniques of image location, that is the cathetus rule, were not adapted to deal with image formation in a lens system like the telescope. No work makes that more abundantly clear than Della Porta's 'De Telescopio'.[21] Della Porta began writing it shortly after Galileo had turned his telescope to the skies. It dealt with image formation in refracting spheres, biconvex and biconcave lenses, to culminate in an account of image formation in a Galilean telescope. As shown in previous chapters, the cathetus rule was used in medieval optics to locate images in plane, convex and concave mirrors, and in plane refracting surfaces.

However, the cathetus rule ran in to problems, when it had to deal with image formation in lenses. Since a ray is refracted twice in a lens, this seemed to imply that also the cathetus rule had to be applied twice. Della Porta did not know how to do this. Already in his 'De Refractione' (1593), he encountered the problem of the multiple application of the cathetus rule when discussing image formation in a refracting sphere.[22] (Figure 6.4)

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[18] Van Helden, *The Invention of the Telescope*, in particular, pp. 16-20.

[19] Lindberg, David C. *Theories of Vision: From Al-Kindi to Kepler*. Chicago London: The University of Chicago Press, 1976, pp. 178-208; Lindberg, David C. 'Optics in Sixteenth-Century Italy.' In *Novità e Crisi del Sapere*, edited by Paolo Galluzzi, 131-48. Firenze: Giunti Barberà, 1984.

[20] The term 'prehistory of the telescope' is borrowed from Van Helden, *The Invention of the Telescope*, p. 20.

[21] For discussion, see Della Porta, Giovan Battista. *De Telescopio*. Edited by Ronchi, Vasco, and Naldoni, Maria Amalia ed. Firenze: Leo S. Olschki, 1962; Ronchi, Vasco. 'Du De Refractione au De Telescopio de G. B. Della Porta.' *Revue d' Histoire des Sciences* 7 (1954): 34-59; Lindberg, 'Optics in Sixteenth Century Italy', pp. 142-8; Dijksterhuis, Fokko Jan. 'Lenses & Waves: Christiaan Huygens and the Mathematical Science of Optics in the Seventeenth Century.' Ph. D., Universiteit Twente, 1999, pp. 32-3.

[22] Porta, Ioan. Baptista. *De Refractione Optices Parte Libri Novem*. Neapoli: Apud Io. Iacobum Carlinum & Antonium Pacem, 1593, p. 49. For discussion, see Lindberg, 'Optics in Sixteenth Century Italy', pp. 144-5.

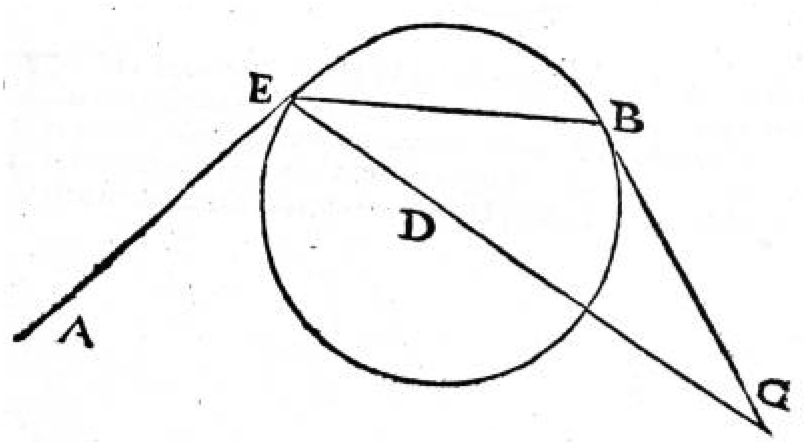

Figure 6.4

For example, Della Porta assumed, without justification, that the ray CB is refracted to E, where it is again refracted to the eye at A. To locate the image of G, he took the cathetus from G through the center D, and, then, located the image in E, at the surface of the refracting sphere, where it intersects the refracted ray BE. The problem is identical with the lenses of his 'De Telescopio'.[23] (Figure 6.5)

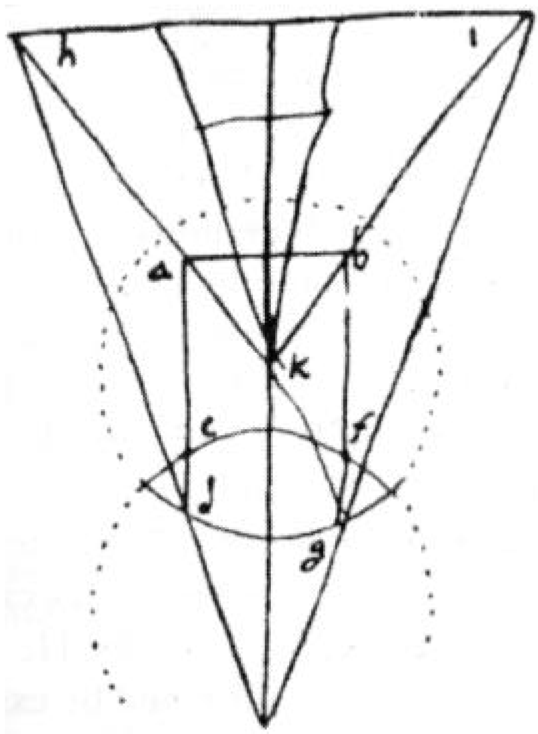

Figure 6.5

[23] Porta, *De Telescopio*, pp. 113-4.

Della Porta built a convex lens dcfg from the intersections of two refracting spheres. To locate the image of the object AB, Della Porta chose, again without any justification, a ray ac that is refracted along cd to the eye. To locate the image of a, he choose the cathetus of the second refracting surface dg of the lens through its center k and the object a. Then, at its intersection with the produced refracted ray at h, the image is located. The image of b is constructed in the same way to be located at i. Consequently, since the two refracting surfaces of a convex lens already presented a problem for the cathetus rule, it is understandable that Della Porta's application of the cathetus rule to the Galilean telescope, with four refracting surfaces, became completely arbitrary.[24] (Figure 6.6)

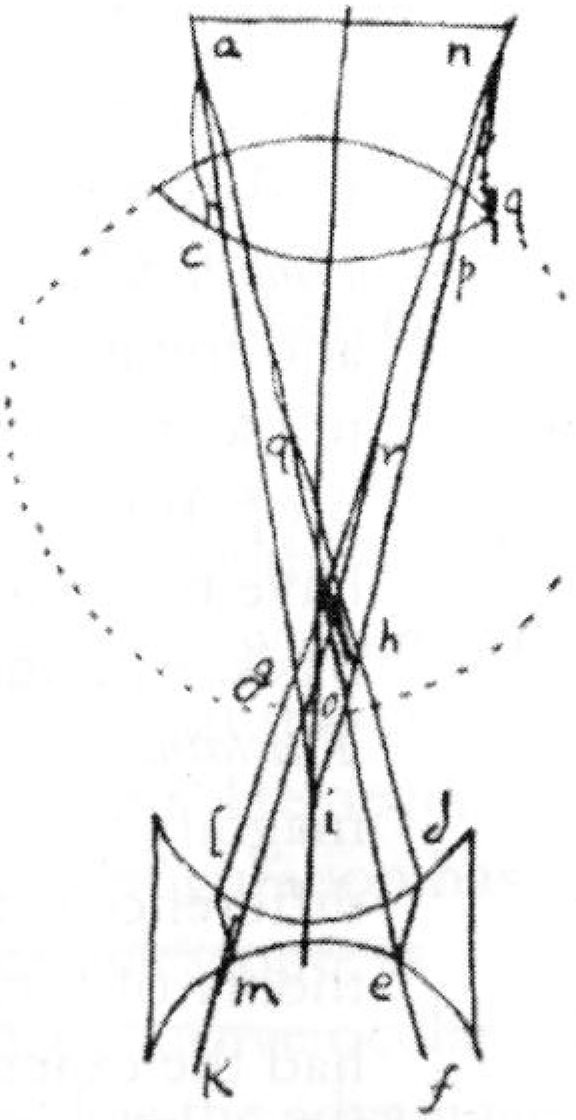

Figure 6.6

[24] Ibid., pp. 141-2. For discussion, see Dijksterhuis, 'Lenses & waves', p. 33.

However, while the geometric techniques of image location of medieval optics as such were not helpful in inventing a telescope, the optics of sixteenth century mathematical practitioners was. In this chapter, it will be argued that the puzzle of the delay of the invention of the telescope resolves when it is realized that, on the one hand, the notion of a point of inversion, as represented in Ausonio's 'Theorica', was, as has been shown, not present in medieval optics, but only came available around 1550, and that, on the other hand, this notion of a point of inversion was strongly connected with the instrumental practice of the prehistory of the telescope. Consequently, it will be argued that without the instrumental practice of the prehistory of the telescope, and its fixation in a notion of point of inversion, the invention of the telescope could not come about. The delay of the invention of the telescope is only a delay when it is assumed either that the invention of the telescope is the result of medieval optics, and that the appropriation of medieval optics by sixteenth century mathematical practitioners was inconsequential, or that the invention of the telescope is the result of trial and error procedures, without any skill involved, once the appropriate optical components became available.

First, it will be argued that the notion of point of inversion in Ausonio's 'Theorica' was connected with the instrumental practice of a concave mirror inside a camera obscura. Second, it will be shown that Ausonio's 'Theorica', and the instrumental practice on which it was based, was highly influential in the sixteenth century. On the one hand, in the optical work of the second half of the sixteenth century, the notion of the point of inversion was transferred from concave spherical mirrors to convex lenses. On the other hand, the instrumental practice on which this transfer was based resulted in the invention of a primitive reflecting telescope in the work of the same mathematical practitioners of the 1570s and 1580s, who adopted the notion of point of inversion. It will be shown that these same mathematical practitioners who appear to have invented a primitive reflecting telescope, like Sarpi and Della Porta in Italy, and Digges and Bourne in Elizabethan England, had access to Ausonio's 'Theorica'. Finally, Galileo's refracting telescope of 1609 will be discussed against the background of this sixteenth century instrumental practice in the footsteps of Ausonio's 'Theorica' from camera obscura to reflecting telescope.

2. Distant City Views: From Ausonio's 'Theorica' to the Reflecting Telescope

The telescopic dream made use of several motives, for example, the possibility of reading letters from large distances and its military application. These motives return in Ausonio's 'Theorica', when he discussed the effects of the concave spherical mirror 'in tenebris'. However, from the context, it is evident that Ausonio did not hint at any telescopic practice as such. In the chart of the 'Theorica', Ausonio claimed that 'in the darkness, the concave mirror causes with burning candles and torches that letters can be read in a dark place or that things which happen in the camps of the enemies can be seen', and also that 'letters are read nearby, while the light is distant'.[25] A line of incidence proceeding from the focal point in the large drawing of the concave mirror details a procedure that makes clear that no telescopic practice was intended. Ausonio claimed that when a candle is placed at the focal point, it would be possible 'to illuminate so clearly that letters can be read at night'.[26] Consequently, reading letters and seeing faraway things, like the movements of inimical troops, was not established by any telescopic practice, but

[25] Ausonio, *Theorica*. See Appendix II.

[26] Ibid. See Appendix II.

by the enhancement of light by means of a concave mirror. In Ausonio's 'Invenzione', his discussion of this procedure was more detailed, in a more than a bit voyeuristic setting of the practice.

> .. with a candle, which when there is much sun can be lighted by means of the mirror, … one can cause the image of the light to extend more than two passes far and to heat it so that anyone can feel it. With this kindled light, one can cause to see at night more than six passes far by placing the candle near the mirror in a certain point, and the image of the light will go about seven passes far, and in the darkness of the night, it makes such a great light that one can discern very well. This summer people slept with their window open without light so they would not be seen, and I quickly set up my mirror, and with a candle the light was sent in the room of my neighbours, and the whole room, and those that were within it, were seen, as if it was day, because where it is darker, sight is better. I have also found myself standing in the dark at night, and when there was a kindled light nearby, that I could see, I placed the mirror opposite this light, and where it made its splendor, I placed a paper with letters, and I could read the letters one by one very well.[27]

While Ausonio's 'Theorica' did not describe any telescopic practice as such, the suggestion of the motive of the telescopic dream was most likely intended. Consequently, Ausonio's 'Theorica' was easily misunderstood as referring to telescopic practice. It is, for example, interesting to note in this respect that, when Magini published Ausonio's 'Theorica', he not only changed the locus of the point of inversion, but also changed Ausonio's 'propè legantur litterae lumine existente remoto' in to 'propè legantur litterae legente existente remoto'.[28] Thus, in Magini's formulation, letters were not read when the light was distant, but when the reader was distant. Consequently, Magini's formulation is more reminiscent of a telescope than of a light-projecting device.[29]
A second effect 'in tenebris' that Ausonio suggested was the use of the camera obscura in combination with a concave spherical mirror to project images. In the chart of the 'Theorica', Ausonio suggested that in the darkness a concave mirror 'paints a marvelous picture on a piece of paper or a screen of those things which are outside, because the Sun illuminates those things

[27] '.. con una candela la quale quando e buon sole si puo accendere per virtù dello specchio, … di far sentire che la imagine del lume esce piu di dui passa lontana e scalda si che ognuno la puo sentire con il lume acceso ancora si puo far vedere la notte piu di sei passa lontano ponendo la candela appresso lo specchio in un certo ponto, e la imagine del lume va lontano circa sette passa, e nella oscurità della notte fa si gran luce che si puo discernere benissimo. Questa estate le persone dormivano con le fenestre aperte senza lume per non essere vedute, et io pigliava, il mio specchio, e con una candela mandava il lume nella camera delli vicini, e vedeva tutta la camera, echi gli era dentro come se fusse stato di giorno perche dove è pui oscuro fa miglior vista. Ancora mi son ritrovato io stare nell' oscuro la notte et essendo nel vicinato un lume acceso, che io poteva vedere posi d' incontro quel lume il specchio, e dove fa un splendore posi la carta scritta e lessi la lettera benissimo a parte à parte'. Ausonio di una nuova invenzione d' uno specchio, B. A. M., A 71 Inf., f. 20r. See Appendix I.

[28] Magini, *Theorica speculi concavi sphaerici*. Compare Ausonio, *Theorica*. See Appendix II.

[29] The invention of a properly working magic lantern involved, just as a primitive reflecting telescope, the appropriate combination of a convex lens and a concave mirror (and a correctly placed slide in the case of a magic lantern). As shown, there is no mention of a convex lens in Ausonio's light-projecting device. The magic lantern appears not to have been developed by Kircher and Huygens before the second half of the seventeenth century. On the early history of the magic lantern, see Hecht, Hermann. 'The History of Projecting Phantoms, Ghosts and Apparitions.' *The New Magic Lantern Journal* 3 (1984): 2-6; Van Nooten, S.I. 'Contributions of Dutchmen in the Beginnings of Film Technology.' *Journal of the SMPTE* 81 (1972): 116-23; Wagenaar, W.A. 'The True Inventor of the Magic Lantern: Kircher, Walgenstein or Huygens?' *Janus* 66 (1979): 193-207.

outside'.[30] The camera obscura itself was a well-known device. First, in medieval optics, the so-called problem of pinhole images was widely discussed in the context of the propagation of light. Lindberg has shown that Bacon, Pecham and Witelo made unsuccessful attempts at answering the question why light passing through apertures of whatever shape always reproduce at a certain distance the shape of the luminous source.[31] Second, since the thirteenh century, the camera obscura was used for astronomical purposes. The astronomer, mathematician and biblical exegete Levi ben Gerson (1288-1344) as well as William of Saint-Cloud, in 1292, used the camera obscura to project an image of the sun on a screen to observe solar eclipses.[32] In this context, the use of the camera obscura demanded a correct method to measure the apparent solar diameter. In the mid-sixteenth century, this use of the camera obscura appears to have come generally in use by astronomers, like Erasmus Reinhold, Gemma Frisius and Tycho Brahe.[33] (Figure 6.7)

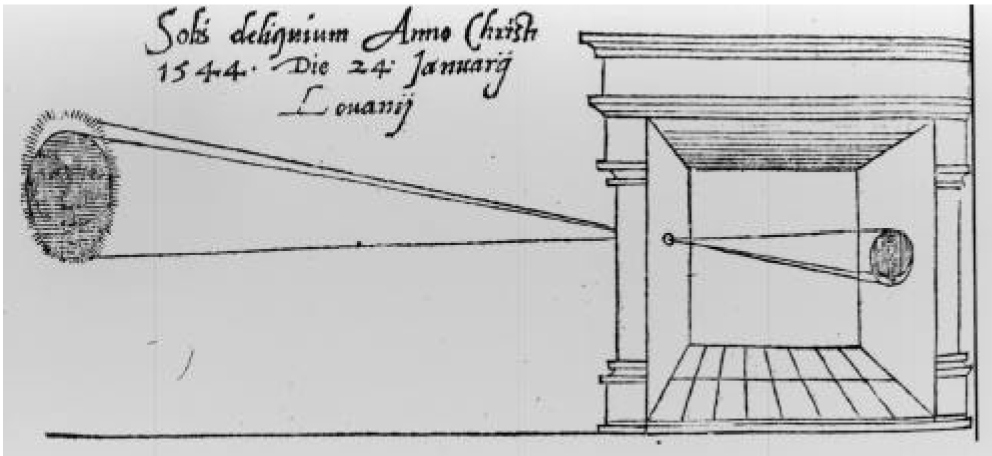

Figure 6.7

In his 'Paralipomena' (1604), most likely independently of Levi ben Gerson, Kepler found a solution for the measurement of the apparent solar diameter when using a camera obscura by subtracting the size of the aperture from the apparent size of the luminary cast on the screen of the camera obscura. Straker has shown that Kepler's distinction between point apertures and apertures of a finite size, made in an astronomical context which Kepler derived from Tycho, was at the basis of his correct solution to the problem of pinhole images, which the perspectivists had

[30] Ausonio, *Theorica*. See Appendix II.

[31] Lindberg, David C. 'The Theory of Pinhole Images from Antiquity to the Thirteenth Century.' *Archive for History of Exact Sciences* 5 (1968): 154- 76; Lindberg, David C. 'A Reconsideration of Roger Bacon's Theory of Pinhole Images.' *Archive for History of Exact Sciences* 6 (1970): 214-23; Lindberg, David C. 'The Theory of Pinhole Images in the Fourteenth Century.' *Archive for History of Exact Sciences* 6 (1970): 299-325.

[32] Goldstein, Bernard R. 'The Physical Astronomy of Levi Ben Gerson.' *Perspectives on Science* 5 (1997): 1-30, pp. 7-9; Goldstein, Bernard R. 'Levi Ben Gerson: On Astronomy and Physical Experiments.' In *Physics, Cosmology and Astronomy, 1300-1700: Tension and Accommodation*, edited by Sabetai Unguru, 75-82. Dordrecht Boston London: Kluwer Academic Publishers, 1991, pp. 78-80; Mancha, J. L. 'Astronomical Use of Pinhole Images in William of Saint-Cloud's Almanach Planetarum (1292).' *Archive for History of Exact Sciences* 43 (1992): 275-98.

[33] Straker, Stephen Mory. 'Kepler's Optics: A Study in the Development of Seventeenth Century Natural Philosophy.' Ph. D., Indiana University, 1971, pp. 304-61.

unsuccessfully tried to answer.[34] Finally, the meridiane provided another astronomical context in which the camera obscura was used.[35] A whole church was turned in to a gigantic camera obscura to study the crossing of the meridian, laid out on the church floor, by the sun's image projected through a small aperture. The first of this kind appears to have been built in 1475 by Toscanelli in the Santa Maria del Fiore in Florence. In mid-sixteenth century Italy, Danti built new meridiane in Florence and Bologna, for the astronomical purposes connected with the calendar reform.

However, the use of the camera obscura in combination with a concave mirror appears to have been a mid-sixteenth century practice. To obtain an image projected by a concave mirror, as in Ausonio's 'Theorica', the concave mirror is placed inside the camera obscura, a darkened room in which the light only enters through a small aperture, opposite the aperture, so that an image of the scene outside is focused on the wall or screen that contains the aperture. The advantage of the use of a concave mirror is that the projected image has the same left-right orientation as the scene outside. An image projected in an ordinary camera obscura is top-down inverted and left-right reversed. When projected by a concave mirror on the screen of the aperture, the image will still be inverted, but the left-right reversal will be undone. This advantage is of little interest, when using the camera obscura in the astronomical context for which it was used since the Middle Ages, that is for measuring the apparent diameter of the sun, but it is of use when the aim is visual representation. The image projected in camera obscura has the disadvantage that when the paper, on which the image is traced after the projected image, is taken from the wall and the paper is turned upside down, so that top and down are as in the original scene outside, left and right will be reversed with respect to the original scene. The only way to avoid this is to use a piece of paper that is not too thick, to hold the paper against the light, and to retrace the picture on the backside of the paper. The use of the concave mirror is a shortcut to this tiresome procedure. When the piece of paper, on which the image projected by a concave mirror is traced, is taken from the wall, and turned upside down, left and right will be as in the original scene.[36]

Thus, the image projected by a concave mirror might have been particularly useful for a painter, who could take the picture traced after the projected image from the wall and immediately compare it with and finish it after the real scene. The use of the concave mirror has a second advantage that, since there was no optical theory available from which it could be derived, was presumably only noted after a concave mirror had been used to revert the image. The image projected by a camera obscura on a screen of a fixed distance from the aperture will become progressively sharper by narrowing the diameter of the aperture.[37] There is a critical limit to this narrowing of the aperture, because beyond this limit the image will become less sharp due to diffraction. However, this limit was presumably never reached, because by progressively narrowing the aperture, the image would become too dim to discern much detail. The use of the concave mirror got around this problem by allowing projecting a sharp and bright image on the screen of the wall containing the aperture with light entering through a larger aperture.

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[34] Straker, Stephen. 'Kepler, Tycho, and the 'Optical Part of Astronomy': The Genesis of Kepler's Theory of Pinhole Images.' *Archive for the History of Exact Sciences* 24 (1981): 267-93.

[35] Heilbron, *The Sun in the Church: Cathedrals as Solar Observatories*, in particular, pp. 62-81.

[36] The problem of left-right orientation of the drawing can be easily verified by studying the drawings made by Hockney with a replication of this catoptrical technique. See Hockney, David. *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*. London: Thames & Hudson, 2001, in particular, pp. 74-7.

[37] Pirenne, M. H. *Optics, Painting and Photography*. Cambridge: Cambridge University Press, 1970, pp. 15-20.

That painters were trying combinations of the camera obscura with optical components around this time is confirmed by the contemporaneous discussion of the camera obscura in combination with a convex lens placed in the aperture of the camera obscura. First mentioned in print by Cardano in 1550, Barbaro discussed its painterly use in his ‘La Pratica della Perspettiva’ (1567).

> If you want to see how nature represents things foreshortened not only as concerns the contours of the whole, and the parts, but also the colors, and the shadows, and the likenesses, make an aperture in the shutter of a window of a room from which you want to look, as large as the lens of a pair of spectacles. And take a pair of spectacles for the elderly, which has a little bit of body in the middle, and which is not concave, like the spectacles for the young, who are nearsighted. Put this lens in the well-tried aperture. Next, close all windows and doors of the room, so that there is no light except the light that enters through the lens. Take a piece of paper, and place it opposite the lens and as much removed [from the lens], that you minutely see on the paper all that is outside the house. This happens most distinctly at a certain distance, which you find by approaching or withdrawing the paper with respect to the lens, until you find the convenient place. Here you will see the forms on the paper as they are, and the foreshortenings, and the colors, and the shadows, and the movements, the clouds, the trembling of the water, the flight of the birds and all those things that one can see. For this experience, it is needed that the Sun is clear and bright, because the light of the Sun has great force in extending the visible species, as to your satisfaction you can try by experience. For this experience, you choose those lenses which are best, and, if you want to cover the lens as much as to leave a bit of circumference in the middle, and that [part] which is clear is not covered, you will see an even more vivid effect. Then, because you will see on the paper the main lines of things, you can indicate the whole Perspective, which appears in this, with a pencil on the paper, and shadow and color it lightly according to what nature shows you, … , until you have finished the drawing.[38]

As the use of a concave mirror to project an image, the use of a convex lens has the advantage that a larger aperture is allowed, so that the image projected by the lens and focused on a piece of paper allows more detail to be seen than if the image is projected through an aperture of the same size without a convex lens applied to the aperture of the camera obscura.[39] Again, this could only

[38] ‘Se vuoi vedere come la natura pone le cose cose digradate ne solamente quanto a i contomi del tutto, & della parti, ma quanti i colori, & le ombre, & le simiglianze, farai uno bucco nello scuro d’ una finestra della stanza di dove vuoi vedere, tanto grande quanto é il vetro d’ un occhiale. Et piglia un’ occhiale da vecchio, cioè che habbia alquanto di corpo nel mezzo, & non sia concavo, come gli occhiali da giovani, che hanno la vista curta. & incassa questo vetro nel bucco assaggiato. Serra poi tutte le finestre, & le porte della stanza siche non vi sia luce alcuna, se non quella, che viene dal vetro, piglia poi uno foglio di carta, et ponlo incontro al vetro tanto discosto, che tu veda minutamente sopra ‘l foglio tutto quello che è fuori di casa, ilche si fà in una determinata distanza più distintamente. ilche troverai accostando, overo discostando il foglio al vetro, finche ritrovi il sito conveniente? Qui vi vederai le forme nella carta come sono, & le digradationi, & i colori, & le ombre, & i movimenti, le nubi, il tremolar delle acque, il volare de gli uccelli, & tutto quello, che si può vedere. A questa isperienza bisogna, che ci sia il Sole chiaro & bello, perche la luce del Sole ha grande forza in cavare le specie visibili, come con tuo piacere ne farai la isperienza, nella quale farai scielta di quelli vetri, che fanno meglio, & se vorrai coprire il vetro tanto, che vi lasci una poca di circonferenza nel mezzo, che sia chiara è scoperta, ne vederai anchora piu vivo effetto. Vedendo adunque nella carta i lineamenti delle cose, tu puoi con uno penello segnare sopra la carte tutta la Perspecttiva, che apparerà in quella. & ombreggiarla, & colorirla teneramente secondo, che la natura ti mostrerà … fin che haverai fornito il disegno’. Barbaro, Daniele. *La Pratica della Perspettiva*. Edited by Roberto Fregna and Giulio Nanetti. Vol. 8, *Biblioteca di Architettura Urbanistica Teoria e Storia*: Arnaldo Forni Editore, 1980, pp. 192-3. For Cardano, see Cardano, Girolamo. ‘De subtilitate.’ In *Opera, tomus tertius quo cintinentur physica*. Edited by Cardanus, Hieronymus. Lugduni: Sumptibus Ioannis Antonii Huguetan, & Marci Antonii Ravaud, 1663, book IV.

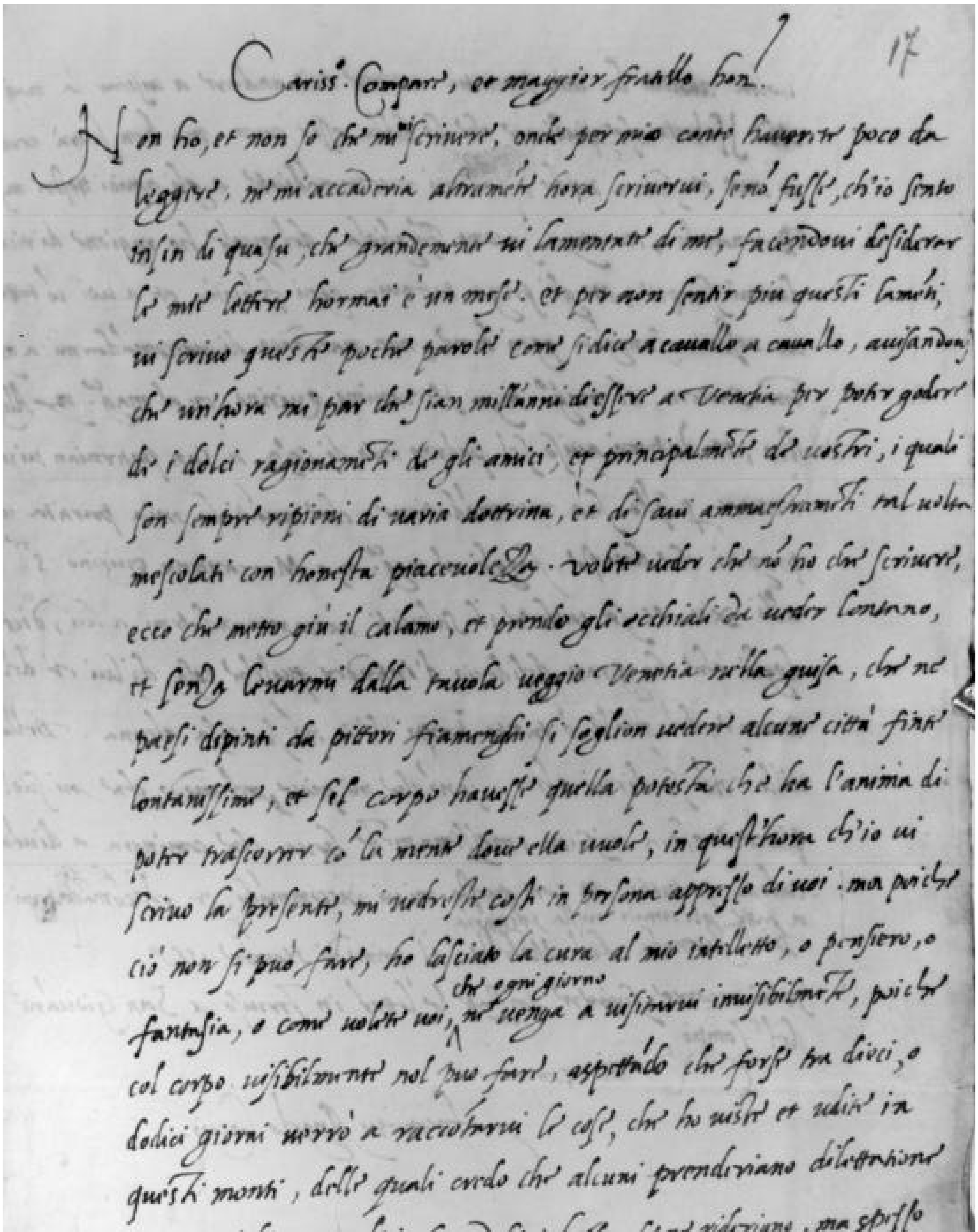
[39] For discussion, see Pirenne, *Optics, Painting and Photography*, pp. 22-4.

have been found out in practice, because no optical theory predicting that lens-images have a higher resolution was available in the fifteenth and sixteenth centuries. Moreover, as will become evident, that convex lenses project images is a notion quite alien to contemporaneous optics.
It is not known when such practices of projecting images were first used by painters. Hockney and Falco have recently suggested that fifteenth century Flemish painters, e.g. Jan Van Eyck and Hans Memling, were not only able to represent the reflected image in a convex mirror, present in several of their paintings, and use those convex mirrors as a drawing aid, but also already used images projected by concave mirrors and convex lenses as a drawing aid.[40] For our purposes, it is however important to point out that the surfacing of such practices in verbal and visual representations in manuscript or print, did not occur before the mid-sixteenth century. To the best of my knowledge, there is no discussion of the use of the concave mirror to project images before the mid-sixteenth century, thus, about the time of Ausonio's 'Theorica speculi concave sphaerici' of circa 1560. The practice of projecting images by a concave spherical mirror is alien to medieval optics. As discussed in chapter 4, medieval optics was interested in what the eye can see in a mirror, not in projecting an image on a piece of paper. As has been shown, only in Ausonio's 'Theorica', with the introduction of the focal point-point of inversion-point of confusion, a representation of this instrumental practice was found, so that images projected by a concave mirror became allowed in optics. Moreover, it will be shown that once this representation was found, which connected the focal point of a mirror with the images perceived in a mirror, the instrumental practice rapidly developed in to a telescopic practice.
Telescopic practice was readily connected with painting and visual representation. In a letter of 1 November 1557, a certain Francesco Angelo Caccio, homesick for Venice and Ausonio's companionship, wrote to him from Treviso, referring to a telescopic practice. (Figure 6.8A-B)

> Because I do not have anything but writing, I put down my pen, and I take my glasses to see far, and without getting up from the table, I see Venice in the way that in the landscapes painted by Flemish painters some cities represented faraway are seen, and if the body had the power of the soul to be able to traverse with the mind wherever it wants, at this moment that I am presently writing to you, you would see me in person close to you, but because this cannot be done, I have left the occupation to my mind.[41]

[40] Hockney, David. *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*, pp. 66-100. For Van Eyck's masterly representation of reflected and refracted light, see De Mey, Marc. 'Late Medieval Optics and Early Renaissance Painting.' In *La Prospettiva: Fondamenti Teorici ed Esperienze Figurative dall' Antichità al Mondo Moderno*, edited by Rocco Sinisgalli, 75-84. Firene: Edizioni Cadmo, 1998. For the use of convex mirrors in painters' workshops, see Schwarz, Heinrich. 'The Mirror in Art.' *Art Quarterly* 15 (1952): 97-118; Schwarz, Heinrich. 'The Mirror of the Artists and the Mirror of the Devout: Observations on Some Paintings, Drawings and Prints of the Fifteenth Century.' In *Studies in the History of Art Dedicated to William E. Suida on His Eightieth Birthday*, 90-105. London: Phaidon Press, 1959. See also Wheelock, Arthur K., Jr. *Perspective, Optics, and Delft Artists around 1650*. New York London: Garland Publishing Inc., 1977. The use of a convex mirror as a drawing aid must have been bothersome, since the painter looking at a scene in the mirror would always be troubled by the mirror image of his own face that impeded a view at the scene he wished to paint. See Magini, *Breve Instruttione*, p. 29.

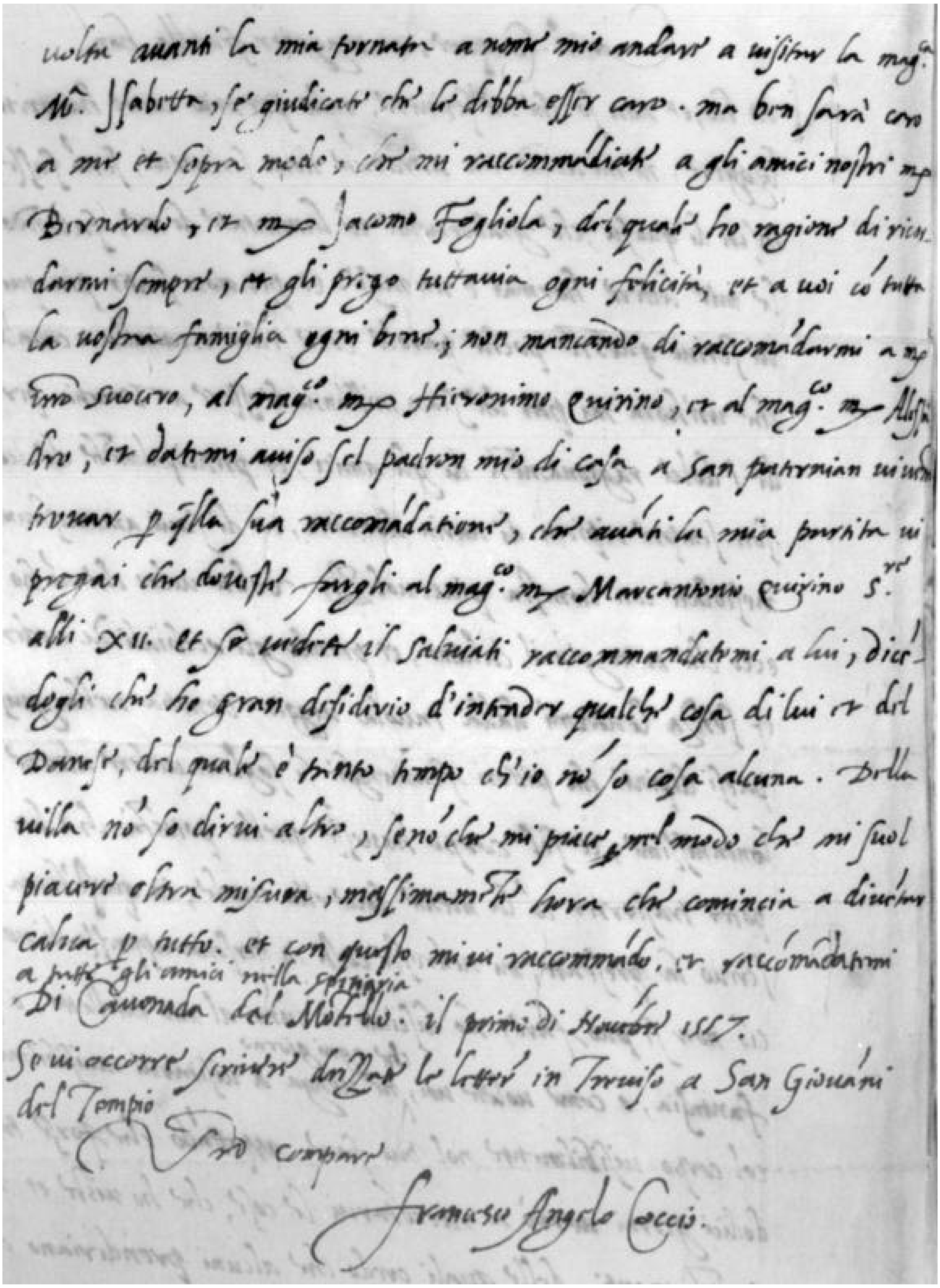
[41] 'volete veder che no ho che scrivere, ecco che metto giù il calamo, et prendo gli occhiali da veder lontano, et senza levarmi dalla tavola veggio Venetia nella guisa, che ne paesi dipinti da pittori fiamminghi si soglion vedere alcune città finte lontanissime, et sel corpo havesse quella potestà che ha l'anima di poter trascorrer co la mente dov' ella vuole, in quest' hora ch' io vi scrivo la presente, mi vedreste cosi in persona appresso di voi, ma poiche ciò non si puo fare, ho lasciato la cura al mio intelletto'. Caccio to Ausonio, 1 November 1557, B. A. M., D 178 Inf., f. 17r.



Cariss.° Compare, et maggior fratello hon. 17

Non ho, et non so che mi scrivere, onde per mio conto haverete poco da
leggere, ne mi accaderia altrimente hora scrivervi, se non fusse, ch'io sento
insin di quasu, che grandemente vi lamentate di me, facendovi desiderar
le mie lettere hormai è un mese. et per non sentir piu questi lamenti,
vi scrivo queste poche parole come si dice a cavallo a cavallo, avisandovi
che un'hora mi par che sian mill'anni d'essere a Venetia per poter godere
de i dolci ragionamenti de gli amici, et principalmente de vostri, i quali
son sempre ripieni di varia dottrina, et di savii ammaestramenti nel volere
mescolati con honesta piacevolezza. volete veder che nõ ho che scrivere,
ecco che metto giu il calamo, et prendo gli occhiali da veder lontano,
et senza levarmi dalla tavola veggio Venetia nella guisa, che ne
paesi dipinti da pittori fiamenghi si soglion vedere alcune città finte
lontanissime, et se'l corpo havesse quella potestà che ha l'anima di
potere trascorrere co'la mente dove ella vuole, in quest'hora ch'io vi
scrivo la presente, mi vedreste costi in persona appresso di voi. ma poiche
ciò non si può fare, ho lasciato la cura al mio intelletto, o pensiero, o
fantasia, o come volete voi, che ogni giorno ne venga a visitarvi invisibilmente, poiche
col corpo visibilmente nol può fare, aspettando che forse tra dieci, o
dodici giorni verrò a raccontarvi le cose, che ho visto et udito in
questi monti, delle quali credo che alcuni prenderiano dilettatione
et maraviglia, et altri facendosene beffa se ne rideriano, ma spesso
aviene che appresso'l riso seguita anche il pianto. ma ponendo hora
da parte queste cose vi voglio pregar siate contento almeno una

Figure 6.8A



uolse avanti la mia partita a nome mio andare a visitare la mag.ca
M.a Isabetta, se giudicate che le debba esser caro. ma ben sarà caro
a me et sopra modo, che mi raccommandiate a gli amici nostri m.r
Bernardo, et m.r Iacomo Feghiola, del quale ho ragione di ricor-
darmi sempre, et gli prego tuttavia ogni felicità, et a voi co' tutta
la vostra famiglia ogni bene; non mancando di raccomandarmi a m.r
vostro suocero, al mag.co m.r Hieronimo Quirino, et al mag.co m.r Alessan-
dro, et datemi aviso sel padron mio di casa a San paternian si vede
trovar p. quella sua raccomandatione, che avanti la mia partita mi
pregai che dovesse farghi al mag.co m.r Marcantonio Quirino S.re
alli XII. et se vedete il Salviati raccommandatemi a lui, dicen-
dogli che ho gran desiderio d'intendere qualche cosa di lui et del
Danese, del quale è tanto tempo ch'io no' so cosa alcuna. Della
villa no' so dirvi altro, se no' che mi piace, nel modo che mi suol
piacere oltra misura, massimamente hora che comincia a diventar
calva p. tutto. et con questo mi vi raccommando, et raccomandatemi
a tutti gli amici nella Spetiaria
Di Camonada del Motello. il primo di Novembre 1557.
Se vi occorre scrivermi drizzate le lettere in Trevisio a San Giovanni
del Tempio
Vostro compare
Francesco Angelo Coccio.

Figure 6.8B

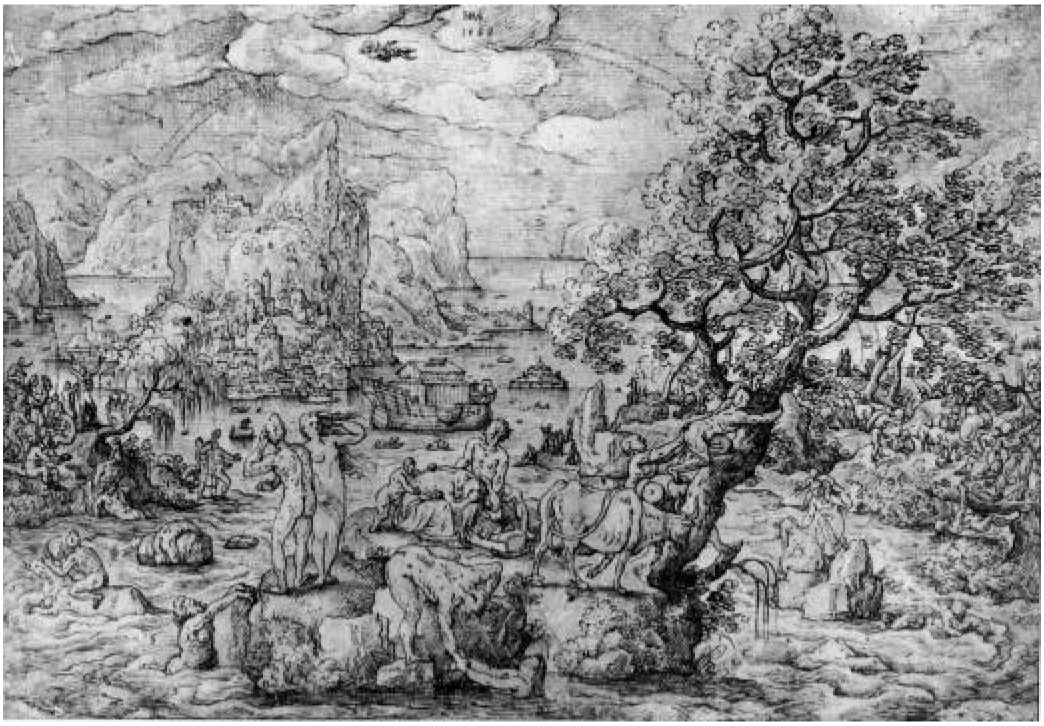


Figure 6.9

Figure 6.10

The 'occhiali da veder lontano' suggests the use of a lens rather than a concave mirror. However, the passage is not necesarrily to be taken at face value as corresponding with an actually existing telescopic device. The context of melancholy and nostalgy suggest that it is rather to be understood metaphorically. The metaphorical understanding of the telescopic practice is highlighted by the analogy that Caccio made between the 'skyline' of Venice that he saw through his 'occhiali da veder lontano' and the cities represented on sixteenth century Flemish landscape paintings. Distant cities are represented on many of the contemporaneous landscape paintings of Hans Bol, Pieter Brueghel the Elder and Joost de Momper.[42] (Figure 6.9 – Figure 6.10) However, different from seventeenth century Dutch landscape painting, sixteenth century Flemish painters often represented imaginary landscapes. These paintings seem to derive from the same dream-like and imaginative context that is suggested by the letter of Caccio. Consequently, the comparison drawn by Caccio between the imaginary distant cities on Flemish landscape paintings and the imaginative context of what he saw through his *occhiali* was appropriate.

However, it is no coincidence that Caccio chose an optical metaphor, while any other metaphor might have been equally appropriate. It appears that an optical method was responsible for the distant city views in contemporaneous landscape paintings, and, consequently, that as imaginary though these paintings qua composition were, parts of these paintings, and the distant city views must have been first choice among the parts, were drawn with an optical aid. In his 'Dialogo di Pittura' (1548), the Venetian painter and art theorist, Paolo Pino claimed that the representation of landscapes was done by 'this method of representing the landscapes in the mirror'.[43] A similar catoptrical method was presumably used in the many contemporaneous landscape paintings of restricted dimensions, which, precisily because of their limited dimensions, were particularly apt at being drawn with the help of a mirror projection.[44] Consequently, the optical method on which Caccio's telescopic metaphor was based, was not telescopic, but referred to the contemporaneous painterly practice of projecting images with a concave mirror. Ausonio must have very well undertood the metaphor, since, as has been shown, he referred to this painterly use of the concave mirror in his 'Theorica'. The telescopic metaphor, based on actual catoptrical practice, in Caccio's letter to Ausonio also shows that in the minds of mid-sixteenth century mathematical practitioners, it would have been evident to search for a telescopic device, starting from the existing catoptrical practice, used in the painterly context of representing distant city views.

The context of the passage from Digges' 'Pantometria' (1571) that Ronan and Rienitz haven taken as referring to the first reflecting telescope, with a convex lens as objective and a concave mirror as ocular, confirms that representing distant city views was the aim of the search for a telescopic device.[45] The complete title of Digges' work, 'A Geometrical Practise, named

[42] For the distant city views in Bol's drawings, see Franz, H. Gerhard. *Hans Bol als Landschaftszeichner*. Vol. 1, *Jahrbuch des Kunsthistorischen Institutes der Universität Graz*. Graz: Akademische Druck, 1965.

[43] 'quel modo di ritrarre li paesi nello specchio'. Pino, Paolo. *Dialogo di Pittura*. Venetia: Paolo Gherardo, 1548, quoted in Damianaki, *Galileo e le Arti Figurative*, p. 47.

[44] For discussion, see Negri-Arnoldi, F. 'Tecnica e Scienza.' In *Storia dell' Arte Italiana*, 205-24. Vol. 4: Ricerche Spaziali e Tecnologie. Torino: Einaudi, 1980, p. 207; Damianaki, *Galileo e le Arti Figurative*, p. 47.

[45] For the Elizabethan reflecting telescope, see Ronan, Colin A. 'The Origins of the Reflecting Telescope.' *Journal for the British Astronomical Association* 101 (1991): 335-42; Ronan, Colin A. 'Leonard and Thomas Digges.' *Endeavour* 16 (1992): 91-94; Ronan, Colin A. 'There Was an Elizabethan Telescope.' *Bulletin of the Scientific Instrument Society* 37 (1993): 2-3; Rienitz, Joachim. ''Make Glasses to See the Moon Large': An Attempt to Outline the Early History of the Telescope.' *Bulletin of the Scientific Instrument Society* 37 (1993): 7-9; Rienitz, Joachim. *Historisch-Physikalischen Entwicklungslinien Optischer Instrumente: Von der Magie zur Partiellen Kohärenz*.

Pantometria divided into three Bookes, Longimetra, Planimetra and Stereometria, containing Rules manifolde for mensuration all lines, Superficies and Solides: with sundry straunge conclusions both by instrument and without, and also by Perspective glasses, to set forth the true description or exact plat of an whole Region', shows that the use of 'perspective glasses' was intended for topographical purposes. The passage that refers to the reflecting telescope also indicates that Digges' most important aim was the representation of distant city views.

> Marveylouse are the conclusions that may be perfourmed by glasses concave and convex of circulare and parabolicall fourmes, using for multiplication of beames sometime the ayde of glasses transparent, which by fraction should unite or dissipate the images or figures presented by the reflection of other. By these kinde of glasses or rather frames of them, placed in due angles, *ye may not only set out the proportion of an whole region, yea represent before your eye the lively image of every towne, village*, &c. and that in as little or great space or place as ye will prescribe, but also augment and dilate any parcell thereof, so that whereas at the first apparance an whole towne shall present it selfe so small and compacte together that ye shall not discerne any difference of streates, ye may by application of glasses in due proportion cause any peculiare house, or roume thereof dilate and shew it selfe in as ample fourme as the whole town firste appeared.[46]

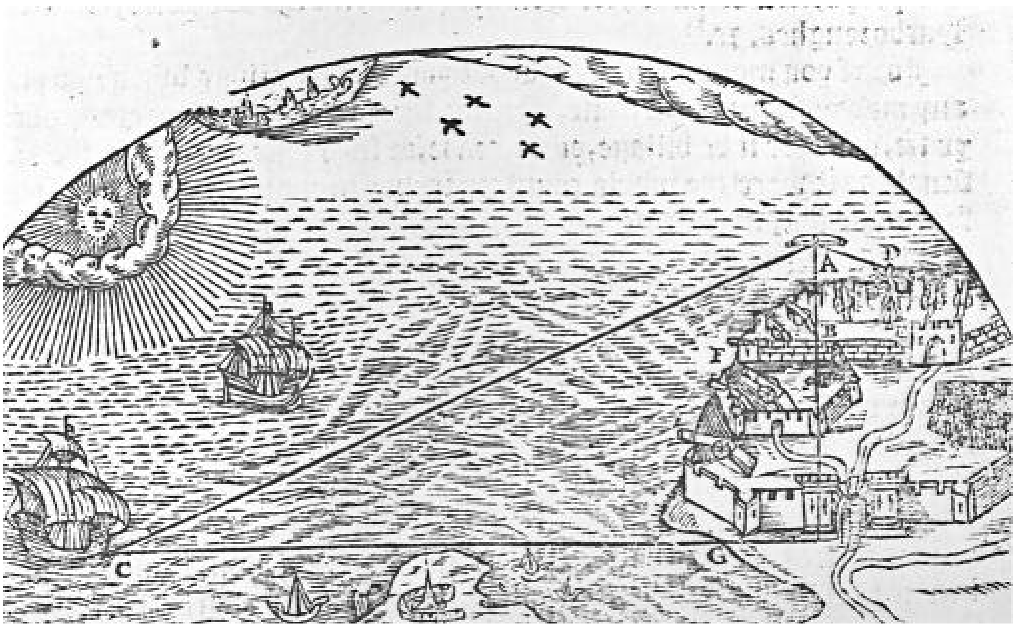


Figure 6.11

Lengerich: Pabst Science Publishers, 1999, pp. 66-132. For the history of the reflecting telescope, see Ariotti, Piero E. 'Bonaventura Cavalieri, Marin Mersenne, and the Reflecting Telescope.' *Isis* 66 (1975): 303-21.

[46] Digges, Thomas. *A Geometrical Practise, Named Pantometria, Divided into Three Bookes, Containing Rules Manifolde for Mensuration of All Lines, Superficies and Solides: With Sundry Straunge Conclusions Both by Instrument and without, and Also by Perspective Glasses, to Set Forth the True Description or Exact Plat of an Whole Region*. London: Henrie Bynneman, 1571, ff. Giv-Giir, my italics.

The broader context of this passage discussed a method to measure with a plane mirror the distance of a ship from a high cliff at the shore, but Digges also indicated that a similar method with a plane mirror could be used to lay out a topographical map of a country. (Figure 6.11)

> You may on this manner from an highe hill or mountayne, having any playne or levell grounde on the toppe, not onely measure the distance of any marke that ye can see, but also set foorth the true platte and proportion of an whole Countrey, with all the Townes, Coastes, Harboronghes, ec. For yf you move circularely about your glasse, alway when you espr any marke, setting up a staffe, writing theuppon the name of the place ye see, whether it be village, porte, roade, or such lyke, ye shall in the end situate as it were the whole countrey in due proportion upon your plat fourme, so that measuring the distance of every staffe set up from the middle lyne perpendiculare falling from the glasse, and the distaunce likewise of every staffe from other, ye may (working by the golden rule) finde out the exacte distaunce of every town, village porte, roade or such lyke from your platfourme, and also how farre every one is distant from other. Thus much is thought good to open concerning the effects of a playne Glasse, very pleasant to practise, yea most exactlye serving for the description of a playne champion countrey.[47]

Consequently, Digges' procedure to make a topographical map is a standard triangulation procedure, based on taking the angles from a sighting point, ideally situated on a hill or tower, to the principal landmarks of a city or countrey, but using a plane mirror instead of a surveying compass, which, as shown in chapter 5, Tartaglia used to make a topographical map. In contemporary Flemish city views, in a context which was less imaginary than contemporaneous Flemish landscape painting, and which were highly influential in the rest of Europe, the complete geometrical control of the space was augmented by profile views of the principal buildings or the 'skyline' of a city.[48] A typical example is the city view of Valencia which the contemporaneous Flemish drafstman Anton van den Wyngaerde drew around 1563. Two of his preparatory sketches show the typical profile view of the principal buildings of Valencia and a profile study of the fortification walls of Valencia. (Figure 6.12 – Figure 6.13)

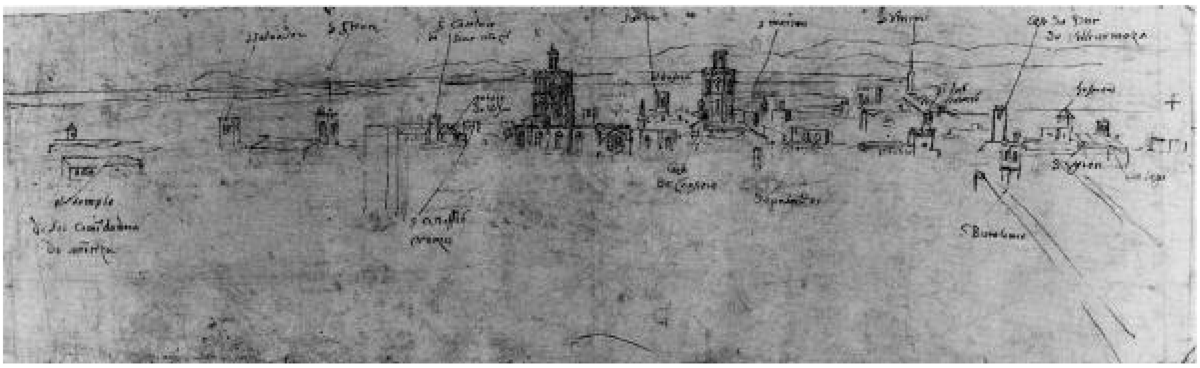


Figure 6.12

[47] Ibid., f. Giv.

[48] Nuti, Lucia. 'The Perspective Plan in the Sixteenth Century: The Invention of a Representational Language.' *The Art Bulletin* 76 (1994): 105- 28, in particular, pp. 109-15. See also Nuti, Lucia. *Ritratti di Città: Visione e Memoria tra Medioevo e Settecento*. Venezia: Marsilio Editore, 1996, pp. 69-100; Schulz, Juergen. *La Cartografia tra Scienza e Arte: Carte e Cartografi nel Rinascimento Italiano*. Ferrara: Franco Cosimo Panini, 1990, pp. 37-41.

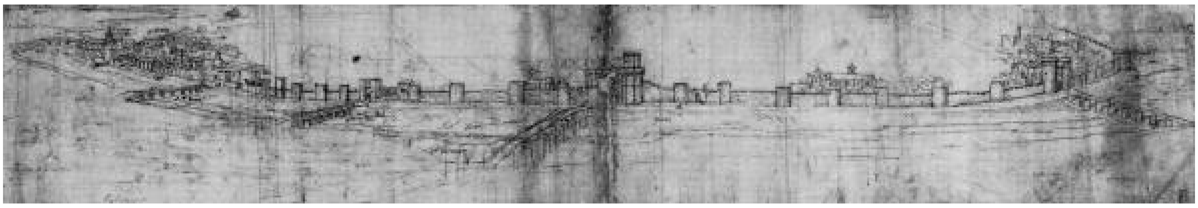


Figure 6.13

Consequently, Digges must have considered his reflecting telescope particularly apt at drawing these profile views, from a distant point of view, which was needed to geometrically control the topographical space.

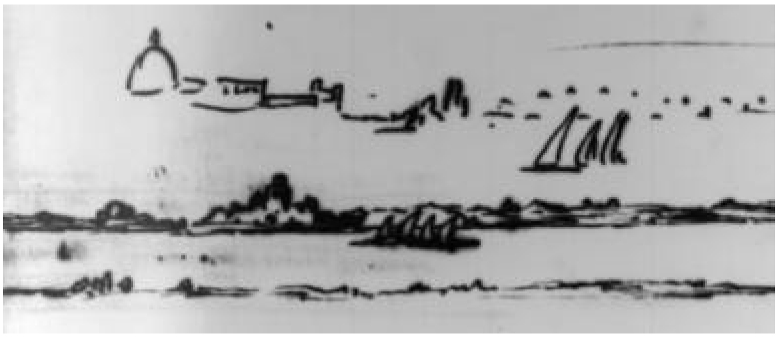


Figure 6.14

Figure 6.15

Finally, the established cultural pattern of using a telescope to draw distant city views influenced none other than Galileo. Among his notes of calculations of the periods of the satellites of Jupiter, between 1610 and 1612, there are two drawings, which surprised Bredekamp, because of their high quality, comparable to his drawings of the moon, which far surpassed Galileo's youthful drawings of female nudes.[49] (Figure 6.14 – Figure 6.15) The first drawing shows the 'skyline' of

[49] Bredekamp, 'Gazing Hands and Blind Spots: Galileo as Draftsman', pp. 439-40.

Venice, seen across the Venetian lagoon, while the second drawing is a view of a townscape extending acrosse two plateaus connected by a fortification wall running upwards and connecting the two plateaus. The similitarities with the sketches of Van den Wyngaerde of the skyline and fortification wall of Valencia are too obvious to escape notice. Although Galileo was presumably not familiar with these specific drawings of Van den Wyngaerde, he must have been acquainted with similar drawings of distant city views. Consequently, considering the cultural background of the telescope, it is most likely that Galileo's drawings are not sketches of an imaginary scene in Galileo's mind, but drawn from life as seen through his recently invented telescope.

Thus, from Ausonio's 'Theorica' to the reflecting telescope of Digges and the refracting telescope of Galileo, there is a fixed cultural setting of visual representation of distant city views in the context of making topographical maps, in which a telescopic device was considered useful. Was the notion of point of inversion in Ausonio's 'Theorica' also at the basis of the attempts to make such a telescopic device? It will be argued that precisily this notion, which in practice linked what is seen through lenses and mirrors with the focal point of these optical devices, made the invention of the telescope in the second half of the sixteenth century possible. First, the influence of Ausonio's 'Theorica' will be discussed. In particular, it will be shown how the knowledge of the point of inversion was transferred from concave mirrors to convex lenses in Della Porta's 'De Refractione' and the more detailed descriptions of Digges' reflecting telescope in the work of Bourne. Second, it will be shown how the point of inversion was used when designing a reflecting telescope, in particular, in the work of the Elizabethans Digges and Bourne.

3. The Point of Inversion, Lenses, and the Reflecting Telescope

Convex lenses show effects similar to concave mirrors. It is possible to obtain them by the experience of varying the distance of a candlelight to a convex lens and placing the eye at a distance exceeding the center of curvature of the lens, similar to the experience described in the context of Ausonio's determination of the point of inversion of a concave mirror in his 'Theorica'.[50] First, if the candlelight is close to the lens, a right oriented virtual image is seen, of approximately the same size as the candlelight itself. When the candlelight is moved farther away from the lens, a virtual image will be seen, in the immediate vicinity of the candlelight, which becomes progressively larger. If the candlelight is at the focal point of the mirror, the image is not seen infinitely large and at an infinite distance, as modern geometrical optics predictes, but, although still growing in size, still more or less at the same distance from the lens as the candlelight. When the candlelight is moved a little bit beyond the focal point of the lens, the image will fill the whole lens surface. As in a concave mirror, this is the point of inversion, because if the candlelight is moved farther away from the lens, the image will be inverted. The image will become progressivily sharper, but also progressively smaller, until it is of the same size as the candlelight when the candlelight is at the center of curvature. Moving the candlelight beyond the center of curvature will result in a progressively smaller inverted image.

As in a concave mirror, a similar effect is obtained by varying the distance between the eye and the lens, when looking at a distant target. If the convex lens is held against the eye, a slighly magnified right oriented image is seen. When the lens is moved away from the eye, the image will become larger until when the eye is at the point of inversion, more or less the focal point of

[50] For a similar 'experiment', see Ronchi, Vasco. *L' Optique, Science de la Vision*, pp. 79-83.

the lens, the image will appear to collapse and fill the whole surface of the lens. When the eye is moved beyond the point of inversion, an inverted image will appear which, though still magnified, will become progressively sharper and smaller. When the eye is at the center of curvature, the inverted image will be of the same size as the distant target. Beyond this center of curvature, the image will always be inverted and become progressively smaller. The point of inversion was introduced in the discussion of convex lenses at the end of the sixteenth century. Moreover, as will become evident, it was most likely introduced for convex lenses by analogy with previous experience with concave mirrors. To make this point, the discussion of lenses by two sixteenth century opticians, Francesco Maurolico and Della Porta will be compared. Maurolico did not use the notion of point of inversion, while Della Porta, who, as will be shown, most likely knew Ausonio's 'Theorica', used it both for concave mirrors and convex lenses.

Francesco Maurolico (1494-1575) was born in Messina in Sicily from Greek immigrants.[51] He devoted his intellectual life to the restoration of mathematics. He prepared editions of the works of the mathematicians of antiquity, for example, of, Euclid, Apollonius and Archimedes, but he also contributed mathematical works of his own. He was well known to Commandino and Clavius. One of these mathematical works of his own was his 'Photismi de lumine et umbra ad perspectivam, et radiorum incidentiam facientes', containing his discussion of reflection, which he appears to have finished at a very young age in 1521. Other optical works of Maurolico are his 'De erroribus speculorum', written in 1555, and his 'Diaphanorum' on refraction, refractive spheres and spectacle lenses, which he finished in 1554.[52] These works were not published during Maurolico's life. Although written in the mid-sixteenth century and at different dates, Maurolico's optical works were only published for the first time, together in one book, in 1611, with annotations by Clavius, who seems to have owned the manuscript, when they appear to have acquired a new importance after Galileo's telescopic discoveries, and, again in 1613.[53]

Maurolico's work has received very different evaluations, depending upon whether or not he is considered the prelude to Kepler's 'Paralipomena'.[54] Leaving this discussion aside, I will focus on Maurolico's involvement with the optical instrumental practice of the sixteenth century. Propositions 30 to 35 of his 'Photismi de lumine et umbra' discuss concave spherical mirrors. First, he discussed the burning effect of a concave spherical mirror. (Figure 6.16) In proposition 30, reminiscent of the pseudo-Euclidean proposition on the concave spherical mirror discussed above, Maurolico demonstrated that two rays issuing from a point C on the axis of the mirror, with its center of curvature at E, and diverging equally from this axis to be reflected in the points F and G of the same circle around the vertex D of the mirror, intersect the axis of the mirror in the same point H.[55] Moreover, he also argued that rays, hitting the surface of the mirror in points

[51] For Maurolico's biography and bibliography, see Rose, Paul Lawrence. *The Italian Renaissance of Mathematics*, pp. 159-84. See also Clagett, Marshall. 'The Works of Francesco Maurolico.' *Physis* 16 (1974): 148-98.

[52] For the dating of different sections of the 'Photismi de Lumine', see Ronchi, *The Nature of Light*, pp. 102-6.

[53] For the publication history of Maurolico's 'Photismi de Lumine et Umbra', see Scaduto, Mario. 'Il Matematico Francesco Maurolico e i Gesuiti.' *Archivum Historicum Societatis Jesu* 17 (1948): 126-41.

[54] Compare Ronchi, *The Nature of Light*, pp. 99-107; Lindberg, 'Optics in Sixteenth Century Italy', pp. 132-41.

[55] Maurolico, Francesco. *Francisci Maurolyci Abbatis Messanensis Mathematici Celeberrimi Theoremata De Lumine, Et Umbra, Ad Perspectivam & Radiorum Incidentiam Facientia. Diaphanorum Partes, Seu Libri Tres: In Quorum Primo, De Perspicuis Corporibus: In Secundo, De Iride: In Tertio, De Organi Visualis Structura, & Conspiciliorum Formis, Agitur. Problemata Ad Perspectivam & Iridem Pertinentia.* Lugduni: Apud Bartholomaeum Vincentium, 1613, pp. 28-9. There is an unreliable English translation, Maurolico, Francesco. *The Photismi de Lumine of Maurolycus*. Translated by Henry Crew. New York: The Macmillan Company, 1940.

of different circles around the vertex of the mirror, also intersect the axis in different points.[56] The more the incoming rays diverge from the axis, the closer to the vertex of the mirror they intersect the axis of the mirror. While recognizing 'spherical aberration', Maurolico was convinced that rays, which he made diverging from a point C on the sun, which are reflected by a small part of the concave mirror, come together in a small zone on the axis.[57] Consequently, Maurolico claimed that these reflected rays are able to kindle fire. Thus, although Maurolico recognized 'spherical aberration' in a concave spherical mirror, he was not able to determine the focal point of this mirror. Maurolico might have thought it to be the center of curvature.

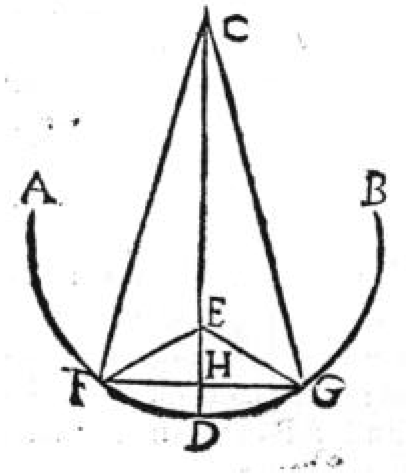


Figure 6.16

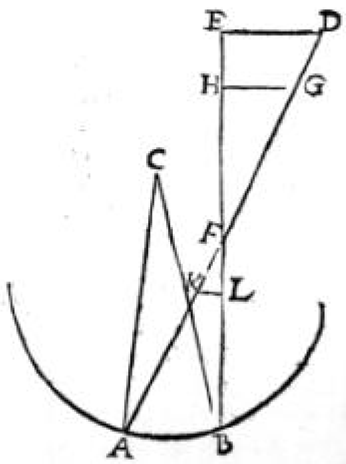


Figure 6.17

[56] Maurolico, *Theoremata de Lumine, et Umbra*, p. 29.

[57] Ibid., p. 29.

Maurolico's discussion of image formation in a concave spherical mirror in proposition 33 of his 'Photismi de lumine et umbra' was separated from his discussion of the burning properties of the mirror. Although Maurolico's discussion of image formation did not make use of an eye, his diagram concerning the 'point of inversion' in proposition 33 is equivalent to Witelo's diagram discussed above in chapter 4, substituting a light source C for the eye.[58] (Figure 6.17) Maurolico argued that 'when the light is reflected by a concave mirror, the shadow [cast by] an object outside the meeting of the rays is like [the object], while if it is within [the meeting of the rays] it is projected inverted'.[59] This is equivalent to the claim that the object KL seen by the eye C yields an inverted image ED, while the object HG, seen along the same rays, results in an image ED that is oriented in the same way as the object HG. However, Maurolico's analysis of the point of inversion F in image formation in a concave mirror was not related to his analysis of the burning properties of these mirrors. He did not suggest that the point of inversion F was in the focal plane of the mirror. Nor did this point function in the same way as in Ausonio's 'Theorica', since there are no eyes and, thus, no images as such in Maurolico's 'Photismi de lumine'.

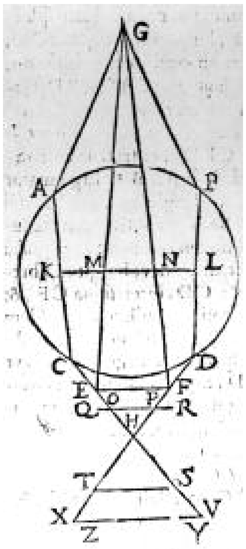


Figure 6.18

Different from his 'Photismi de lumine et umbra', there are eyes in Maurolico's 'Diaphanorum', written more than thirty years later. Maurolico first discussed image formation in a refractive sphere before discussing convex and concave lenses. Proposition 15 is most important as concerns the point of inversion.[60] In Maurolico's diagram, ABCD is the refractive sphere and G is the eye that sees the object EF along the rays GACE and GBDF. (Figure 6.18) Maurolico

[58] Ibid., pp. 32-3.

[59] 'Cavo speculo lucem reflectente, à re extra radiorum congressum umbra sicut est: intra verò, conversa proiicietur'. Ibid., p. 32.

[60] 'Quod per diaphanam sphaeram transparet intra radiorum congressum, maius vero, quàm si inter sphaeram esset, & eo maius, quo ad signum congressus propius accesserit, spectatur, & in vero situ. Extra vero congressum protinus omnifariam conversum apparebit: & prope congressum quidem, maius quam sit, ac recedens minus, & eo minus, quo longius recesserit'. Proposition 25, in Ibid., p. 47.

argued that the object EF will appear right oriented and larger. Moreover, the object will become progressively larger as it approaches the meeting point [congressum radiorum] H of the refracted rays issuing from the eye G. The image of an object TS beyond the meeting point H will appear inverted and become progressively smaller as the object moves away from the meeting point H. However, again, Maurolico did not relate this meeting point with a focal point, although, as for concave spherical mirrors, he showed him again aware of 'spherical aberration'. In proposition 18, he argued that if parallel rays are refracted in a sphere, the farther from the axis of the sphere such a ray is, the closer to the sphere, the refracted ray will intersect the axis of this sphere.[61]

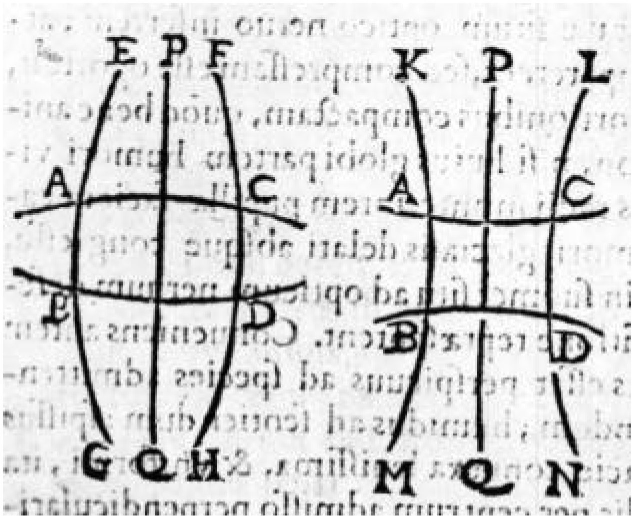


Figure 6.19

Maurolico was however well aware of contemporaneous optical instrumental practice. In proposition 23, he argued 'if whatever light or illuminated [object] shines through a transparent sphere, it projects its inverted image on a surface near the endpoints of the meeting [rays]'.[62] This proposition was based on the contemporaneous practice of projecting an image inside a camera obscura with a convex lens applied to the aperture of the camera obscura. The corollary to this proposition stated that 'it is therefore clear why light or any illuminated object extends its inverted image through the transparent glass of lenses to the endpoint'.[63] Maurolico considered such a convex lens to yield a more distinct image, because a convex lens was only part of a larger sphere. Consequently, a lens succeeded better than a sphere in making the rays issuing from a point converging again in one point.[64] Maurolico also took up lenses in his section 'on the organ

[61] Ibid., p. 48.

[62] 'Si quod lumen, vel illuminatum quodpiam per sphaericum transpareat diaphanum, ad superficiem iuxta concursuum appositum, conversam sui formam proiicet'. Ibid., p. 54.

[63] 'Patet ergo ratio, quare lux vel aliquod illuminatum per conspiciliorum vitrum transparens ad terminum quendam conversam porrigit effigiem'. Ibid., p. 55.

[64] 'quandoquidem conspicilia superficiem habent vtrinque conuexam. Immò in huiusmodi vitro talis conuersa effigies expressior transparet, quàm si vitrum ipsum sphaericum esset. Et hoc quia vitrum illud habet superficies, quae sunt paruae sphaerae portiones, quare fit, vt parum absit quin omnes ab vno signo radij in vnum congrediantur signum, & ideò per singula puncta, distincta imprimatur effigies'. Ibid., p. 55.

of sight and the forms of lenses'.[65] However, Maurolico's principal aim in this context was to explain the functioning of eyeglasses.[66] There is no discussion of image formation in lenses. Maurolico argued that a convex lens makes the rays converge, while a concave lens makes the rays diverge. It is interesting to note that in his account of lenses Maurolico retreats to talking about visual rays [radij visuales], which are used nowhere else in his treatise. Maurolico explained that 'just as visual rays transmitted through a transparent [lens], convex on both sides, more rapidly come together in a narrow [space], the visual rays that go through a [lens], concave on both sides, spread out more'.[67] Maurolico's diagram shows the symmetrical case in which rays parallel to one other inside a lens, converge and diverge from it to the same extent as they arrive at the lens before being refracted. (Figure 6.19) Maurolico was also aware of the effect of the curvature of the lens. He argued that 'in a more curved convex [lens] the refracted rays come more together, while in a concave [lens] which is more concave, the refracted rays are spread out more'.[68] Finally, Maurolico brought his knowledge of lenses to the problem of the cause of the visual defects of myopia and presbyopia.[69] He recognized that the crystalline humour in the eye is a biconvex lens, and that myopia occurs when the curvature of this crystalline lens is too great, and presbyopia when the curvature is insufficient. In the case of myopia the rays converge too rapidly, while in the case of presbyopia they converge too slow. Thus, from Maurolico's analysis of the passage of visual rays through convex and concave lenses, it followed that to treat myopia a biconcave lens is required and for presbyopia a biconvex lens is needed.

Maurolico's 'De conspiciliis' (1554) is the first analysis in the optical literature of the passage of rays through convex and concave lenses. He was also the first to analyze the cause of visual defects and to relate his knowledge of lenses to spectacle glasses to remedy these defects. However, the point of inversion did not play any part in his analysis of convex lenses. Fourty years later, when Della Porta published his 'De refractione' (1593), the point of inversion had become the principal notion to analyze image formation in convex lenses. First, it will be shown that Della Porta most likely had access to Ausonio's 'Theorica'. It is possible to derive Della Porta's debt to Ausonio's 'Theorica' by comparing his analysis of concave mirrors in his 'Magiae naturalis' (1589) as well as in his 'De refractione'. Second, it will be argued that in his 'De refractione' he transferred his knowledge of concave mirrors to convex lenses. As will become evident, Della Porta's analysis of convex lenses was not based on his analysis of image formation in refractive spheres, but on his analysis of image formation in concave mirrors.

Della Porta's 'Natural Magick' discussed catoptrical effects that appear to have been derived from Ausonio's 'Theorica'. Ausonio's editor, Magini, was convinced that Della Porta's catoptrical effects were indebted to Ausonio's 'Theorica'. In his 'Breve instruttione', he suggested that Ausonio's catoptrical effects 'were extensively discussed by Mr. Batista dalla

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[65] 'de organi visualis structura, & conspiciliorum formis'. Ibid., p. 77.

[66] For the discussion of lenses in the context of Maurolico's visual theory, see Lindberg, 'Optics in Sixteenth Century Italy', pp. 138-41; Frangenberg, Thomas. 'Perspectivist Aristotelianism: Three Case-Studies of Cinquecento Visual Theory.' *Journal of the Warburg and Courtauld Institutes* 54 (1991): 137-58, pp. 145-50.

[67] 'Sicut radij visuales per convexum utrinque diaphanum transmissi citius in angustum coëunt: ita per concavum utrinque trasecti magis dilatantur'. Maurolico, *Theoremata de Lumine, et Umbra* p. 81.

[68] 'In convexo conglobatiori magis couniri fractos radios: in concavo autem magis cavo, magis dilatari'. Ibid., p. 82.

[69] Ibid., pp. 85-7.

Porta in his Natural Magick'.[70] As discussed in chapter 2, Della Porta was in Venice in 1580 to attend the construction of a parabolic mirror. At that occasion, he not only met Contarini, who supervised the construction, but also Paolo Sarpi.[71] As shown in chapter 4, Sarpi was well acquainted with Ausonio's 'Theorica'. His copy might have been as early as 1578. Consequently, it is most likely that Della Porta became acquainted with Ausonio's 'Theorica' through his Venetian contacts on his visit in 1580. The similarities of Della Porta's optical work with Ausonio's 'Theorica' are too close to be explained by independent discovery. Beside the catoptrical effects 'in tenebris', Della Porta's determination of the focal point of a concave spherical mirror at half its radius of curvature and its identification with the point of inversion, as has been shown highly innovative, strongly suggest that Della Porta was indebted to Ausonio.

As concerns the catoptrical effects 'in tenebris', beside the camera obscura in combination with a concave mirror and with a convex lens, which Della Porta might have borrowed more generally from contemporaneous optical instrumental practice, Della Porta also discussed the use of candlelight, placed in the focal point of the mirror, to enhance the light and read letters at night.[72] It is most likely that Della Porta was indebted to Ausonio's 'Theorica' for this catoprical effect.

> Take the Glass in your hand, and set a candle to the point of Inversion, for the parallel beams will be reflected to the place desired, and the place will be enlightened above sixty paces, and whatsoever falls between the parallels, will be clearly seen: the reason is, because the beams from the Centre to the circumference, are reflected parallel, when the parallels come to a point; and in the place thus illuminated, letters may be read, and all things done conveniently, that require great light.[73]

Although in this quote of his 'Natural Magick', Della Porta appears to confuse the focal point of a concave spherical mirror with its center of curvature, on other occasions in his 'Natural Magick', he showed himself aware of the locus of the focal point of a concave spherical mirror, when avoiding 'spherical aberration' by using an aperture to obstruct the incoming light.

> In a Concave spherical Glass the beams meeting together, kindle fire in a fourth part of the diameter under the Centre, which are directed within the side of a Hexagon from the superficies of the circle. But a Parabolic Section, is, wherein all the beams meet in one point from all the parts of its superficies.[74]

Moreover, in his 'Natural Magick', Della Porta claimed that the focal point was also the 'point of inversion' of the concave spherical mirror. He proposed the reader to 'hold your Glass against the Sun, and where you see the beams unite, know that to be the point of Inversion'.[75] Again, Della

[70] 'lequali sono state poi molto copiosamente distese dal Signor Battista dalla Porta nella sua Magia naturale'. Magini, *Breve Instruttione*, p. 31.

[71] On the relationship between Della Porta and Sarpi, see Gliozzi, M. 'Relazioni Scientifiche tra Paolo Sarpi e GB Porta.' *Archives Internationales d' Histoire des Sciences* 1 (1947): 395-433.

[72] Porta, *Natural Magick*, pp. 363-5. Porta discussed three types of camera obscura: (1) with a simple aperture, (2) with a convex lens, (3) with a concave mirror. See also Muraro, Luisa. *Giambattista della Porta Mago e Scienziato*. Milano: Feltrinelli, 1978, p. 121. For a contemporaneous discussion of the camera obscura in combination with plane mirrors, see also Danti, *La prospettiva di Euclide*, pp. 81-4.

[73] Porta, *Natural Magick*, p. 362.

[74] Ibid., p. 371.

[75] Porta, *Natural Magick*, p. 360.

Porta showed his debt to Ausonio's 'Theorica' in his determination of the locus of the focal point of the concave spherical mirror, and its identification with the point of inversion of this mirror.
Della Porta's 'De Refractione' was a discussion of optics, more along the lines of a traditional treatise on optics than his 'Natural Magick'. In this work, Della Porta is very clear about the locus of the focal point of a concave spherical mirror. In the first proposition of the second book of 'De refractione', he demonstrated that 'in the reflections of concave mirrors, the line of reflection does not ascend beyond the fourth [part] of the diameter'.[76] Moreover, Della Porta's demonstration was based on a thorough understanding of the mathematics involved. He demonstrated that, when BC is the side of a hexagon inscribed in the circle of which the mirror is a part, the ray AB is reflected to the vertex C of the mirror.[77] (Figure 6.20)

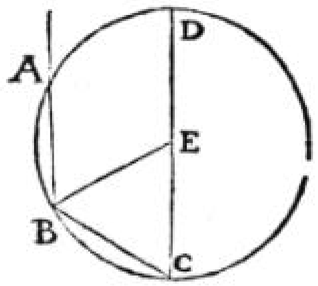

Figure 6.20

Solar rays that are farther away from the axis of the mirror than this 'hexagonal ray' intersect the axis of the mirror outside the mirror. He also showed that, when BF is the side of an octagon inscribed in the circle of which the mirror is a part, the ray AB is reflected perpendicular to the axis of the mirror, which it intersects in E.[78] (Figure 6.21) Finally, he showed that the closer the rays are to the axis of the mirror, the closer to the point at half the radius of curvature, their reflections intersect the axis.

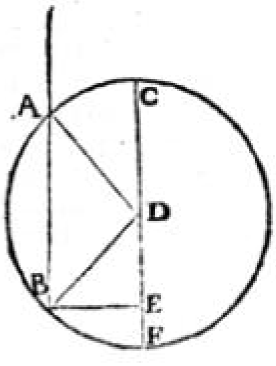

Figure 6.21

[76] 'In reflexionibus speculorum concavorum reflexionis linea non ascendet ultra quartam diametri'. Porta, *De refractione optices parte libri novem*, p. 39.

[77] Ibid., pp. 36-7. For discussion, see Lindberg, 'Optics in Sixteenth Century Italy', pp. 142-3.

[78] Porta, *De refractione*, pp. 37-8.

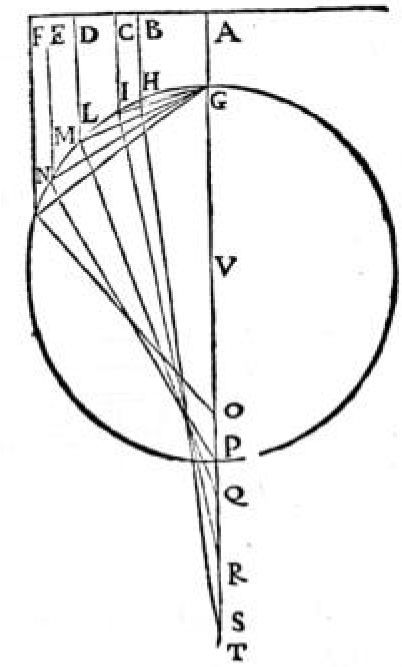

Figure 6.22

Next, Della Porta applied his analysis of reflection in a concave spherical mirror to refraction of solar rays in a refractive sphere. This transfer was problematic. In his diagram, parallel solar rays are refracted in a sphere.[79] (Figure 6.22) GM is the side of a hexagon inscribed in the circle of the sphere. Consequently, the ray EM is equivalent to the 'hexagonal ray' in a concave spherical mirror. Following the analogy between concave mirrors and refractive spheres, Della Porta argued that the ray EM is refracted to the 'vertex' P. Next he argued that rays closer to the axis of the sphere than the hexagonal ray intersect the axis in different points, not on the side of the refractive sphere, as in the case of concave mirrors, but outside the sphere. He also held the converse to be true, that is, that rays farther from the axis than the hexagonal ray intersect the axis of the sphere inside the refractive sphere. Della Porta did not give any justification whatsoever for these claims, beside their analogy with reflection in a concave mirror. From his knowledge of refraction, he should have realized that whether the ray EM intersects the axis in P is dependent upon the optical density of the sphere. The ray EM is refracted differently in refractive spheres of different optical density. It will not necessarily be refracted to the 'vertex' P of the sphere.

As concerns image formation in refractive spheres, it has already been noted that Della Porta was in trouble with the cathetus rule (and likewise in trouble when he tried to apply it to lenses in his 'De Telescopio'). Proposition 16 of book II of his 'De refractione' dealt with the point of inversion.[80] In Della Porta's diagram, BCED is a refractive sphere, GF the object and the eye A. (Figure 6.23) He argued that GE is refracted through C to the eye A, and that FL is refracted through D to the eye A. Della Porta considered the lines CM and BM through the center M to be the catheti. He argued that the object GF is seen inverted, since it is located outside the point of

[79] Ibid., pp. 41-3. For discussion, see Lindberg, 'Optics in Sixteenth Century Italy', pp. 143-4.

[80] Porta, *De refractione*, pp. 56-8.

inversion L. However, he did not provide any justification whatsoever for the location of this point of inversion. Propositions 17 to 20 provide variations on this theme, showing for example that when located inside the point of inversion, an object is seen right oriented, but these propositions contribute nothing fundamentally to the discussion.[81] Consequently, Della Porta's analysis of image formation in a refractive sphere is equivalent to Witelo's discussion of image formation in a concave mirror as concerns the point of inversion. Proposition 22 shows Porta's attemp at an analysis of a burning sphere, but this analysis was not related to his discussion of image formation and the point of inversion in the preceding theorems.[82]

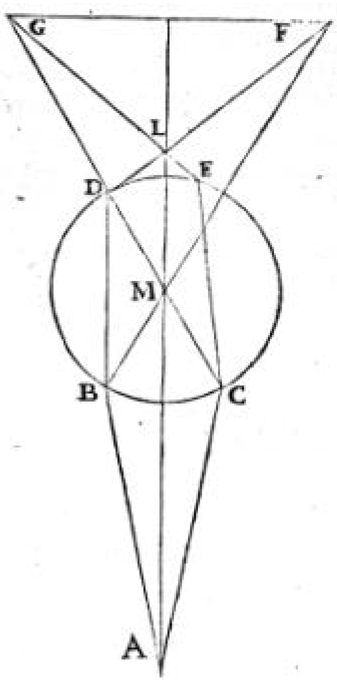

Figure 6.23

Della Porta discussed image formation in lenses in book VIII of his 'De refractione'. In the second proposition of this book, Della Porta applied his knowledge of reflection in concave mirrors to refraction in lenses.[83] Just as Ausonio had identified the point of inversion with the focal point in his 'Theorica' and Della Porta himself identified both points in his 'Natural Magick', he argued that parallel solar rays converge in the point of inversion [punctum inversionis]. His diagram is again fraud with difficulties, as it is based on the analogy of hexagonal rays reflected in a concave mirror to the vertex of the mirror, but while missing from his analysis of refracting spheres, the focal point of a convex lens is considered its point of inversion.[84] (Figure 6.24)

[81] Ibid., pp. 58-62.

[82] Ibid., pp. 63-4.

[83] Ibid., pp. 175-6.

[84] 'Puncta concursus secundo libro prop. 3. determinauimus in crystallina pila, & punctus inuersionis non alius, quàm concursus radiorum ex oppositis in circulo partibus, vti ex sinistra in dextram, aut ex dextra in sinistram, & sursum, deorsumq;'. Ibid., p. 176.

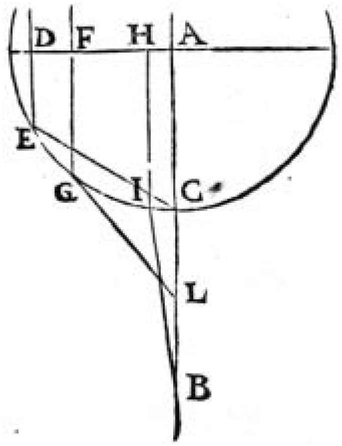

Figure 6.24

Moreover, proposition 3 and 4 identify the point of inversion with the point of confusion. In proposition 3, Della Porta argued that when the eye is placed in the point of inversion, the lens surface is filled completely by the image.[85] Proposition 4 placed the object instead of the eye at the point of inversion. It stated that, in this case, again, the image of the object fills the lens completely.[86]

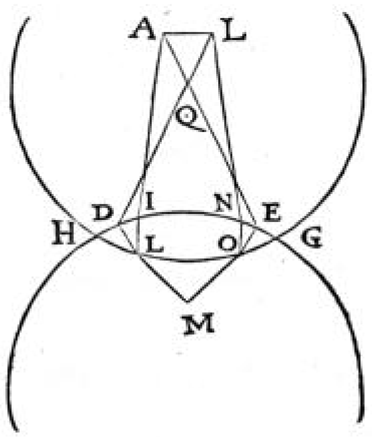

Figure 6.25

However, Della Porta's geometrical 'proofs' of these propositions are not satisfactory. As concerns proposition 3, the eye is at the point of inversion M. (Figure 6.25) The object AL is seen by this eye through the convex lens LOIN. The ray AI is refracted through L to the eye at M. Likewise, LN is refracted through O to M. Della Porta argued that the image of the object AL is seen at the back surface of the lens DE, constructed by drawing the catheti LQ and AQ through

[85] 'In convexis specillis oculo in puncto inuersionis magnitudinis, imagine specillumcomplebitur'. Ibid., p. 176.

[86] 'In convexis specillis magnitudo posita in puncto inuersionis, eius imagine totum specillum occupabitur'. Ibid., p. 177.

the center of one of he spheres from which the convex lens is made up. Consequently, the image DE fills the complete surface of the lens LOIN. It is not clear why Della Porta selected the rays AI and LN among the possible trajectories of rays coming toward the lens. There is no justification. The remainder of the propositions argued in more detail, considering several cases, that an object between the point of inversion and the lens is seen right oriented, while an object beyond the point of inversion is seen inverted. However, the proofs of these propositions are along the same problematic lines. Thus, Della Porta recognized that Ausonio's analysis of image formation in a concave spherical mirror was equally applicable to image formation in convex lenses. Consequently, Della Porta introduced the point of inversion-focal point-point of confusion for convex lenses. His proofs however only look like geometry. In fact, they are, as Ausonio's 'Theorica', the direct representation of practice with concave mirrors and convex lenses.

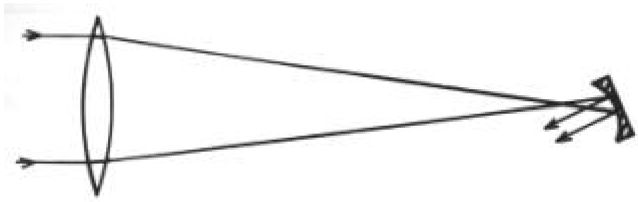

Figure 6.26

How was this point of inversion used in the invention of the reflecting telescope? The reflecting telescope that Ronan and Rienitz have considered to be invented by Digges and Bourne consists of a convex lens of great diameter and of a large focal length as objective and a concave mirror, likewise of a great diameter.[87] (Figure 6.26) The lens is fixed in a frame that is set in the aperture made in the window of a room and targetted at some distant objects. The concave mirror is hold more or less at the focal point of the convex lens and relatively inclined with respect to the lens. The observer, looking in the mirror with his back toward the lens, will have a magnified, wide-angle view of the distant objects that fills the whole surface of the concave mirror. Ronan and Rienitz have both constructed such a telescopic device according to the specifications given by Digges and Bourne. Although the device works well, there have been raised some doubts concerning the technical capacities of contemporaneous lens-makers to make convex lenses of the large focal lenghts and large diameters that are needed for this reflecting telescope.[88] Convex lenses to correct presbyopia, readily available in the shops of spectacle-makers, had focal lengths not exceeding 50 centimeters or 2 diopters and must have had small diameters.[89] However, Bourne mentioned that the convex lens for a reflecting telescope 'must bee made very large, of a foote, or 14. or 16. inches broade, and the broader the better'.[90] Thus, the convex lens suggested

[87] See, in particular, Ronan, 'There Was an Elizabethan Telescope', p. 3; Rienitz, *Historisch-Physikalischen Entwicklungslinien Optischer Instrumente: Von der Magie zur Partiellen Kohärenz*, pp. 110-3.

[88] See, in particular, from a technological point of view, Turner, Gerard L'E. 'There Was No Elizabethan Telescope.' *Bulletin of the Scientific Instrument Society* 37 (1993): 3-5, p. 5.

[89] Van Helden, *The Invention of the Telescope*, p. 11.

[90] William Bourne, *Inventions or devices. Very necessary for all generalles and captaines, or leaders of men, as well by sea as by land*, reproduced in Van Helden, *The Invention of the Telescope*, p. 30.

by Bourne had a diameter of 30-40 centimeters. Bourne confirmed the large diameters of his convex lens in another report, while he implicitely specified the large focal length of his lens.

> And now allso in lyke manner for to make a glasse for perspective, for to beholde, and see any thinge, that ys of greate distance from yow, which ys very necessary: for to viewe an army of men, or any castle, or forte, or such other lyke causes. Then they must prepare very cleare, and white Glasse that may bee rounde, and beare a foote in diameter; as fyne and white Venysse Glasse. And the larger, the better: and allso yt must bee of a good thickness, and then yt must bee grounde uppon a toole fitt for the purpose. Beynge set fyrst uppon a syman block, and firste grinde on the one syde, and then on ye other syde, untill that the sydes bee very thynn, and the middle thicke. And for that yf the glasse bee very thicke, then yt will hynder the sighte. Therefore yt must bee not above a quarter of an ynche in thickness: and the sydes or edges very thynne, and so polysshed or cleared. And so sette in a frame meete for the purpose for use.[91]

It has been calculated that a lens with a diameter of a foot and a central thickness of a fourth of an inch has a focal length of about 5,5 meters.[92] Such a lens was not to be found in the regular shop of a contemporaneous spectacle-maker. However, it was very well possible to make such lenses for specific purposes. That the lens was fixed in a frame suggests that the origin of this instrument was a camera obscura with a convex lens applied in its aperture. As has been noted for a camera obscura, it made good sense to make a convex lens of a large diameter, because it allowed for the brightness of the image on the screen. Contemporaneous reports of the practice with camera obscura's confirm that, in this context, convex lenses with large diameters and large focal lengths were used. In his 'Paralipomena', Kepler mentioned having seen a camera obscura 'at Dresden in the Elector's theater of artifices'.[93] Of this instrument, Kepler wrote that 'a disk thicker in the middle, or a crystalline lens, a foote in diameter, was standing at the entrance of a closed chamber against a little window, which was the only thing that was open'.[94] Consequently, this camera obscura was equipped with a convex lens of such a large diameter. Moreover, in his 'Dissertatio cum Nuncio Sidereo' (1610), Kepler mentioned to have made observations of the moon by projecting the image of the moon on a screen placed 12 foot from a convex lens 'of a very large circumference'.[95] Thus, Kepler's convex lens was not only very large, but also must have had a focal length of about 3,60 metres. Consequently, Bourne's report gains credibility by observing that contemporaneous practitioners, like Kepler, working with a camera obscura-cum-lens, used similar convex lenses of a large diameter and a large focal length.

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[91] William Bourne, *A treatise on the properties and qualities of glasses for optical purposes, according to the making, polishing, and grinding of them*, reproduced in Van Helden, *The Invention of the Telescope*, p. 33.

[92] Von Rohr, 'Aus der Geschichte der Brille', p. 41.

[93] Kepler, Paralipomena, p. 181. Translation in Kepler, *Optics*, p. 194.

[94] Kepler, Paralipomena, p. 181. Translation in Kepler, *Optics*, p. 194.

[95] 'Id autem hoc pacto me spero perfecturum mea observandi ratione, vultu a Luna averso; si lunae lumen per foramen in tabellam pertica circumlatam intromisero, sic tamen, ut foramen obvallet lens crystallina, sphaerico maximi circuli gibo, et tabella ad locum collectionis radiorum accommodetur. Sic in pertica 12 pedes longa, lunae corpus perfectissime depingetur quantitate monetae argenteae maioris'. Kepler, Dissertatio cum Nuncio Sidereo, in Galileo, *Opere*, Vol. 3.1, p. 112. It is wrongly translated in Rosen, Edward. *Kepler's Conversation with Galileo's Sidereal Messenger.* Translated by Edward Rosen. Vol. 5, *The Sources of Science*. New York London: Johnson Reprint Corporation, 1965. There is also a French translation, Kepler, Johannes. *Dissertatio cum Nuncio Sidereo - Discussion avec le Messager Céleste; Narratio de Observatis Jovis Satellitibus - Rapport sur l' Observation des Satellites de Jupiter.* Translated by Isabelle Pantin, *Science et Humanisme*. Paris: Les Belles Lettres, 1993.

Beside Digges and Bourne in the 1570s in England, Della Porta and Sarpi were working on a reflecting telescope in Italy in the 1580s. In 1586, Della Porta wrote to his patron Cardinal d' Este that he was making '*occhiali* that make it possible to represent a man some miles away'.[96] The way that Della Porta described this telescopic device is reminiscent of Caccio's reference in his letter to Ausonio to use 'occhiali' to see distant cities. While I have suggested that Caccio's metaphor was based on an actual catoptrical practice in a painterly context, 'occhiali' as such conventially referred to lenses. What Della Porta might have meant was that he was trying to make a convex lens with a large focal length. Convex lenses with such large focal lengths (and, likewise, concave mirrors with large focal distances) yield a small telescopic effect, even if no second optical component is used.[97] However, a contemporaneous note of Sarpi, who, as has been noted, was in contact with Della Porta, shows that they might have rather been thinking of a combination of a lens and a mirror than of a magnifier made of one optical component.

> One or more mirrors can be positioned so that one sees a man as much outside [the mirror]. The same can de done with lenses. To make letters readable 50 passes far: I have tried it with a spherical [mirror?], and/or the lens, but better with the parabolic [mirror?] or/and its lens, and read them while the light is faraway.[98]

Sarpi's note is fraud with difficulties of interpretation regarding the actual composition of this instrument. Not the least of the difficulties is that the passage has been read as 'a lens or a mirror' as well as 'a lens and a mirror'.[99] Obviously, only the latter reading might refer to a reflection telescope with a convex lens and a concave spherical or parabolic mirror. Moreover, the ending of the note that the 'light is faraway' suggests rather some kind of light-projecting device than a telescope, just as in Ausonio's 'Theorica'. However, in a letter of 6 January 1609 to Groslot de l' Isle, after the refracting telescope had appeared in Venice, Sarpi interpreted his earlier efforts with a parabolic lens as relating to the making of some kind of reflecting telescope.

> The news of the new *occhiali* that I saw already more than a month ago, and I believed it as much that it was sufficient not to search further, nor to philosophize on it, because Socrates prohibits to philosophize on experience that one has not seen himself. When I was young, I thought about such a thing, and it crossed my mind that a lens [occhial] made of a parabolic shape might cause such an effect; and I had reason to make the demo nstration. But because these are abstract things, and they bring not into account the resistance of the materials, I felt some opposition. Therefore, I was not much inclined to the work, because it would have been tiring. Henceforward, I did confirm nor tried my thought with experience.[100]

[96] 'occhiali che possini raffigurare un huomo alcune miglia lontano'. Quoted from Gliozzi, 'Relazioni Scientifiche tra Paolo Sarpi e GB Porta', p. 419, my translation.

[97] Rienitz, ''Make Glasses to See the Moon Large'', p. 7; Rienitz, *Historisch-Physikalischen Entwicklungslinien Optischer Instrumente: Von der Magie zur Partiellen Kohärenz*, pp. 94-8. However, if noted accidentally, this did not entail, as will become evident, that they understood the relationship between magnification and focal length.

[98] 'Uno o molti specchi si posson talmente accomodare che vegga l' uomo quanto di fuori si fa, lo stesso con occhiali avenendo. Far legger lettere di lontano 50 passi: l' ho io provato collo sferale, o/e con la lente, ma meglio colla parabola o sua lente, e leggerle stando lontano il lume'. Sarpi, *Pensieri Naturali, Metafisici e Matematici*, p. 436.

[99] Compare Ibid., p. 436; Gliozzi, 'Relazioni Scientifiche tra Paolo Sarpi e GB Porta', p. 415.

[100] 'L' avviso delli nuovi occhiali l' ho veduto già più d' un mese, e lo credo per quanto basta a non cercar più oltre, non per filosofarci sopra, proibendo Socrate il filosofare sopra esperienza non veduta da sè proprio. Quando io ero giovane, pansai ad una tal cosa, e mi passò per la mente che un occhial fatto di figura di parabola potesse far tale effetto; e avevo ragione di farne la dimostrazione. Ma perchè queste son cose astratte, e non mettono in conto la

From this letter, it is evident that Sarpi did not try to make a telescopic device. He considered the use of a parabolic mirror only a thought experiment, which could not be brought in to practice, most likely because Sarpi considered a mirror of an exact parabolic shape that focuses the reflected rays in one point, out of the range of technical capacities of contemporary mirror-makers. Moreover, nothing in this passage suggests that Sarpi thought of combining this mirror with a convex lens to obtain a design of the kind proposed by Digges and Bourne. However, Della Porta appears to have considered a combination of a concave mirror and a lens, and Sarpi was considered by his contempories to be the only one to be able to understand Della Porta's obscure description of such design in his 'Natural Magick'. On 4 October 1624, Bartolomeo Imperiali asked for Galileo's help in understanding this obscure passage of Della Porta.

> It is my desire that Your Lordship applies his thought to chapter 11 of book 17 of the Magia of Gio. Batta della Porta, a passage of which Kepler confessed to Your Lordship that he did not understand it, nor do I know of any mathematician able to explain it; as I know that Magino has confessed that also Porta, no matter the high esteem in which he was held by princes and men of letters, was never able to explain his mind. He just said that Master Paolo of Venice, the Servite friar, had understood it.[101]

The passage in Della Porta's 'Natural Magick' of which Sarpi was claimed to have been the exception who understood it, is on 'spectacles whereby one may see very far, beyond imagination'.[102]

> I will not omit a thing admirable and exceeding useful; how bleare-ey'd people may see very far, and beyond that one would believe. I spake of Ptolemies Glass, or rather spectacle, whereby for six hundred miles he saw the enemies ships coming; and I shall attempt to shew how that might be done, that we may know our friends some miles off, and read the smalles letters at a great distance, which can hardly be seen. A thing needful for mans use, and grounded upon the Opticks. And this may be done very easily; but the matter is not so to be published too easily; yet perspective will make it clear. Let the strongest sight be in the Centre of the Glass, where it shall be made, and all the Sun beams are most powerfully disperst, and unite not, but in the Centre of the foresaid Glass: in the middle of it, where diameters cross one the other, there is the concourse of them all. Thus is a Concave pillar-Glass made with sides equidistant: but let it be fitted by those Sections to the side with one oblique Angle: but obtuse Angled Triangles, or right Angled Triangles must be cut here and there with cross lines, drawn from the Centre, and so will the spectacle be made.[103]

Della Porta appears to suggest the use of a concave parabolic or hyperbolic mirror, since it is cut from an obtuse angled or right-angled triangle, according to the prevailing contemporaneous knowledge of conic sections, with which Della Porta, as seen in chapter 2, was familiar.

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repugnanza della materia, sentivo qualche opposizione. Per questo non sono molto inchinato all' opera, e questa sarebbe stata faticosa: onde nè confermai nè riprovai il pensier mio con l' esperienza'. Quoted from Ibid., p. 416.

[101] 'è il desiderio che V. S. applichi il pensiero al capitolo 11° del libro 17° della Magia di Gio. Batta della Porta, passo di cui confessa a V. S. il Ceplero che non l' intende, nè ho io saputo già mai che matematico alcuno l' abbia saputo dichiarare; come so che l' istesso Magino ha confessato, nè il Porta, per quanta istanza li sia stata fatta da prencipi e letterati, si è potuto già mai inchinar a dichiarar l' animo suo; solo che disse che Mastro Paolo da Venetia, Servita, l' aveva capito'. Galileo, *Opere*, Vol. 13, p. 212.

[102] Porta, *Natural Magick*, p. 369.

[103] Ibid., p. 369.

Moreover, he appears to suggest the combination of this mirror with a lens with which people with dim sight are able to see far. This appears to refer to a concave lens to correct myopia rather than a convex lens. In the next section, it will become clear that such a combination, in the minds of sixteenth century optical practitioners, might have made sense. However, it should also be noted that Della Porta did not seem to suggest the Elizabethan design of a reflecting telescope.

Thus, it can hardly be denied that Della Porta and Sarpi were searching for a telescopic device in the 1580s. Moreover, it also appears that to obtain such a telescopic device they were looking for a concave mirror. They might have thought to combine this mirror with a lens, but there is no evidence that they thought it to be a convex lens. If they were looking for a combination of the concave mirror with a lens, they appear to have rather thought of a concave lens. However, given the obscurity of their descriptions, it can, on the other hand, not be excluded that they tried such a combination of convex lens and concave mirror. What's more important is that their search for a telescopic device appears to have been based on the notion of a point of inversion. In their search for a telescope, they appear to have started from the observation that when the eye is located in the point of inversion of a convex lens or a concave mirror, a magnified image fills the whole surface of the convex lens or concave mirror. This is why a concave mirror or a convex lens as such, without a second optical component, appears to have been considered to be possibly telescopic. However, Sarpi and Della Porta must also have noted that this magnified image is much blurred. This presumably inspired the search for a parabolic instead of a spherical concave mirror, which by making the rays converge in one point was also thought to allow for clearer sight. This conjecture as concerns Della Porta's and Sarpi's procedures might seem to stretch the evidence from their obscure passages a little too far. Fortunately, Bourne left a much clearer account of his successful procedure of making a reflection telescope that allows confirming that the point of inversion was at the basis of their attempts to make a telescopic device.

Leonard Digges (ca. 1520- ca. 1559) was a mathematical practitioner, mostly active in the fields of surveying, navigation, military engineering, mathematical instrument design and astronomy.[104] He was a close friend of John Dee. Thus, when he died, he entrusted his son Thomas to Dee to teach him mathematics. Thomas Digges (ca. 1546-1595) continued the practical mathematical bent of his father and his teacher Dee. He was mostly active in surveying, astronomy, military engineering and ballistics. Most of Leonard's work had to wait for publication by his son Thomas. In these editions it is often impossible to discriminate between the contributions of father and son. However, in Thomas' edition of his father's 'Pantometria' (1571), no matter the changes Thomas might have made, he attributed the invention of a telescopic device to his father.

> My father by his continual painfull practises, assisted with demonstrations Mathematicall, was able, and sundrie times hath by proportionall Glasses duely situate in convenient angles, not onely discovered things farre off, read letters, numbred peeces of money with the very coyne and superscription thereof, cast by some of his freends of purpose uppon Downes in open fieldes, but also seven myles of declared wat hath beene doon at that instante in private places.[105]

This would place the invention of the reflecting telescope before the death of Leonard Digges in 1559. However, since there is no corraborating evidence that links the invention to the father, it might very well be possible that the invention was made at a later date, but most likely before

[104] For the biography of Leonard and Thomas Digges, see Ronan, 'Leonard and Thomas Digges', pp. 91-2.

[105] Digges, *A Geometrical Practise, Named Pantometria*, f. Aiiiv.

1571, by the son, who might have taken more obscure references of his father to be about a device he himself invented. There is however no doubt that the invention occurred in the circle of John Dee. A later report of another mathematical practitioner, William Bourne, of circa 1585, with the title 'A treatise on the properties and qualities of glasses for optical purposes, according to the making, polishing and grinding of them', is to be considered as about the invention made in the circle of Dee and Digges, made presumably in the 1560s, according to Bourne's own words.

> For that there ys dyvers in this Lande, that can say and do the knowe much more, in these causes, then I: and especially Mr. Dee, and allso Mr. Thomas Digges, for that by theyre Learninge, they have reade and seene moo auctors in those causes: And allso, theyre ability ys suche, that they may the better mayntayne the charges: And also they have more leysure and better tyme to practyze those matters.[106]

As will become evident, Bourne's explanation of the reflecting telescope in the circle of Dee and Digges was highly based on the point of inversion. It is not unlikely that knowledge of the point of inversion in concave mirrors was transmitted from Ausonio to Dee. As discussed in chapter 2, Dee was in Venice in 1563, and, at that occasion, he might have met Ausonio. Consequently, the the procedure of designing a reflecting telescope, based on Ausonio's point of inversion, as described by Bourne around 1585, is most likely a representation of the actual procedure followed by Digges to design the reflecting telescope, described in Digges' 'Pantometria' (1571).
Bourne discussed the properties of concave mirrors and convex lenses. In chapter 4 of his 'Treatise on the properties and qualities of glasses for optical purposes', he discussed 'in what maner of forme to make Lookinge Glasses, to make any thinge shewe bigger than yt ys'.[107] Bourne noted that when the eye is placed 'at some appoynted distance' the image fills the whole surface of the mirror. As is known from the discussion of Ausonio's 'Theorica speculi concave sphaerici', this is the point of inversion of the concave spherical mirror.

> To make lookinge Glasses for to shewe any thinge bigger than yt ys, That Glasse must bee made very large, for elles yt will not conteyne any quantitye in sighe; and this glasse must bee Concave inwardes, and well pollyshed of the hollowe or concave syde: and then the foylle must be layde on that syde that doth swell, as a hyll, and bosse outwarde. And then this glasse, the property of yt ys, to make all thinges which are seen in yt to seem muche bigger then yt ys to the syghte of the Eye, and at some appoynted distance, from the glasse, accordinge to the forme of the hollowness, the thinge will seem at the biggest, and so yow standinge nearer the thinge will seeme less, unto the sighte of the eye: so that, accordinge unto the forme of the concavity or hollowness, and at some appointed distance from hym that looketh into the glasse, And yf that the glasse were a yearde broade, the beame that shoulde come unto his eye, shall showe his face as broade, as the whole Glasse.[108]

While the exact place of this point, where the image fills the whole surface of the mirror, might seem not to be well defined in Bourne's description of the concave mirror, as he not explicitly stated that this distance is close to the point of inversion of the mirror, his description of this distance is much more precise in the case of convex lenses, called 'perspective glasses'.

[106] William Bourne, *A treatise on the properties and qualities of glasses for optical purposes, according to the making, polishing, and grinding of them*, reproduced in Van Helden, *The Invention of the Telescope*, p. 34.

[107] Ibid., p. 32.

[108] Ibid., p. 32.

> And the propertie of this glasse, is this, if that you doo behold any thing thorow the glasse, then your eye being neare unto it, it sheweth it selfe according unto the thing, but as you doo goe backwardes, the thing sheweth bigger and bigger, untill that the thing shall seeme of a monstrous bignesse: but if that you doo goe to farre backe, then it will debate and be smal, & turne the fashion downewards.[109]

Moreover, this point of inversion of a convex lens, where things become smaller and inverted, was located at the focal point of the mirror, called the 'burninge beame' by Bourne.

> And yf that yow doo beholde any thinge thorowe this Glasse, and sette the glasse furder from yowe then the burning beame, and so extendinge after that what distance that yow list, all suche thinges, that yow doo see or beholde, thoroughe the glasse, the toppes ys turned downwardes. Whether that yt bee trees, hilles, shippes on the water, or any other thinge whatsoever that yt be: As yf that yt were people, yow shall see them thoroughe the Glasse, theyre heades downwardes, and theyre feete upwardes, theire righte hande turned to theyre lefte hande, &c. [110]

Bourne made a distinction between the 'burninge beame' and the 'perspective beam'. He was particularly interested in this 'perspective beame' and its location with respect to the focal point.

> The quality of this Glass, ys, if that the sunne beames do pearce through yt, at a certayne quantity of distance, and that yt will burne any thinge, that ys apte for to take fyer: And this burnynge beame, ys somewhat furder from the glasse, then the perspective beame.[111]

Why was Bourne interested in the 'perspective beame'? It locates the point where the eye is to be placed to get as large an image as possible, filling the whole diameter of the lens, before the image completely collapses, which he considered the focal point and the point of inversion.

> That (beholdinge any thing thorowe shall seeme very large and greate: and more perfitter withall. And allso standing further from the glasse yow shall discerne nothing thorowe the glasse: But like a myst, or water: And at that distance ys the burninge beame, when that yow do holde yt so that the sunne beames doth pearce thorowe yt. And allso yf that yow do stande further from the glasse, and beholde any thinge thorowe the glasse, The you shall see yt reversed and turned the contrary way, as before ys declared.[112]

Again, Bourne emphasized that of the eye is placed at the location of this 'perspective beame', the image will be at its largest, filling the whole surface of the convex lens.

> The quality of the Glasse, (that ys made as before ys rehearsed) ys, that in the beholding any thinge thorowe the glasse, yow standinge neare unto the Glasse, yt will seeme thorow the glasse to bee but little bigger, then the proportions ys of yt: But as yow do stande further, and further from yt, so shall the perspective beame, that commeth through ye glasse, make the thinge to seeme bigger and bigger, untill such tyme, that the

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[109] William Bourne, *Inventions or devices*, reproduced in Van Helden, *The Invention of the Telescope*, p. 32.

[110] William Bourne, *A treatise on the properties and qualities of glasses for optical purposes, according to the making, polishing, and grinding of them*, reproduced in Van Helden, *The Invention of the Telescope*, p. 33.

[111] Ibid., p. 33.

[112] Ibid., p. 33.

> thinge shall seeme of a marvellous bignes: Whereby that these sortes of glasses shall much proffet them, that desyer to beholde those things that ys of great distance from them.[113]

Given the magnifying properties of concave mirrors and convex lenses, Bourne must have thought the effect to be additive if a concave mirror and a convex lens are combined. The location of the point of inversion, or the 'perspective beame', indicated the distance between the convex lens and the concave mirror, as the distance between them must be so chosen that the images are largest. Thus, the distance was determined by the focal points of mirror and lens.

> Wherefore yt ys to be supposed, and allso, I am of that opinyon, that having dyvers, and ondry sortes of these concave lookinge glasses, made of great largeness, That suche the beame, or forme and facyon of any thinge beeyinge of greate distance, from the place, and so reseaved into another of these concave glasses: and so reseaved from one glasse into another, beeyinge so placed at such a distance, that every glasse dothe make his largest beame. And so yt ys possible that yt may bee helpped and furdered the one glass with the other, as the concave lookinge glasse with the other grounde and polysshed glasse. That yt ys likely yt ys true to see a small thinge, of very greate distance.[114]

Rienitz has suggested that Digges and Bourne chose a concave mirror, because of the relativily wider field of view that can be obtained with it.[115] However, this explanation of Digges' and Bourne's choice for a concave mirror is a bit wanting. There were no theoretical means available to them that suggest this wide field of view obtained by a concave mirror. First, if a wider field of view with respect to naked eye observation is suggested, the choice would have been rather for a convex mirror that, as for example is evident from the scheme with the mirror on top of the tower in one of the already discussed letters to Ausonio, was associated with a relatively wider field of view with respect to the naked eye and a plane mirror. Second, if a relatively wider field of view with respect to the 'Galilean' design of a telescope is suggested, the evidence that Digges and Bourne considered such a combination of a convex lens with a concave lens is missing. With no other evidence available than Digges' description in the 'Pantometria' and Bourne's report, it is more likely that Digges and Bourne did not know of such a telescopic combination. Third, if a relatively wider field of view with respect to a single convex lens is suggested, then there is no apparently good reason why Digges and Bourne should have arrived at this particular combination of a convex lens as objective and a concave mirror as ocular of a telescope.

The use of this particular telescopic combination with a concave mirror is better explained by considering the origin of the reflecting telescope in the use of camera obscura in the context of surveying and map-making that, as has been shown, presented distant city views. The starting point of the reflecting telescope was most likely the camera obscura-cum-lens. This origin is also suggested by Bourne's recommendation to 'set that glasse fast'.[116] As has been shown, concave mirrors were used in combination with a camera obscura without a lens to project an image that has the same left-right orientation as the scene outside. This might have suggested the use of a concave mirror to change the orientation of the image projected by a camera obscura with a

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[113] Ibid., p. 33.

[114] Ibid., p. 34.

[115] Rienitz, *Historisch-Physikalischen Entwicklungslinien Optischer Instrumente*, p. 113.

[116] William Bourne, *Inventions or devices*, reproduced in Van Helden, *The Invention of the Telescope*, p. 30.

convex lens. If the reflecting telescope is used by looking in the mirror with the back to the convex lens, as has been suggested, the image is inverted. However, when the mirror is hold under an angle before the chest of the observer and by bending the head downwards, an apparently upright image is obtained, as Rienitz himself has noted.[117] Consequently, the origin of the reflecting telescope in the camera obscura practice, including the experience with the problem of the orientation of the image, explains why, beside that the magnification of convex lens and concave mirror was considered additive by Digges and Bourne, this particular combination of convex lens as objective and concave mirror as ocular was chosen by Digges and Bourne.

Thus, the notion of point of inversion-focal point-point of confusion goes a long way in explaining the specific properties of the reflecting telescope as designed by Digges and Bourne. First, it explains why a convex lens and a concave mirror were used, since these were the two kinds of optical instruments for which the point of inversion was developed. Second, it explains why Bourne thought that the convex lens and the concave mirror needed to have a large diameter. If the eye is placed at the point of inversion, or shortly closer to the lens or mirror, the magnified image fills the complete surface of the lens. It is then reasonable to consider magnification dependent upon the diameter of the lens and the mirror. Third, the locus of the point of inversion, and thus the focal point, also gave indications about the distance between the convex lens and the concave mirror. Thus, since the invention of the reflecting telescope depends upon a notion of point of inversion, and this notion was only developed for concave spherical mirrors around 1560, and most likely somewhat later for convex lenses, the absence of the notion of point of inversion before the second half of the sixteenth century explains why a telescopic device was not developed prior to this period, although convex lenses and concave mirrors were available.

Henceforward, the notion of point of inversion set limits to sixteenth century optical practice eventually resulting in a telescopic practice around 1570. This is not to say that the notion of point of inversion gave a complete blueprint of a well-working reflecting telescope. Trial and error adjustements to obtain the best possible telescope were still needed. In this context, one of the most vexing problems was presumably magnification. It is unclear whether Digges and Bourne considered magnification to be dependent upon focal length. From Bourne's use of the point of inversion, it is only to be derived that these Elizabethan telescope-makers considered magnification to be dependent upon the diameter of the convex lens. However, the technology of glass-making presumably set limits upon the possibilities as concerns magnification. On the one hand, since the point of inversion suggests magnification to be dependent upon the diameter of the lens, it would have made sense to make larger and larger lenses to obtain higher and higher magnification. On the other hand, Bourne himself indicated, as has been shown, that the quality of contemporaneous glass dictated that the central thickness of the lens must be restricted to the minimum, 'for yf the glasse bee very thicke, then yt will hynder sight'.[118] Combining these two precepts in to one precept of making a convex lens of a diameter as large as possible and a central thickness as thin as possible, results in making a lens with a focal length as long as possible.

There are no indications that the Elizabethan reflecting telescope shaked the world. This is rather awkward, because it appears to be the fulfillment of the telescopic dream, so widely dispersed and cherished for such a long time. The incomplete control of magnification, not considered to be dependent upon focal length, but on the diameter of the lens, together with the technological

[117] Rienitz, ''Make Glasses to See the Moon Large'', pp. 8-9.

[118] William Bourne, *A treatise on the properties and qualities of glasses for optical purposes, according to the making, polishing, and grinding of them*, reproduced in Van Helden, *The Invention of the Telescope*, p. 33.

problems associated with making lenses of larger diameters, must be one of the main causes of the silence outside the circle of Digges and Bourne as concerns the apparent unpopularity of this design of reflecting telescope. Of course, the Elizabethan design of reflecting telescope was also of very low practicality, since it is difficult to handle and not enclosed in a tube. The refracting telescope with a convex lens as objective and a concave lens as ocular, invented somewhat later, and most likely not much earlier than 1608, was, as it was made of spectacle lenses of much smaller diameters, and enclosed within a tube, of much higher practicality to handle. As the reflecting telescope, its invention could benefit from the notion of point of inversion.

4. Workshop Practices, Galileo's Telescope and Magnification

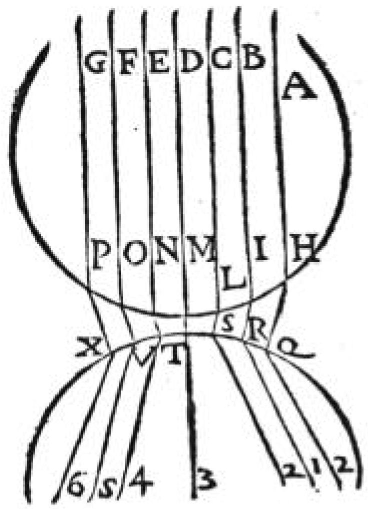

Figure 6.27

The properties of concave lenses were very little understood at the end of the sixteenth century. Exemplary, only six propositions of Della Porta's 'De Refractione' discussed concave lenses. Propositions 15 to 17 of book VIII of Della Porta's 'De Refractione' showed that the image in a concave lenses is always, no matter the distance of the eye from the lens, smaller than the object.[119] In proposition 18, Della Porta argued that a concave lens does not kindle fire.[120] (Figure 6.27) Consequently, a concave lens was understood to have no focal point. Finally, proposition 19 discussed that 'those with weak vision see sharper with concave lenses'.[121] This last proposition, which must have been very well known, since it is obvious to anyone with concave eyeglasses, must have been at the basis of the invention of the refracting telescope. When the eye is placed at the point of inversion, or a little bit closer to the lens, the convex lens shows a magnified image. However, this image is blurred, and, when the eye is progressively farther

[119] Porta, *De Refractione Optices Parte Libri Novem*, pp. 185-7.

[120] Ibid., pp. 187-8. For discussion, see Lindberg, 'Optics in Sixteenth Century Italy', p. 146.

[121] 'Visu debiles concavis specillis acutiùs vident'. Porta, *De Refractione Optices Parte Libri Novem*, p. 188.

removed from the lens, the image is not only progressively more magnified, but also it becomes progressively more blurred until it collapses when the eye is at the point of inversion. This must have given rise to the idea of combining a concave lens with a convex lens in order to sharpen by looking through a concave lens, the blurred magnified image in a convex lens. That Della Porta was thinking of a concave lens to sharpen the blurred magnified image, is evident from the passage in his 'Natural Magick', quoted above, about a telescopic instrument in which he appears to propose to combine a concave lens to sharpen the blurred magnified image, obtained, not in a convex lens, but in a concave mirror which, as has been noted at the beginning of this chapter, was culturally more readily associated with telescopic vision.

Van Helden has shown that by randomly probing all different kinds of combinations of convex and concave lenses the most obvious telescope that results from such a trial and error procedure is the 'Galilean' combination of a convex lens of longer focal length and a concave lens of shorter focal length.[122] My argument is not that it was impossible for Dutch spectacle-makers like Lipperhey, Metius and Janssen to invent a refracting telescope by such a trial and error procedure, but that sixteenth century practice with lenses, as represented in the notion of point of inversion, strongly inclined them to invent a telescope of such a combination of lenses and that familiarity with the point of inversion set limits to the actual practice that allowed them to not randomly try all kinds of combinations of lenses. The model for inventing a 'Galilean' telescope, based on the notion of point of inversion, is then somewhat different from the model proposed by Van Helden. He started from a passage of Della Porta about the combination of a convex and a concave lens.

> Concave Lenticulars will make one see most clearly things that are afar off; but Convexes, things neer hand; so you may use them as your sight requires. With a Concave you shall see small things afar off, very clearly; with a Convex, things neerer to be greater, but more obscurely: if you know how to fit them both together, you shall see both things afar off, and things neer hand, both greater and clearly.[123]

Since no distance between both lenses is specified, Van Helden has interpreted the passage as if Della Porta was starting from a convex and concave lens laying on top of each other near the eye, without any distance in between them.[124] Readily available spectacle lenses must have been used. As already noted, convex lenses had focal lengths not exceeding 50 centimeters. Concave lenses had focal lengths not exceeding 20 to 30 centimeters.[125] Consequently, if spectacle lenses were used, its magnification would not have exceeded 2.5. If the concave lens is closer to the eye than the convex lens, the image will be sharply defined. The erect image will grow in size, but always be sharply defined, when the convex lens is progressively moved away from the concave lens. If a convex lens of 50 centimeters focal length and a concave lens of 20 centimeters focal length are used, the sharp image will be largest, that is 2.5 times magnified, when the convex lens is 30 centimeters from the concave lens. Beyond this point the image will become blurred until it explodes. Moving the convex lens even further results in an inverted image that becomes progressively smaller.

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[122] Van Helden, *The Invention of the Telescope*, pp. 16-20.

[123] Porta, *Natural Magick*, p. 368.

[124] Van Helden, *The Invention of the Telescope*, p. 18.

[125] Ibid., p. 11.

Considering the familiarity of sixteenth century optical practitioners with the point of inversion, they would not have started from a convex lens and a concave lens on top of each other. First, the point of inversion told them to start from a single convex lens, with the eye placed close to the point of inversion, where the result is a maximal magnified but blurred image. Second, since a concave lens was considered to allow sharper vision of things at large distances, they tried a concave lens to make the blurred magnified image sharper. Third, it is reasonable to start by placing the concave lens near the point of inversion, where the image is largest, and to move forwards not backwards, because, as the location of the point of inversion of a convex lens tells, beyond this point the image becomes smaller. By gradually moving forward, the point to place the concave lens to give a most sharply defined and most magnified image would have been quickly found, considering the short focal lengths of contemporary concave lenses. Thus, sixteenth century familiarity with the point of inversion gave good indications, limiting practice, about the distance needed between the convex and concave lens to obtain a telescope.

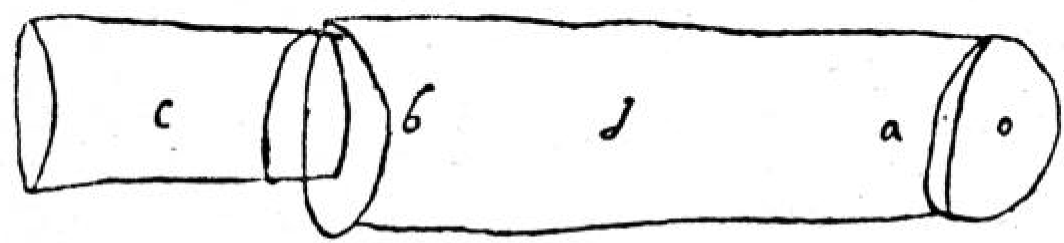

Figure 6.28

In any case, with hindsight, after the refracting telescope invented in the Netherlands, had made its way to Italy, it was interpreted to depend upon the point of inversion. On 28 August 1609, Della Porta made the first drawing of the Dutch telescope and he wrote to Federico Cesi that 'about the secret of the *occhiale*, I have seen it, and it is a studipity, and it is taken of book 9 [sic, book 8] of my *De Refractione*'.[126] (Figure 6.28) Another source, Kepler who cannot be suspected to twist the facts for reasons of priority, made a similar claim in his 'Dissertatio cum Nuncio Sidereo', when he wrote, citing the passage from Della's Porta's 'Natural Magick' quoted above, that 'so powerful a telescope seems an incredible undertaking to many persons, yet it is neither impossible nor new. Nor was it recently produced by the Dutch, but many years ago it was announced by Giovanni Battista della Porta in his 'Natural Magick', Book XVII, Chapter 10'.[127] Galileo never denied the claim, as he must have understood the telescope in the same way, when it came first in to his hands. Galileo's copy of Ausonio's 'Theorica' shows that Galileo was familiar with sixteenth century optical instrumental practice and the point of inversion. Since this notion was considered to be at the basis of telescopic practice, it would have been quite naturally for Galileo to consider the telescope, at his first acquaintance, on the same grounds.

[126] 'Del secreto dell' occhiale l' ho visto, et è una coglionaria, et è presa dal mio libro 9 *De refractione*'. Giambattista Della Porta to Federico Cesi, 28 August 1609, in Galileo, *Opere*, Vol. 10, p. 252.

[127] 'Incredibile multis videtur epichirema tam efficacis perspicilli, at imposibile aut novum nequaquam est; nec super a Belgis prodiit, sed tot iam annis antea proditum a Io: Baptista Porta, Magiae naturalis libro XVII. Cap: X'. Kepler, Dissertation cum Nuncio Sidereo, in Galileo, *Opere*, Vol. 3.1, p. 108, translation in Rosen, *Kepler's Conversation with Galileo's Sidereal Messenger*, p. 15.

It was however left to Galileo to improve the magnification of the telescope. As discussed in chapter 1, Galileo succeeded in making telescopes that magnified twenty to thirty times. If the first telescopes were made of lenses, available in spectacle-makers' shops, the resulting telescope only magnified about 2.5 times. Making telescopes that yielded higher magnifications was not only a technological problem of making convex lenses of longer focal lengths than available in the spectacle-makers' shops. It was also necessary to find out that magnification was depended upon focal length. Van Helden has suggested that Galileo found out that the magnification of a telescope equals the proportion between the focal length of the objective, the convex lens, and the focal length of the ocular, the concave lens.[128] However, as is evident from the practice of the reflecting telescope, finding a way to control magnification was not evident, and problems with understanding magnification most likely were the cause that practitioners, like Della Porta, not recognizing the telescopes of low magnifications as up to their definitions of a telescope, informed by the exaggerations of the telecopic dream, were not able to make telescopes of higher magnifications that would come closer to the standards set by the telescopic dream.

However, it is highly unlikely that Galileo, informed by sixteenth century optics, considered magnification depending upon the proportion of the focal lengths of the convex lens and the concave lens, as Van Helden has suggested. Unlike convex lenses, concave lenses did not make rays converge to cause burning, and, consequently, according to the sixteenth century standards, as evident in Della Porta's 'De refractione', concave lenses were not considered to have a focal point. In the context of sixteenth century optics, it is much more likely that magnification was considered to depend solely on the convex lens, since the concave lens was only considered to make the image of the convex lens sharper. However, even when only the convex lens was taken in to account, it was not evident that the magnification of the telescope increases with a longer focal length of the convex lens. Practice with a single convex lens suggests that shortening the focal length of a convex lens yields higher magnifications. Moreover, the point of inversion in sixteenth century optics appears to have suggested, as is evident from Bourne's decription of the reflecting telescope, that magnification is a function of the diameter of the convex lens rather than its focal length. It will be shown that Galileo considered the magnification solely dependent upon the convex lens, and that he found out that it was its focal length that mattered.

Galileo was certainly not the only one to have considered the magnification of the telescope solely depended upon the convex lens, not even to have figured out that the focal length of the convex lens determined magnification. On 26 February 1610, prior to the publication of Galileo's 'Sidereus Nuncius' in March, Sergio Venturi (1584-1646), a Sienese architect and engineer, sent a letter, presumably to Nicola Antonio Stigliola, about the construction of a 'Galilean' telescope.[129] Venturi suggested that the origin of the telescope was in the work of Della Porta and that the distance between the convex and the concave lens was most important for a telescopic effect. However, his diagram shows him to have little understood the concave lens. (Figure 6.29)

> I apply these principles to the instrument and its parts, that are two spherical sections, one concave and the other convex of that kind of eyeglasses that are said [to be] for seeing sharp and for seeing large, that is, they represent the visible things smaller and larger than natural vision. Because Mr. Giovanni Battista della

[128] Van Helden, Albert. 'Galileo and the Telescope.' In *Optics in Sixteenth-Century Italy,* edited by Paolo Galluzzi, 149-58. Firenze: Giunti Barberà, 1984, p. 152; Van Helden, *The Invention of the Telescope*, pp. 11-2.

[129] Palladino, Franco. 'Un Trattato sulla Costruzione del Cannocchiale ai Tempi di Galilei: Principi Matematici e Problemi Tecnologici.' *Nouvelles de la République des Lettres* 1 (1987): 83-102, p. 83.

Porta diffusively deals with these things in his work on refraction, and Your Lordship has this author with him, a man much esteemed in these matters, I refer in this part to this work. But for the composition of this instrument these two sections are to be placed at a given distance in a concave cylinder.[130]

Figure 6.29

While Venturi did not discuss the properties of the concave lens, he indicated the importance of the distance of the point E, that is, of the focal length of the convex lens. After reviewing the problems of lens making that did not allow determining the focal length of a convex lens at demand before making it, he pointed out that magnification was a function of this focal length.

Since the [convex lens] is part of a ball not much larger than a cane, it makes its effect at a lesser distance, namely the images are rather less magnified than those made with [lenses] that are part of a larger ball.[131]

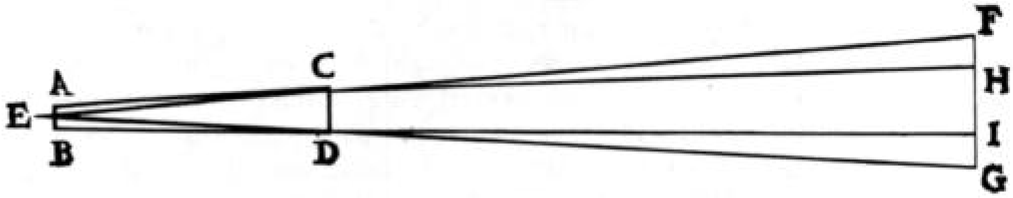

Figure 6.30

That Galileo likewise only considered the convex lens to be important for the magnification of the telescope is evident from his drawing of the telescope in the 'Sidereus Nuncius'. (Figure 6.30) The drawing only shows refraction, and a change of the visual angle, at the surface of the convex lens CD while the rays go unrefracted through the concave lens to the eye at E. In the 'Sidereus Nuncius', Galileo also described a procedure to measure the magnification of the telescope.

[130] 'Farmàti questi prìncipii, gl' applicherò allo strumento e parti di esso, le quali sono due settioni sferiche, una concava e l' altra convessa di quella sorte d' occhiali che si dicono di vista acuta e di vista grossa, cioè che rappresentino le cose visibili minori e maggiori della vista naturale. Della quale materia perchè il S.r Gio. Batt. della Porta ne tratta diffusamente nella sua opera de refractione e V. S. Ill.ma ha costì il medesimo autore, huomo in queste cose stimato singulare, in questa parte a quella mi rimetto. Ma per la compositione dello strumento queste due setioni devono esser poste in una data distanza dentro un cilindro concavo'. Ibid., p. 101.

[131] 'per che è parte di palla non maggiore di molto che sia il canello, per questo fa il suo effetto in distanza menore. Cioè ci accresce le immagini assai meno di quello che faccino quelle che son parte di palla maggiore'. Ibid., p. 102.

> Indeed, in order that anyone may, with little trouble, make himself more certain about the magnification of the instrument let him draw two circles or two squares on paper, one of which is four hundred times larger than the other, which will be the case when the larger diameter is twenty times the length of the other diameter. He will then observe from afar both sheets fixed to the same wall, the smaller one with one eye applied to the glass and the larger one with the other, naked eye. This can easily be done with both eyes open at the same time. Both figures will then appear of the same size if the instrument multiplies objects according to the desired proportion.[132]

This procedure is based on the merging of the images of the naked eye and the eye applied to the telescope, so that it becomes very easy to test the angular magnification of the telescope by comparing these two images.[133] A telescope of the desired magnification is found, when the two images match. Galileo's procedure to test the magnification of a telescope was not original. It will be shown that it was based on a procedure used by contemporaneous spectacle-makers to test the curvatures of single convex lenses, and that with this test procedure from workshop practice at hand, Galileo was able to find out the relationship between the focal length of a convex lens and the angular magnification, not of a single convex lens, but of a 'Galilean' telescope.

It was an old practice to classify spectacle lenses according to their strength in terms of the age of the bearer of the spectacles of a certain strength.[134] At the end of the sixteenth century, this practice appears to have changed. In his 'Uso de los Antojos' (1623), the Andalusian licenciate Benito Daza de Valdés discussed another procedure to classify lenses according to their strength.[135] The procedure was no longer based on the age of the patient, but on the 'grado' of the lens. The 'grado' provided a practical means to determine the curvature of a lens. The procedure of Daza de Valdés shows a remarkable similarity to Galileo's procedure to determine the magnification of a telescope in the 'Sidereus Nuncius'. Daza de Valdés began by drawing two circles X and Q of unequal diameter on a sheet of paper. (Figure 6.31) To the smaller circle, a scale was attached with a scale in 'grados'. A convex lens was positioned along the scale so that the two circles X and Q were seen equally large. The position of the lens along the scale gave its 'grado'. Although the 'Uso de los Antojos' was published in 1623, it appears to go back on an older practice that was current at the end of the sixteenth century in Italy. Von Rohr and Pflugk have shown that the punti-system, discussed by Tommaso Garzoni in his 'La Piazza Universale di Tutte le Professioni del Mondo' (1585) was identical to the grados-system of Valdés.[136]

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[132] 'Ut autem de multiplicatione instrumenti quilibet parvo negotio certior reddatur, circulos binos aut quadrata bina chartacea contornabit, quorum alterum quatercenties altero maius existat; id autem erit tunc, cum maioris diameter ad diametrum alterius longitudine fuerit vigecupla: deinde superficies anbas in eodem pariete infixas simul a longe spectabit, minorem quidem altero oculo as Perspecillum admoto, maiorem vero altero oculo libero; commode enim id fieri licet uno eodemque tempore, oculis ambobus adapertis: tunc enim figurae ambae eiusdem apparebunt magnitudinis, si Organum secundum optatam proportionem obiecta multiplicaverit'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 61, translation in Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 38.

[133] Compare Kepler, Dioptrice, proposition 124, in Kepler, Johannes. *Gesammelte Werke*. Edited by Max Caspar and Franz Hammer. Vol. 4. München: C.H. Beck'sche Verlagsbuchhandlung, 1941, pp. 404-5.

[134] Ilardi, 'Eyeglasses and Concave Lenses in Fifteenth-Century Florence and Milan', p. 349.

[135] Márquez, Manuel. *El Libro del Lic. Benito Daza de Valdés Uso de los Antojos*. Vol. 4, *Biblioteca Clasica de la Medicina Espanola*. Madrid: Real Academia Nacional de Medicina, 1923, pp. 129-46.

[136] Garzoni, Tomaso. *La Piazza Universale di Tutte le Professioni del Mondo*. Edited by Giovanni Battista Bronzini, Pina De Meo and Luciano Carcereri. Vol. 49, *Biblioteca Di "Lares"*. Firenze: Leo S. Olschki Editore, 1996. See Pflugk, A. v., and M. v. Rohr. 'Beiträge zur Entwickelung der Kenntnis von der Brille.' *Zeitschrift für*

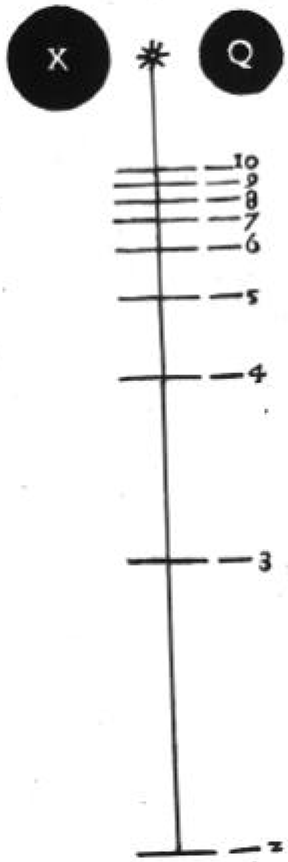

Figure 6.31

Galileo was no doubt a regular visitor of spectacle-makers' shops, certainly once he became involved in making telescopes. As early as 1602, Galileo's workshop appears to have functioned as an intermediary between spectacle-makers' shops and clients, as one of his correspondents mentions that he received a pair of *occhiali*, most likely eyeglasses, for his parents from Galileo.[137] Consequently, Galileo must have known the Garzoni-Valdés-system that spectacle-makers used, and he might have used a similar system when selecting the appropriate lenses for his telescope. What's interesting about the system is that it embodied a relationship between lens curvature and magnification. Galileo was certainly well enough acquainted with sixteenth century optics to know that the curvature of a lens (or a concave mirror) determined its focal length. With the procedure of the spectacle-makers at hand, he would have quickly found out, by trying several convex lenses in combination with a standard concave lens, that convex lenses with longer focal lengths resulted in higher magnifications, and fixed the procedure of the spectacle-makers, adapted to be appropriate to test the magnification of telescopes, as described in his

Augenheilkunde 40 (1918): 50-77, pp. 50-63; Von Rohr, Moritz. 'Ein Versuch zur Ermittelung der Optischen Kenntnisse der Brillenhersteller um das Jahr 1600.' *Zeitschrift für ophthalmologische Optik* 10 (1919): 1-8, pp. 2-7.

[137] 'et con la più vecchia ho havuto la scatola di occhiali, quali son stati a sodisfatione de' parenti, et ringratio V. S. della briga presasi in farmeli haver boni et del presente che V. S. mi fa del prezzo di essi'. Paolo Pozzobonelli to Galileo, 12 September 1602, in Galileo, *Opere*, Vol. 10, p. 93.

'Sidereus Nuncius'. Then, Galileo thought of magnification as exclusivily a function of the visual angle subtended at the eye, like the field of view of a telescope, that Galileo qualitatively described in his 'Sidereus Nuncius', with reference to his drawing of the telescope.

> After such an instrument has been prepared, the method of measuring distances is to be investigated, which is achieved by the following procedure. For the sake of easy comprehension, let ABCD be the tube and E the eye of the observer. When there are no glasses in the tube, the rays proceed to the object FG along the straight lines ECF and EDG, but with the glasses put in they proceed along the refracted lines ECH and EDI. They are indeed squeezed together and where before, free, they were directed to the object FG, now they only grasp the part HI. Then, having found the ratio of the distance EH to the line HI, the size of the angle subtended at the eye by the object HI is found from the table of sines.[138]

In his 'Il Saggiatore' (1619), Galileo explicitly stated the principle that 'the telescope enlarges the objects by bringing them under a larger angle; most correct is the proof that the perspectivists give for this'.[139] The passage in the 'Sidereus Nuncius' has been taken as evidence that Galileo adopted an extramissionist theory of vision, since he stated that the rays proceed from the eye to the object.[140] However, Galileo was not an extramissionist. In a letter of 1611 to Piero Dini, Galileo clearly stated, in a context in which he adopted the theory of multiplication of species, that the light of the satellites of Jupiter 'spreads, very vividly, to the earth'.[141] The passage cannot be interpreted otherwise than being intromissionist. The passage in the 'Sidereus Nuncius', relating magnification to visual angle, is rather to be considered in agreement with the Euclidean framework, provided by sixteenth century optics, as discussed in chapter 2, in particular, the manuals on perspective and measurement of sixteenth century mathematical practitioners, with which Galileo, as shown in chapter 5, was well acquainted.[142] Also, it has already been noted that sixteenth century opticians, like Maurolico, thought the Euclidean framework applicable to lenses, because they discussed lenses in terms of visual rays. Consequently, Galileo must have been much pleased with the principle that magnification was dependent upon the visual angle subtended at the eye, because it did only provide the means to improve the magnification of the telescope, it was also in agreement with the optical tradition of the mathematical practitioners.

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[138] 'Consimili parato Instrumento, de ratione distantiarum dimetiendarum inquirendum erit: quod tali artificio assequemur. Sit enim, facilioris intelligentiae gratia, tubus ABCD. Oculus inspicientis esto FG secundum lineas rectas ECF, EDG ferrentur; sed, appositis Perspecillis, ferantur secundum lineas refractas ECH, EDI: coarctantur enim, et qui prius liberi ad FG obiectum dirigebantur, partem tantummodo HI compraehendent. Accepta deinde ratione distantiae EH ad lineam HI, per tabulam sinuum reperietur quantitas anguli in oculo ex obiecto HI constitui'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, pp. 61-2, translation in Galileo, *Sidereal Messenger*, p. 39.

[139] 'il telescopio ingrandisce gli ogetti col portargli sotto maggior angolo; verissima è la prova che n' arrecano i prospettivi'. Galileo, Il Saggiatore, in Galileo, *Opere*, Vol. 6, p. 254, my translation, different from the translation in Galilei, Galileo, Horatio Grassi, Mario Guiducci, and Johann Kepler. *The Controversy on the Comets of 1618*. Translated by Stillman Drake and C. D. O' Malley. Philadelphia: University of Pennsylvania Press, 1960.

[140] See, for example, Pedersen, Olaf. 'Sagredo's Optical Researches.' *Centaurus* 13 (1968): 139-50, p. 141.

[141] 'il lume loro assai vivamente sino in terra si diffonda'. 21 May 1611, in Galileo, *Opere*, Vol. 11, p. 115. For the context of Galileo's discussion, see Redondi, Pietro. 'Galilée aux Prises avec les Théories Aristotéliciennes de la Lumière (1610-1640).' *XVIIe Siècle* 34 (1982): 267-83; Gómez, Susana. 'The Bologna Stone and the Nature of Light: The Sciences Academy of Bologna.' *Nuncius* 6 (1991): 3-32; in particular, for Galileo's intromissionist theory, see Dupré, *De Optica van Galileo Galilei: Interactie Tussen Kunst en Wetenschap*, pp. 52-67.

[142] For the sixteenth century interpretation of the Euclidean visual angle axiom, see Panofsky, Erwin. *Perspective as Symbolic Form*. Translated by Christopher S. Wood. New York: Zone Books, 1997, pp. 27-36.

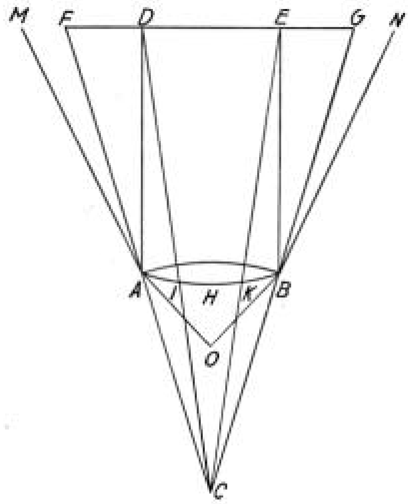

Figure 6.32

Galileo's principal of angular magnification was highly influential. Kepler adopted it in his 'Dioptrice' (1611). In proposition 80, Kepler explained why a convex lens enlarges when the eye is placed between the lens and its focal point.[143] In his diagram, the eye is at C, the object is DE, and the lens AB. (Figure 6.32) The rays CI and CK are refracted at the lens towards E and D. Therefore, the angle DCE is the angle under which a smaller part of DE is perceived which is estimated to be as large as DE. Consequently, to apprehend DE, the rays from D and E that reach the eye must come from outside CI and CK, for example, CA and CB. Since the angle ACB is larger than the angle DCE, under which the object is seen by the naked eye, the object is estimated to be larger than it is. Thus, just as Galileo, Kepler considered magnification a function of the visual angle subtended at the eye. Moreover, Kepler, like Galileo, considered magnification to be primarily determined by the focal length of the convex lens. Propostion 115 of the 'Dioptrice' stated that 'when the concave lens is near the eye, convex lenses, of a smaller convex circle, represent the visible things smaller, those of a larger convex circle, represent them larger'.[144] Consequently, also Kepler did not think of magnification in terms of the proportion between the focal lengths of the convex and the concave lens. Moreover, in propostion 124, Kepler described the method that Galileo had appropriated from the spectacle-makers' workshops, to measure the magnification of the telescope.[145] The notion that the magnification of the telescope was dependent upon the convex lens was to dominate telescopic practice for a very long time. It is at the basis of the practice of the seventeenth century to improve the magnification of the telescope by making objectives of convex lenses of progressively longer focal lengths.[146]

[143] Kepler, Dioptrice, in Kepler, Johannes. *Gesammelte Werke*. Edited by Max Caspar and Franz Hammer. Vol. 4. München: C.H. Beck'sche Verlagsbuchhandlung, 1941, pp. 381-2. There is a reliable German translation, Kepler, Johannes. *Dioptrik*. Translated by Ferdinand Plehn. Leipzig: Verlag von Wilhelm Engelmann, 1904.

[144] 'Proposita lente cava proximè oculum, convexarum lentium, quae minori circulo convexa est, minora repraesentat visbilia, quae majori, majora'. Ibid., p. 401.

[145] Ibid., pp. 404-5.

[146] For seventeenth century practice of improving the magnification of the telescope by making telescopes with ever longer focal lengths of the objective, the convex lens, see the dimensions given in Van Helden, Albert. 'The Telescope in the Seventeenth Century.' *Isis* 65 (1974): 38-58; Van Helden, Albert. 'The 'Astronomical Telescope'

Thus, in this chapter, it has been shown that the notion of point of inversion, developed in mid-sixteenth century optics, was at the basis of the telescopic practice of the end of the sixteenth century. Both the reflecting telescope and the refracting telescope benefited from the notion. The absence of this notion prior to the second half of the sixteenth century must be the cause for the delay of the invention of the telescope, although both the promising idea of a telescope as well as the optical components had been available. Control of the magnification of the telescope appears to have been the main problem, not only for technological reasons, but also because the notion behind what would make such technological improvements for making convex lenses of progressively longer focal lengths necessary was not evident. Galileo, informed by this sixteenth century optical practice, was able to find out that magnification depended upon focal length, with the help of practical procedure borrowed from the spectacle-makers' workshop. He also considered, determined by the sixteenth century state of optical knowledge, magnification to be solely a function of the focal length of the convex lens, not of the concave lens. In the next chapter, it will be shown that magnification was not the only problem. The problems of 'optical aberrations' of the telescope resulted in an extensive exploration of light by Galileo.

1611-1650.' *Annali dell' Istituto e Museo di Storia della Scienza* 1 (1976): 13-35; Van Helden, Albert. 'The Development of Compound Eyepieces, 1640-1670.' *Journal for the History of Astronomy* 8 (1977): 26-37.

VII. 'As Little Moons': Galileo's Exploration of Light

1. The Aberrations of Galileo's Telescope

Improving the magnification of the telescope was only one problem that Galileo had to solve. His second problem was to guarantee, in anachronistic terms, the 'image quality' of the telescope by means of the control of, in again anachronistic terms, the 'aberrations'. Galileo's most important mean in the fine-tuning of the optical performance of his telescopes was the application of a diaphragm or aperture stop to the objective lens. As discussed in chapter 1, the still preserved instruments of Galileo show that the diameters of the objective lenses were stopped down with a cardboard ring. By evaluating the performance of Galileo's telescopes with and without diaphragms with optical design software, Zik has shown that, besides using lenses with different radii, the diaphragm was indeed the single most important element in Galileo's control of the optical aberrations of his telescopes.[1] With the diaphragm, spherical aberration and lateral chromatic aberrations are reduced to a minimum. Only the longitudinal chromatic aberrations would still have troubled the optical performance of Galileo's telescopes. It cannot be doubted that the fine-tuning of the optical performance of the telescope was a matter of trial and error, choosing the best lenses from a whole set of pre-made lenses and applying aperture stops of different radii to the objective lens in order to get the best result when observing a target.[2]
However, this chapter is concerned with when Galileo started applying aperture stops to his objective lens, why he introduced the diaphragm and how he understood the way such a diaphragm worked. In a recent examination of the telescopes by Van Helden, a piece of paper was found glued to the end of the tube of the second telescope with an inscription, most likely in Galileo's handwriting.[3] This inscription shows a measurement in piedi, the Paduan length measurement. Thus, this second instrument, with a diaphragm, is to be dated to Galileo's Paduan period. Henceforward, it must have been made at the end of 1609 or the beginning of 1610. That Galileo was using diaphragms at an early date is confirmed by his correspondence. The diaphragm is first mentioned in a letter on his telescopic observations to Antonio de' Medici of 7 January 1610.

> It is good that the convex glass, which is the one far from the eye, should be partly covered, and that the opening left should be oval in shape, since thus are objects seen much more distinctly.[4]

Consequently, Drake has linked the introduction of the diaphragm directly with Galileo's discovery of the satellites of Jupiter only a few days later.[5] As the context of Galileo's

[1] Zik, Yaakov. 'Galileo and the Telescope: The Status of Theoretical and Practical Knowledge and Techniques of Measurement and Experimentation in the Development of the Instrument.' *Nuncius* 14 (1999): 31-67, p. 51.

[2] For Galileo's use of lenses with different radii, see Ibid., p. 49. For the practice of choosing the best lens among a much larger set of lenses, see Sagredo's letters to Galileo, in Galileo, *Opere*, Vol. 12, pp. 316, 376.

[3] Van Helden, Albert. *Catalogue of Early Telescopes*. Firenze: Giunti, 1999, p. 32.

[4] 'E bene che il vetro colmo, che è il lontano dall' occhio, sia in parte coperto, et che il pertuso che si lascia aperto sia di figura ovale, perchè cosi si vedranno li oggetti assai più distintamente'. Galileo to Antonio de' Medici, 7 January 1610, in Galileo, *Opere*, Vol. 10, p. 278, translation in Drake, Stillman. 'Galileo's First Telescopic Observations.' *Journal for the History of Astronomy* 7 (1976): 153-68, p. 158.

introduction of the diaphragm, Drake has argued that 'the answer, 2I think, lies in the excessive spherical and chromatic aberration of the 20-power telescope when it was first used at the beginning of December. These defects would not materially disturb lunar observations, but they would make planets indistinguishable from stars and would make them all hazier and less distinct than to the naked eye. Even the fact that previously unseen stars were now made visible would not have been at all obvious without troublesome special study. Meanwhile the splendid and exciting progress of lunar illumination would suffice to keep Galileo's attention fixed on those phenomena'.[6] Drake is correct in linking the introduction of the diaphragm to the observation of the planets and the stars. However, his explanation is wanting, because he did not make clear why Galileo would be interested in observing a difference between the planets and the stars. Moreover, as will be shown, that there is such a difference was not at all evident a priori.

As to why Galileo introduced the diaphragm, Drake has suggested that Galileo incidentally hit on the diaphragm, not from reasoning from any theoretical foundation or knowledge obtained from other sources, but from his own experience as a patient, suffering from an eye disease, that made him see bright lights with colored rings. Drake has concluded that 'a person so afflicted quickly learns that he can see quite well by squinting or by holding a pinhole before the eye'.[7] Drake's observation that observing through a pinhole is helpful in obtaining a sharper image is of course correct. However, it is unclear when Galileo developed this eye disease and to what extent it troubled him.[8] Most important, Drake's explanation lacks context that links Galileo's experience to his telescopic observations. It might be rightly asked why Galileo came to the procedure of looking through a pinhole to see sharper, even if he had an eye disease, and how he understood it. Moreover, if it were in particular colored haloes that plagued him, it is very unlikely, as will become evident, that he would have connected such 'chromatic aberrations' with looking through a pinhole. In Drake's account, the discovery of the diaphragm appears as an isolated discovery.

Galileo discussed the diaphragm in his 'Sidereus Nuncius', but not for its effect on the image quality, but as a method for measuring angular distances between Jupiter and its satellites in order to find their periods. After introducing how the field of view of the instrument was dependent upon the convex lens, as discussed in the last chapter, with reference to the drawing presented there, Galileo proposed to use plates of different apertures applied to the convex lens.

> If over the glass CD we fit plates perforated some with larger and some with smaller holes, putting now this plate and now that one over it as needed, we form at will angles subtending more or fewer minutes. By this means we can conveniently measure the spaces between stars separated from each other by several minutes with an error of less than one or two minutes.[9]

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[5] Ibid., pp. 158-9.

[6] Ibid., p. 158.

[7] Ibid., pp. 158-9. See also Drake, Stillman. *Galileo at Work: His Scientific Biography.* Chicago London: The University of Chicago Press, 1978, p. 148.

[8] In the passage of Galileo's 'Il Saggiatore' to which Drake refers, Galileo speaks of this illness as only temporary. Galileo, Il Saggiatore, in Galileo, *Opere*, Vol. 6, p. 357, translation in Galilei, Galileo, Horatio Grassi, Mario Guiducci, and Johann Kepler. *The Controversy on the Comets of 1618*. Translated by Stillman Drake and C. D. O' Malley. Philadelphia: University of Pennsylvania Press, 1960, p. 319.

[9] 'Quod si Specillo CD bracteas, alias maioribus, alias vero minoribus perforatas foraminibus, aptaverimus, modo hanc, modo illam, prout opus fuerit, superimponentes, angulos alios atque alios pluribus paucioribusque minutis subtendentes, pro libito constituemus; quorum ope Stellarum intercapedines, per aliquot minuta adinvicem

The method was not success, because, on the one hand, Galileo underestimated the complexity of the relationship between the size of the aperture and the field of view, and, on the other hand, the image would have become too dim when using apertures of such a small size.[10] Zik has argued that 'as a result of the investigations he made, for the first time Galileo gained some knowledge of the way diaphragms could affect the image in an optical device'.[11] This might be correct, in the sense that Galileo's experience with aperture stops of different diameters to measure angular distances made him aware that there was a certain minimum limit to the size of the aperture. However, Galileo's application of diaphragms of different diameters is not to be considered at the basis of his introduction of a diaphragm. Galileo was well aware of the effect of the diaphragm on the image, as is evident from the already quoted letter to Antonio de' Medici in which he argued that with the aperture stop 'objects are seen much more distinctly'.[12] Moreover, this effect was not evident to all contemporaneous observers, as is shown by Clavius' inquiry.

> Here in Rome, there are seen some *occhiali* sent by Your Lordship, that have rather large convex lenses, but covered, so that only a small aperture is uncovered. I would desire to know the purpose of this largeness [of the lens], if one has to cover it in this way. Some think that they are made large, so that when uncovered completely at night, they are better able to see the stars.[13]

In his answer, Galileo gave two reasons. First, the lenses are made with large diameters to minimize the degrading effects of wedge-shaped lenses by making lenses of large diameters instead of smaller ones. Second, a large part of the lens needs consecutively to be covered, because larger apertures give larger fields of view, but unfortunately also more cloudy images.

> It is only left to explain how I have made some rather large glasses, even though I have next covered a large part. This is done for two reasons: first, to be able to work more precisely, because a more spacious surface is better maintained in the desired shape than a small surface; the other reason is that when one wants to see a larger space in one gaze, one can uncover the glass: but one needs to place a less acute glass near the eye and to shorten the tube, otherwise they will see the objects rather cloudy.[14]

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dissitarum, citra unius aut alterius minuti peccatum, comode dimetiri poterimus'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 62, translation in Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 39.

[10] For Galileo's underestimation of the complexity of the relationship between the size of the aperture and the field of view of his telescope, see North, John. 'Thoms Harriot and the First Telescopic Observations of Sunspots'. In *Thomas Harriot: Renaissance Scientist*, edited by John W. Shirley, 129-65. Oxford: Clarendon Press, 1974.

[11] Zik, 'Galileo and the Telescope: The Status of Theoretical and Practical Knowledge and Techniques of Measurement and Experimentation in the Development of the Instrument', p. 53.

[12] Galileo to Antonio de' Medici, 7 January 1610, in Galileo, *Opere*, Vol. 10, p. 278.

[13] 'Si sono visti qui in Roma alcuni occhiali mandati da V. S., i quali hanno li vetri convessi assai grandi, ma coverti, con restarvi solamente un bucco piccolo libero. Desiderarei di sapere che serve tanta grandezza, se ha da coprirsi in queto modo. Pensano alcuni, che siano fatti grandi, acciò tutti la notte, si possono meglio vedere le stelle'. Clavius to Galileo, 17 December 1610, in Galileo, *Opere*, Vol. 10, p. 485.

[14] 'restami di dirgli come ho fatto alcuni vetri assai grandi, benchè poi ne ricuopra gran parte, et questo per 2 ragioni: l' una, per potergli lavorar più giusti, essendo che una superficie spaziosa si mantiene meglio nella debita figura, che una piccola; l' altra è, che volendo veder più grande spazio in un' occhiata, si può scoprire il vetro: ma bisogna presso all' occhio mettere un vetro meno acuto et scorciare il cannone, altramente si vedrebbono gli oggetti assai annebbiati'. Galileo to Clavius, 30 December 1610, in Galileo, *Opere*, Vol. 10, pp. 501-2. In the light of the discussion of the magnification of the telescope in the previous chapter, remark how Galileo argued that the

Zik has also suggested an alternative explanation of the origin of the diaphragm on Galileo's telescopes. He has claimed that 'Galileo was familiar with the phenomena of the pupil's enlargement at night and shrinkage at daytime, and knew that these must affect the quantity and the angle by which light rays impinge the eye. From his experience Galileo learned that the use of a shutter (aperture stop) may improve the telescope's performance at daytime'.[15] This suggestion is however misleading. Galileo was well aware that the size of the pupil is to be taken into account when making measurements. In his 'Dialogo intorno alle due massime sistemi del mondo' (1632), and, again, in his 'Le operazioni astronomiche', a late work, never published during his life, Galileo discussed a method to correct the error of angular size measurement as a consequence of considering 'the rays of sight as deriving from one indivisible point, which is false, since they are produced by the whole small circle of the pupil of the eyes'. Consequently, Galileo argued that 'the observer needs to have a refined measure of the diameter of the pupil of the eye, of which the size has to be taken into account'.[16] This problem of the 'eccentricity of the eye', consisting of determining the locus of the apex of the visual triangle between, for example, two stars, was known since Archimedes.[17] It was central to the design of mathematical instruments. For example, Levi ben Gerson considered this apex to be at the center of the eye about one centimeter of the end of his measuring instrument.[18] However, other designers of mathematical instruments had ignored this problem of the 'eccentricity of the eye'. Galileo's method to measure the diameter of the pupil consisted of taking two strips of cardboard, the one black, the other white and twice the size of the black one.[19] The larger white piece of cardboard is placed behind and parallel to the black one at a distance of ten braccia. The eye is placed at the summit of the visual triangle thus formed. Galileo noted however that, if the eye is placed at this summit, the piece of black cardboard did not completely hide the white one. To completely hide the piece of white cardboard, the eye needs to be brought forward, closer to the black piece of cardboard. The difference between the first and the second positions of the eye is measured. Then, by simple triangulation, the size of the diameter of the pupil is found.

Galileo was also well aware of the contraction and dilation of the pupil depending on the light conditions. His source was most likely one of his Venetian friends, either Paolo Sarpi who

curvature of the concave lens, not expressed in its focal length, affects the distance of the concave lens from the convex lens, while he is silent about how the curvature of the concave lens affects the magnification of the telescope.

[15] Zik, 'Galileo and the Telescope: The Status of Theoretical and Practical Knowledge and Techniques of Measurement and Experimentation in the Development of the Instrument', p. 58.

[16] 'raggi della sua vista come derivanti da un punto solo indivisibile; il che è falso, atteso che vengono prodotti da tutto 'l piccolo cerchio della pupilla de gli occhi: onde fa di bisogno che il riguardante abbia una squisita misura del diametro della pupilla del proporio occhio, la cui grandezza si deve mettere in conto'. Galileo, Le Operazioni Astronomiche, in Galileo, *Opere*, Vol. 8, p. 456.

[17] Lejeune, Albert. 'La Dioptre d' Archimède.' *Annales de la Societé Scientifique de Bruxelles* 61 (1947): 27-47.

[18] Goldstein, Bernard R. 'Remarks on Gemma Frisius's De Radio Astronomico et Geometrico.' In *From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences Presented to Asger Aaboe*, edited by J. L. Berggren and B. R. Goldstein, 167-80. Copenhagen: University Library, 1987, p. 168.

[19] Galileo, Le Operazioni Astronomiche, in Galileo, *Opere*, Vol. 8, pp. 456-7; Galileo, Dialogo sopra i Due Massimi Sistemi del Mondo, in Galileo, *Opere*, Vol. 7, pp. 390-1, translation in Galilei, Galileo. *Dialogue Concerning the Two Chief World Systems - Ptolemaic & Copernican*. Translated by Stillman Drake. Berkeley Los Angeles London: University of California Press, 1967, pp. 363-4. For discussion, see Hamou, Philippe. *Lu Mutation du Visible: Essai sur la Portée Épistémologique des Instruments d' Optique au XVIIe Siécle*. 2 vols. Vol. 1: Du Sidereus Nuncius de Galilée à la Dioptrique cartésienne. Villeneuve d' Ascq: Presses Universitaires du Septentrion, 1999, pp. 103-5.

appears to have been readily connected with the size variation of the pupil, or Fabricius d' Acquapendente, Galileo's personal physician, who had discussed it in his medical work.[20]

> I'll wager anything that not more than one out of every thousand people who have observed the extreme contraction and dilation of the pupil in a cat's eye have observed a like effect in the human pupil, depending upon whether it is looking through a well or a poorly lighted medium. In daylight the circle of the eye is much diminished; when looking at the disc of the sun, it is reduced to a size smaller than a millet seed; but when looking at nonshining objects in a dark medium, it dilates to the size of a pea or larger. In general this enlargement and reduction varies in much more than a tenfold ratio, from which it is obvious that when the pupil is much dilated, the angle of intersection of the rays must be farther away from the eye, as happens when looking at poorly lighted objects.[21]

However, for Galileo, the size variation of the pupil was marginal to his thought. He argued that 'in order to reveal the error of the astronomers, you do not need such accuracy; for even if we favor them by assuming that the intersection is made right at the pupil itself, it does not much matter, their error being so enormous'.[22] That the size variaton of the pupil was marginal to Galileo, is most clearly evident from him never bringing in this size variation to account for the difference between day and night observations of planets, for example Venus. Galileo noted that Venus is seen more according to the true size of its image during daylight observations, but he never explained it by variation of the size of the pupil. Consequently, Galileo did not arrive at the application of the diaphragm from an analogy with the size variation of the pupil of the eye.

It is important to point out that Galileo did not think of the cloudy and colored images that he saw through his telescope in terms of 'aberrations' of his lenses in a sense similar to what is understood under 'optical aberrations' today. As concerns chromatic aberration, colour was an important subject of research in contemporaneous discussions of the rainbow. In his treatment of color, Theodoric of Freiberg (ca. 1305), in an important but little influential treatise, deviated from the Aristotelian colour theory by no longer considering black and white as the generators of

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[20] Sarpi was credited with the discovery that the pupils of the eyes of cats dilate and contract by Fabricius d' Acquapendente. Acquapendente, Girolamo Fabricio d'. 'De visione'. In *Opere Omnia*. Leipzig: Sumptibus Johannis Friderici Gleditschi, Excudebat Christianus Goezius, 1687, p. 229. For Sarpi, see Gliozzi, M. 'Relazioni Scientifiche tra Paolo Sarpi e GB Porta.' *Archives Internationales d' Histoire des Sciences* 1 (1947): 395-433, in particular, pp. 398-403. For Fabricius d' Acquapendente, see De Nil, Erwin, and Marc De Mey. 'Hieronymus Fabricius d' Acquapendente: De Visione, Ending of the Perspectivistic Tradition.' In *Optics and Astronomy*, edited by Gérard Simon and Suzanne Débarbat, 51-82. Turnhout: Brepols, 2001, in particular, p. 55.

[21] 'e farei ben qualsivoglia scommessa, che tra mille che hanno osservato ne' gatti strignersi ed allargarsi assaissimo la pupilla dell' occhio, non ve ne sono due, nè forse uno, che abbia osservato, un simile effetto farsi dalle pupille de gli uomini nel guardare, mentro il mezo sia molto o poco illuminato, e che nella aperta luce il cerchietto della pupilla si diminuisce assai, si che nel riguardare il disco del Sole si riduce a una piccolezza minore di un grano di panico, che nel mirare oggetti non risplendenti, e dentro a mezo men chiaro, si allarga alla grandezza di una lente o più; ed in somma queto allargamento e strignimento si diversifica più assai che in decupla proporzione: dal che è manifesto che quando la pupilla è dilatata molto, è necessaro che l' angolo del concorso de' raggi sia più remoto dall' occhio, il che accade nel riguardare gli oggetti poco luminosi'. Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, p. 390, translation Galileo. *Dialogue Concerning the Two Chief World Systems - Ptolemaic & Copernican*, p. 363.

[22] 'ma in questa, per manifestar l' errore de gli astronomi, non vi è necessaria tanta accuratezza, perchè, quando anc a favor della parte noi supponessimo tal concorso farsi sopra l' istessa pupilla, poco importerebbe, per essere la fallacia loro tanto grande'. Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, p. 390, translation in Ibid., p. 363.

colours.[23] Aristotle considered all colours as mixtures of black and white, resulting in a scale of colours depending on the ratio of black and white.[24] From around 1400, starting with Cennino Cennini and Alberti, it were in particular painters, based on their experience with the mixing of pigments, who contributed to the establishment of a colour theory different from Aristotle's.[25] Artistic practice suggested that there were some primary colours, although there was disagreement about their number, from which other colours were to be made. Black and white were no longer considered real colours, but adding black or white was considered to alter the 'species', to use Alberti's terminology, or the tone of a colour. Galileo was most likely acquainted with contemporaneous painterly practice as concerns colour. Reeves has suggested that Galileo's familiarity with contemporaneous painterly practice with colour, in particular, the tonal modelling of colours, was an important factor in his recognizing the 'ashen light' of the moon as the light reflected from the earth.[26] However, before Newton's discovery of the different refrangibility of light, with each degree of refrangibility corresponding with a colour, it would have been impossible to recognize the colored images as the effect of 'chromatic aberration'.[27]

While the absence of a concept of 'chromatic aberration' to Galileo might be pointing out the obvious, it is more important to note that Galileo also had no concept of 'spherical aberration' at his disposal. As has been shown, sixteenth century optics recognized that parallel rays passing through a refractive sphere or reflected by a concave spherical mirror do not converge in one point. As shown in the discussion of Galileo's numbers on his copy of Ausonio's 'Theorica', Galileo recognized the octogonal ray in Ausonio's drawing. Consequently, it can hardly be denied that Galileo was familiar with this property of concave spherical mirrors, refractive spheres and convex lenses. However, there is a difference between familiarity with rays not converging in one point and 'spherical aberration'. A concept of spherical aberration assumes that the 'cloudiness' of the images seen through a telescope is attributed to the non-focusing of the rays in one focal point. Only Kepler's 'Paralipomena', as shown, unknown to Galileo prior to 1610, connected this property of refractive spheres with a concept of an image (pictura) that makes an evaluation of the influence of 'spherical aberration' on the quality of an image optically meaningful.[28] However, in a practical context, it was recognized that limiting the aperture of a convex lens results in a 'more vivid' image, however without linking this to 'spherical

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[23] Boyer, Carl B. *The Rainbow: From Myth to Mathematics*. Princeton: Princeton University Press, 1987, pp. 113-4. See also Wallace, William A. *The Scientific Methodology of Theodoric of Freiberg: A Case Study of the Relationship between Science and Philosophy*. Fribourg: The University Press, 1959.

[24] Kemp, Martin. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*. New Haven London: Yale University Press, 1990, pp. 264-5.

[25] Edgerton, Samuel Y., Jr. 'Alberti's Colour Theory: A Medieval Bottle without Renaissance Wine.' *Journal of the Warburg and Courtauld Institutes* 32 (1969): 109-34; Hall, Marcia B. *Color and Meaning: Practice and Theory in Renaissance Painting*. Cambridge: Cambridge University Press, 1992, pp. 14-46.

[26] Reeves, Eileen. *Painting the Heavens: Art and Science in the Age of Galileo*. Princeton: Princeton University Press, 1997, pp. 94-104. In his library, Galileo had one book on color, namely Morato, Fulvio Pellegrino. *Del Significato de Colori e de Mazzolli*. In Vinegia: Gioanne Padoano, 1551. See Favaro, Antonio. *La Libreria di Galileo Galilei*. Vol. 10, *The Sources of Science*. New York London: Johnson Reprint Corporation, 1964, p. 261.

[27] For a discussion of Newton's color theory against the background of the Renaissance color theory of painters, see Shapiro, Alan E. 'Artists' Colors and Newton's Colors.' *Isis* 85 (1994): 600-30.

[28] Kepler, Paralipomena ad Vitellionem, book V, proposition 9, translation in Kepler, *Optics: Paralipomena to Witelo & Optical Part of Astronomy*, pp. 198-9. See also Kepler's treatment of eyeglasses, in Ibid., pp. 216-8.

aberration'. As discussed in chapter 6, Barbaro recommended the use of the camera obscura-cum-lens to painters. He also advised them 'to cover the lens as much as to leave a bit of circumference in the middle, and that [part] which is clear is not covered, you will see an even more vivid effect'.[29] Consequently, Galileo's application of a diaphragm to the convex lens of his telescope might have derived from his acquaintance with the optical instrumental practice of the sixteenth century. As Barbaro, Galileo never linked this practice with 'spherical aberration'.

However, as will become evident, Galileo never linked the use of the diaphragm with an optical defect of the lenses of his telescope. Instead, he connected the diaphragm with the phenomenon of irradiation, the bright rays around light sources that disappear when looking through a pinhole. It will be shown that Galileo's application of the diaphragm did not arise suddenly and isolated, as Drake has suggested, but as part of an exploration of celestial light that extended over the period from around 1604, when a nova appeared, to around 1611, when his thoughts on the light of the moon, the planets and the stars and his concept of irradiation, as they were eventually published in the 'Dialogo' (1632), had fully matured. First, I will give an overview of the history of celestial light to show that it was not a settled issue at the beginning of the seventeenth century whether the moon, the planets and the stars reflect solar light or shine with their own light. Second, Galileo's explorative trajectory as concerns the question of celestial light will be traced. Galileo's trajectory can be divided in to four phases, (1) the initial position of 1604 that both planets and stars reflect solar light, (2) the investigations with mirrors and reflection of 1607 concerning the light of the moon, (3) the telescopic observation of a difference of appearance of the stars and the planets of the 'Sidereus Nuncius', published in March 1610, and (4) the discovery of the phases of Venus, announced in December 1610, and the development of his final position that while the stars shine with their own light, the planets only reflect solar light.

2. Celestial Light: Moon, Planets, and Stars

The question of celestial light was a much-debated problem among the medieval perspectivists as well as among the scholastic commentators on Aristotle.[30] Among the heavenly bodies, the moon held, in this respect, a special position, because it was the only heavenly body on which surface features, large dark spots, were seen with the naked eye. The medieval commentators on Aristotle, from Averroës on, argued that the moon did not reflect light like a mirror. If the moon is a mirror, it would hardly lit up, because only a small amount of the reflected rays reaches the eye of the observer. To hold the reflection theory of the moon's light, medieval commentators would have had to admit that the moon's surface was rough. However, since convential Aristotelian cosmology demanded the perfection of the heavenly bodies, the moon could not have a rough surface. Consequently, to save the Aristotelian perfection of the heavenly bodies, scholastic commentaries on Aristotle argued that the moon was translucent. The dark spots on the

[29] coprire il vetro tanto, che vi lasci una poca di circonferenza nel mezzo, che sia chiara è scoperta, ne vederai anchora piu vivo effetto'. Barbaro, Daniele. *La Pratica della Perspettiva*. Edited by Roberto Fregna and Giulio Nanetti. Vol. 8, *Biblioteca di Architettura Urbanistica Teoria e Storia*: Arnaldo Forni Editore, 1980, pp. 192-3.

[30] For the scholastic discussion of the light of the moon, see Ariew, Roger. 'Galileo's Lunar Observations in the Context of Medieval Lunar Theory.' *Studies in the History and Philosophy of Science* 15 (1984): 213-26.

moon were explained by differences in optical density. This explanation is first found in a little work of Alhazen, from whom it passed on to Averroës and the Aristotelian commentaries.[31]
In his 'Paralipomena' (1604), Kepler revived Plutarch's 'On the face that appears on the orb of the moon', in which Plutarch had argued that the moon was just like another earth.[32] As is well known, Kepler was a Copernican, and, consequently, there was no need for him to save the Aristotelian perfection of the heavenly bodies by positing that the moon is translucent, as the scholastic tradition and Witelo had argued, or that the moon shines with light of its own, as Reinhold had argued.[33] Kepler argued that the moon only reflects solar light. However, to maintain the earthlike nature of the moon, Kepler needed to explain the naked eye observations of the dark spots on the moon, the reddish light of the moon during lunar eclipses and the moon's ashen light. Kepler observed the moon with a camera obscura. His drawing shows a large dark spot and a sinuous terminator. (Figure 7.1)

Figure 7.1

As concerns the dark spots, Kepler agreed with Plutarch that the moon has a mountainous surface, but, unlike Plutarch, he considered the brighter parts of the moon to be seas and the dark spots to be land, similar to continents on earth.[34] Kepler argued that 'watery surfaces most of all become radiant with light, if they are placed next to the land, I believe because of the uniformity of the surface as a whole, but the roughness and rippled state of the tiny parts, or because they participate less in dark color than the earth. The former of these makes it suited for reflecting the

[31] For Alhazen's discussion of the light of the moon, see Kohl, Karl. 'Uber das Licht des Mondes: Eine Untersuchung von Ibn Al Haitham.' *Sitzungsbericht der Physikalisch-medizinischen Sozietät in Erlangen* 56-57 (1922): 305-98; Alhazen. *Uber die Natur der Spuren <Flecken>, die man auf der Oberfläche des Mondes Sieht.* Translated by Carl Schoy. Hannover: Orient Buchhandlung Heinz Lafaire, 1925.

[32] Plutarch. 'Concerning the Face Which Appears in the Orb of the Moon (De Facie Quae in Orbe Lunae Apparet).' In *Plutarch's Moralia*, 1-223. London Cambridge, Massachusetts: William Heinemann Ltd Harvard University Press, 1962. For discussion of Kepler's appropriation of Plutarch, see Booth, Sara, and Albert Van Helden. 'The Virgin and the Telescope: The Moons of Cigoli and Galileo.' *Science in Context* 13 (2000): 463-88, pp. 468-70.

[33] Kepler, Paralipomena ad Vitellionem, pp. 226-9, translation in Kepler, *Optics*, pp. 241-4.

[34] Kepler, Paralipomena ad Vitellionem, pp. 246-51, translation in Ibid., pp. 259-63.

sun's light in almost all directions'.[35] Thus, Kepler argued that seas reflect more light, because the roughness of the water, like waves, reflects the solar light in all directions. As concerns the reddish light of the moon during lunar eclipses, Kepler attributed it to the refraction of solar light in the atmosphere of the earth in to the earth's shadow cone.[36] Kepler argued that the ashen light of the moon, best visible before and after the quadratures, was not the moon's own light, as Witelo and Reinhold had argued, but he conceded with his teacher Maestlin that the moon's ashen light was caused by the reflection of solar rays by the earth to the moon.[37]

As is evident from Kepler's argument in the 'Paralipomena', there is a strong connection between Copernicanism and the position that the moon only shines with reflected solar light. However, unlike the case of the light of moon, Copernicanism did not entail the Galilean argument of 1611 that planets only reflect solar light, while the stars only shine with their own light. As will become evident, if any position as concerns the light of the planets and the stars was determined by Copernicanism, it was rather that planets also have light of their own. The question of the light of the planets and the stars was a particularly vexed problem, because, before telescopic observation, the only observable difference between fixed stars and planets was in their motion and in the fact that the former twinkle, while the latter do not. Consequently, as concerns the question of celestial light, most often there was not made a difference between planets and fixed stars. In his work 'On the light of the stars', Alhazen argued that the fixed stars as well as the planets shine with their own light.[38] As evidence for his argument, Alhazen pointed in particular to the observational absence of the phases of Venus. He argued that, if the planets and fixed stars only reflect light, Venus would show phases like the moon, and, since this is evidently not the case, planets and stars shine with their own light. Alhazen's argument passed on to the scholastic Aristotelian commentary tradition.[39] Avicenna and Macrobius, argued that stars and planets, unlike the moon, shine with their own light, again, referring to the observational absence of the phases of Venus (and Mercury) as evidence. However, opinions diverged. Albertus Magnus argued that neither the planets nor the stars have their own light. Consequently, for Albertus Magnus, the sun is the only light source, a position that was also attributed to Aristotle and Averroës. He claimed that planets and stars were transparent and became visible to us, because their bodies are impregnated with solar light that is subsequently transmitted to us. Finally, Albert of Saxony in his 'Questiones super quatuor libros de Celo et Mundo', adopted a mixed position. He conceded that the planets and stars also have some light of their own.

This mixed position was also adopted by some of the perspectivists, like Pecham. In the book on catoptrics of his 'Perspectiva communis', Pecham claimed that stars, fixed or wandering, have light of their own, but also reflect light of the sun.[40] Moreover, he claimed that the twinkling of the stars is due to this reflection of solar light and the movement of the stars themselves.

[35] Kepler, Paralipomena ad Vitellionem, p. 251, translation in Ibid., p. 262.

[36] Kepler, Paralipomena ad Vitellionem, pp. 272-6, translation in Ibid., pp. 282-7.

[37] Kepler, Paralipomena ad Vitellionem, pp. 2252-6, translation in Ibid., pp. 263-8.

[38] Arafat, W., and H.J.J. Winter. 'The Light of the Stars - a Short Discourse by Ibn Al-Haytham.' *British Journal for the History of Science* 5 (1971): 281-88.

[39] For the scholastic discussion of the light of the planets and the stars, see Grant, Edward. *Planets, Stars, and Orbs: The Medieval Cosmos, 1200-1687*. Cambridge: Cambridge University Press, 1996, pp. 390-421; Ariew, Roger. 'The Phases of Venus before 1610.' *Studies in the History and Philosophy of Science* 18 (1987): 81-92.

[40] Lindberg, David C. *John Pecham and the Science of Optics*. Translated by David C. Lindberg. Madison, Milwaukee London: The University of Wisconsin Press, 1970, p. 209.

> For since stars are solid bodies of uniform surface, their surfaces must be reflective, and consequently they reflect solar rays. But since celestial bodies are continually moved, the angle of incidence is continually varied, and the resulting sensible variation in reflection produces a certain appearance of vibration.[41]

That the planets do not twinkle while the stars do, Pecham attributed to the difference in distance, affecting the angle of incidence and the angle of reflection of the light of the sun.

> Planets do not twinkle because they are nearby, for a solar ray falling on the body of a fixed star has a large angle of incidence, because of the remoteness of the star, and consequently a large angle of reflection. Therefore because of the elongation of the ray from the star, sight is somewhat capable of perceiving the difference between stellar light and solar light reflected from the star. Conversely, the angle formed by the incident and reflected rays at the surfaces of planetary bodies is smaller because they are nearby; consequently our sight does not distinguish between light of the planetary star itself and the solar light reflected from that star.[42]

Unlike Pecham, Witelo argued that all the stars, fixed and wandering, receive their light from the sun or from other stars. The fact that they did not show phases like the moon Witelo attributed to their distance that did not allow perceiving their shape.[43] Kepler in his 'Paralipomena' again criticized Witelo's argument. Kepler argued that planets and stars both shine with their own light.[44] First, Kepler excluded the possibility that the planets and stars are translucent, as some had claimed the moon to be. Kepler had studied comets, which he considered, like Apianus and Pena, as discussed in chapter 1, to be burning spheres. He had refined Apianus' and Pena's argument by showing the tail of the comet to be the caustic of such a refractive sphere.[45] As concerns the light of the planets and the stars, Kepler argued that if they would be translucent, they would show a tail not unlike a comet.[46] Second, Kepler's main argument against Witelo's position that planets and stars only reflect light was again the observational absence of the phases of Venus.[47] Since Venus did not show such phases, Venus had to have some light of its own. After Galileo had revealed that the telescope shows that Venus went through phases, Kepler admitted that he was much surprised, 'because, due to the enormous brightness of Venus, I was

[41] 'Cum enim stelle sint corpora solida equalis superficiei necesse est ut habeant superficies speculares; reflectunt ergo radios solares. Sed quia continue moventur corpora celestia variatur continue angulus incidentie, et per consequens reflexionis sensibilis variatio facit quandam vibrationis apparentiam'. Pecham, Perspectiva Communis, book II, Proposition 56, translation in Ibid., pp. 208-9.

[42] 'planete non scintillant quia prope sunt, radius enim solis cadens super corpus stelle fixe propter remotionem stelle facit angulum magnum incidentie stella et per consequens angulum magnum reflexionis, ita quod propter elongationem radii a stella potest visus advertere aliquo modo diversitatem luminis stellaris et solaris reflexi a stella. Econtra autem in corporibus planetarum quia prope sunt angulus minor est quem constituit radius incidentie et reflexionis cum superficie stelle, et propter hoc aspectus noster non distinguit inter lumen ipsius stelle et lumen solare reflexum a stella'. Pecham, Perspectiva Communis, book II, proposition 56, translation in Ibid., pp. 210-1.

[43] Witelo, Perspectiva, book X, proposition 55, in Lindberg, *Opticae Thesaurus*, pp. 449-50.

[44] Kepler, Paralipomena ad Vitellionem, pp. 260-3, translation in Kepler, *Optics*, pp. 271-4.

[45] Kepler, Paralipomena ad Vitellionem, pp. 264-7, translation in Ibid., pp. 274-8. For discussion, see Barker, Peter. 'The Optical Theory of Comets from Apian to Kepler.' *Physis* 30 (1993): 1-25, pp. 12-5.

[46] Kepler, Paralipomena ad Vitellionem, p. 260, translation in Kepler, *Optics*, p. 272.

[47] Kepler, Paralipomena ad Vitellionem, pp. 261-2, translation in Ibid., pp. 272-3.

of the opinion that it had its own light'.[48] As concerns the twinkling of the stars, in his 'De stella nova' (1607), on the appearance of the nova of 1604, Kepler argued that the rotating movement of the star itself caused the twinkling of the nova.[49] He also adduced a second explanation that the twinkling was caused by the movement of light at the firmament, much as looking through a trembling flame. Kepler's arguments were the starting point for Galileo's discussion of the light of the stars and the planets (and twinkling) in 1604.

Thus, at the beginning of the seventeenth century, when Galileo entered the field, the question of celestial light was a much-debated issue. Moreover, the observational absence of the phases of Venus was a key argument in the debate that adepts of the Ptolemaic as well as the Copernican system forced to adopt the self-luminosity of Venus, and, by extension, of all the planets and the stars.[50] Ptolemaic as well as Copernican astronomy required the self-luminosity of Venus to explain the observational absence of its phases. Both systems demanded that Venus shows phases if the planets only shine with reflected solar light. There were two variants of the Ptolemaic system, one in which Venus was considered to be always above the sun, and one in which Venus was always considered to be below the sun. In the former case, apparently the less popular one, Venus would hardly show phases, as it will appear almost always circular, with only a slighly gibbous phase when Venus approaches inferior conjunction. However, in the latter case, Venus presents a very outspoken phase cycle, although a different one from its phase cycle in the Copernican system. In the Ptolemaic system, with Venus always below the sun, Venus would show a cycle of varying crescent phases, tending toward semicircularity at maximum elongation, but never reaching semicircularity. In the Copernican system, Venus is sometimes above and sometimes below the sun, showing a full pattern of phases from circularity at superior conjection over gibbous and semicircularity to crescent at inferior conjunction. Consequently, if Galileo was a Copernican even before 1610, it should have rather forced him to argue that planets shine with their own light, rather than only reflect the light of the sun.[51] However, as will be shown, Galileo's position around 1604 was a rather different one than self-luminosity.

3. Leonardo's Eye, Irradiation and the Light of the Planets and the Stars

It has escaped notice that Galileo had already an outspoken opinion on the light of the planets and the stars around 1604 that was quite different from his eventual position on celestial light in 1611. His lecture notes and publications following the appearance of a nova in 1604 provide the context in which Galileo discussed celestial light. In his three public lectures, Galileo argued against

[48] 'nam propter ingentem claritatem Veneris opinabar proprium in illa lumen inesse'. Kepler to Galileo, 28 March 1611, in Galileo, *Opere,* Vol. 11, p. 78. For discussion of the Kepler-Galileo correspondence, see Chevalley, Catherine. 'Kepler et Galilée dans la Bataille du 'Sidereus Nuncius' (1610-1611).' In *Optics in Sixteenth-Century Italy*, edited by Paolo Galluzzi, 167-75. Firenze: Giunti Barberà, 1984.

[49] Kepler, De stella nova, chapter XVIII, in Kepler, Johannes. *Gesammelte Werke*. Edited by Max Caspar. Vol. 1. München: C.H. Beck'sche Verlagsbuchhandlung, 1938, pp. 241-5.

[50] Goldstein, Bernard R. 'The Pre-Telescopic Treatment of the Phases and Apparent Size of Venus.' *Journal for the History of Astronomy* 27 (1996): 1-12; Palmieri, Paolo. 'Galileo and the Discovery of the Phases of Venus.' *Journal for the History of Astronomy* 32 (2001): 109-29, in particular, pp. 109-17.

[51] For the problem whether Galileo was a Copernican prior to 1610, see Drake, Stillman. 'Galileo's Steps to Full Copernicanism and Back.' *Studies in the History and Philosophy of Science* 18 (1987): 93-105.

convential Aristotelian cosmology that the nova was well situated beyond the lunar sphere based on the absence of parallax. Also concerning the nature of the nova, Galileo's opinion diverged from Aristotelian cosmology. As discussed in chapter 1, Aristotelian cosmology considered comets, with which novas were traditionally assimilated, slowly burning vapors arising from a sphere of fire. For Galileo, the nova was also made of exhalations or vapors rising from the earth to the skies well above the moon, but these vapors were not slowly burning.[52] The nova was visible by its reflection of solar light. That Galileo was influenced by sixteenth century optics for his view that the heavens were subject to change and his denial of the Aristotelian origin of the nova is evident from his reference to 'the view of Jean Pena, namely that the air extends all the way to the fixed stars, which is also the view of Pythagoras and the Stoics, who believed that such celestial phenomena could be created'.[53] Galileo was well acquainted with the cosmological discussions that arose out of sixteenth century optics. For example, in his 'Le operazioni astronomiche', Galileo discussed the tables of atmospheric refraction of Tycho Brahe, about the precision of which he was sceptical, in particular, for low altitudes.[54] However, he conceded with the Tychonian view, rejected by Kepler, that both the earth's atmosphere and terrestrial vapors close to the earth are responsible for the refractions.[55]

That Galileo considered the nova to shine with reflected solar light might have made him highly critical of the view that stars shine with their own light. In his lecture notes, Galileo referred to Kepler's argument that the twinkling of the nova was caused by the rotating movement of the star itself.[56] As shown, Kepler also adduced a second explanation of the twinkling in his 'De stella nova'. In the 'Considerations of Alimberto Mauri' (1606) attributed to Galileo, he referred to this second explanation, and subsequently rejected it. The position of 'another worthy man', to which Galileo referred, is, without doubt, the position of Kepler in his 'De stella nova'.

> Another worthy man thought that twinkling was caused by movable bodies between us and the eighth heaven. For, he said, if we look through a large fire at an object behind it, then because the fire is mobile and trembling, the object also appears to us vacillating and movable, as our sight passes in this way through this variety of motions. Thus the stars of the firmament may very well appear to tremble and twinkle. Truly a very subtle idea, and one that at first glance contains much of the probable. Yet consider that if it is so,

[52] Galileo, Frammenti di lezioni e di studi sulla nuova stella dell' ottobre 1604, in Galileo, *Opere*, Vol. 2, pp. 277-84.

[53] 'opinionem Iohannis Penae, unam esse aëris continuationem usque ad fixas, et Pythagorae et Stoicorum, qui in caelo talia creari posse existimarunt'. Galileo, Frammenti di lezioni e di studi sulla nuova stella dell' ottobre 1604, in Galileo, *Opere*, Vol. 2, p. 284. For the neostoic context of Galileo's discussion of the nova, see Reeves, *Painting the Heavens: Art and Science in the Age of Galileo*, pp. 57-90.

[54] 'Il negozio delle rifrazzioni resta per ancora appresso di me assai ambiguo, nè vi so discernere precisione alcuna, fondata sopra stabili e certe osservazioni: e veramente confesso di non restar capace come la struttura delle tavole di esse rifrazzioni, portata come assai risoluta da Ticone, sia veramente tanto sicura, che di essa si possa fare assoluto capitale nel calcolare le elevazioni delle stelle, in particolare ne' luoghi non molto alti sopra l' orizzonte'. Galileo, Le Operazioni Astronomiche, in Galileo, *Opere*, Vol. 8, p. 461.

[55] 'Da questa osservazione mi pare che si possa, in certo modo, introdurre due sorti di refrazioni: cioè, la prima, fatta dal grand' orbe vaporoso, che circonda quasi che immutabilmente la Terra, mercè del quale nascono i crepuscoli; e l' altra sia effetto d' altri più grossi vapori, che in minor altezza si distendono sopra qualche parte del globo terrestre e che forse non si elevano più in alto che sormentino gli altri vapori grossi, circoscrivendo quella parte vicina dove si producono le nuvole, le pioggie, i venti etc.'. Galileo, le operazioni astronomiche, in Galileo, Opere, Vol. 8, p. 461.

[56] 'Kepplerus, De stella nova, car. 95, de scintillatione ait, fieri posse ex rotatione fixarum'. Galileo, Frammenti di lezioni e di studi sulla nuova stella dell' ottobre 1604, in Galileo, *Opere*, Vol. 2, p. 280.

> [the disturbance] will necessarily happen either through the diurnal movement [of the skies] or through the proper motions of the planets.[57]

Galileo rejected both. He argued that, on the one hand, also the sphere of the fixed stars participates in the diurnal movement of the skies, and that, on the other hand, the movement of the planets is to slow to cause the rapid twinkling of the stars.[58] Instead, Galileo attributed the twinkling of the stars to the reflection of solar light. Moreover, he adopted the more general position of Witelo about the question of celestial light, specifically rejected by Kepler. Galileo claimed that the stars reflect light from the sun, just as the moon reflects solar light.

> Indeed, I am of the opinion that the whole effect should be attributed to the sun, which is very distant, striking with its light thinly and weakly on the stars, which Witelo says receive their light from this no otherwise than does the moon. Rays are contributed to these by sun-blows, so to speak, whence is caused that weakness, like panting or tired breathing.[59]

As concerns the light of the moon, it has already been pointed out in chapter 5 that already in his 'Considerations of Alimberto Mauri', Galileo disagreed with the Aristotelian perfection of the moon, arguing for its rough surface instead on the basis of its sinuous terminator. Moreover, Galileo argued that 'there arise in the moon scabby little darknesses, because greatly curved mountains (as Perspectivists teach) cannot receive and reflect the light of the sun as does the rest of the moon, flat and smooth'.[60] Thus, Galileo claimed that the dark spots on the moon are mountains, while the brighter part of the moon is flat and smooth. Galileo's explanation of the light of the moon was the first among the questions of celestial light to be subjected to a change.
In his 'Sidereus Nuncius', Galileo again argued that the moon does not shine with light of its own, but only reflects solar light. Therefore, Galileo had to explain the ashen light of the moon, not as the moon's own light, but as a reflection of solar light by the earth to the surface of the moon.

> Since, therefore, this secondary light is not intrinsic and proper to the Moon, and is borrowed neither from any star nor from the Sun, and since in the vastness of the world no other body therefore remains except the Earth, I ask what are we to think? What are we to propose – that the lunar body or some other dark and gloomy body is bathed by light from the Earth? But what is so surprising about that? In an equal and grateful exchange the Earth pays back the Moon with light equal to that which she receives from the Moon almost all the time in the deepest darkness of the night.[61]

[57] Galileo, Considerations of Alimberto Mauri, translation in Drake, Stillman. *Galileo against the Philosophers in His Dialogue of Cecco Di Ronchitti (1605) and Considerations of Alimberto Mauri (1606)*. Translated by Stillman Drake. Los Angeles: Zeitlin & Ver Brugge, 1976, pp. 90-1.

[58] Ibid., p. 91

[59] Ibid., p. 91.

[60] Ibid., p. 104.

[61] 'Cum itaque eiusmodi secundarius fulgor nec Lunae sit congenitus atque proprius, nec a Stellis ullis nec a Sole mutuatus, cumque iam in Mundi vastitate corpus aliud supersit nullum, nisi sola Tellus, quid, quaeso, opinandum? Quid proferendum? Nunquid a Terra ipsum lunare corpus, aut quidpiam aliud opacum atque tenebrosum lumine perfundi? Quid mirum? Maxime: aequa grataque ermutatione rependit Tellus parem illuminationem ipsi Lunae, qualem et ipsa a Luna in profundioribus noctis tenebris toto fere tempore recipit'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 74, translation Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 55.

Reeves has argued that Galileo arrived at this explanation of the secondary light of the moon prior to his telescopic observations, in 1607, at the same time his friend Cigoli depicted the moon's ashen light in his 'Deposition'.[62] Moreover, she has shown that Galileo arrived at this explanation of the secondary light of the moon after a study of reflection together with Sarpi, who himself prior to 1606 had found the earthly nature of the moon's ashen light.[63] Consequently, the basis of Galileo's discussion of reflection, mirrors and the light of the moon, eventually published in the 'Dialogo' (1632) was already established in 1607. In particular, Reeves has stressed the influence of Leonardo's notebooks, which are known to have circulated in the circle of Pinelli as well among Florentine painters, like Cigoli, on Galileo's discussion of the light of the moon.[64] I will further develop Reeves' argument, showing that during Galileo's study of reflection and the lunar light of the moon in 1607, he was confronted with the phenomenon of irradiation that would prove very infuential to his application of the diaphragm to the objective lens of his telescope and his continued exploration of celestial light. Moreover, I will argue that Galileo arrived at an understanding of irradiation similar to the one found in Leonardo's notebooks.
Leonardo's notebooks show an analysis of the reflection of solar light in water, comparing the single image of the sun in the water to the multiple radiance from the ruffled surface of the sea.[65] (Figure 7.2)

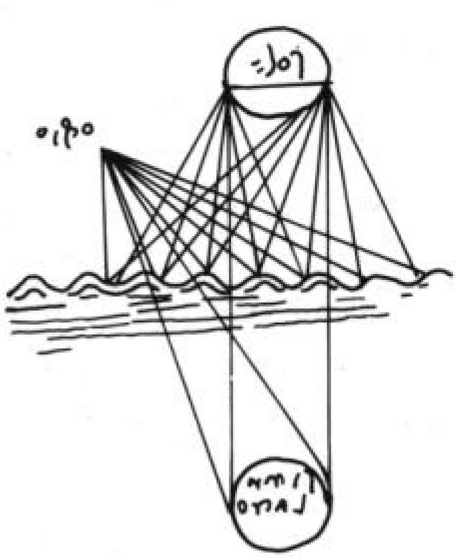

Figure 7.2

[62] Reeves, *Painting the Heavens: Art and Science in the Age of Galileo*, pp. 125-37.

[63] Ibid., pp. 104-12.

[64] Ibid., pp. 29-34, 113-8.

[65] For discussion, see Kemp, Martin. *Leonardo da Vinci: The Marvellous Works of Nature and Man*. Cambridge, Massachusetts: Harvard University Press, 1981, pp. 324-5

Leonardo applied his knowledge of reflection by rough surfaces to the light of the moon. He argued that the moon did not reflect the solar light as a smooth and polished mirror. Instead, the moon's surface consisted of areas of land and sea, and the rough surface, considered to be the better reflector, caused by the waves of the moon's seas reflects the light of the sun.

> The surface of the moon is furrowed; and this roughness only exists in liquid bodies when they are stirred by the wind, as we have seen with the sea how the sun is reflected by tiny waves near to the eye, and stage by stage over a distance of more than forty miles these illuminated waves grow larger. Wherefore we conclude that the luminous part of the moon is water, which if it were not in movement would not be luminous to the same degree; but by the movement of this water which has been stirred up by the winds it becomes filled with waves; and every wave takes the light from the sun; and the great multitude of waves beyond number reflect the solar body an infinite number of times.[66]

Leonardo's passage is in disagreement with Galileo's discussion of the light of the moon in his 'Considerations of Alimberto Mauri'. On the one hand, both Leonardo and Galileo agreed that the moon only shines with reflected solar light. On the other hand, while Leonardo considered a rough surface, caused by the waves of the seas, a better reflector than a flat and smooth surface, Galileo argued that a rough surface, caused by mountains, is a worse reflector than a flat and smooth surface. However, contrary to his earlier opinion in the 'Considerations of Alimberto Mauri', Galileo adopted Leonardo's analysis of reflection in his 'Dialogo', when he argued that a smooth surface like a mirror is not as good a reflector as a rough surface. Consequently, he claimed that 'what is clearly seen in the moon is that the darker parts are all plains, with few rocks and ridges in them, though there are some. The brighter remainder is all full of rocks, mountains, round ridges, and other shapes'.[67] Only when it is established that rough surfaces are better reflectors, the moon's ashen light can be thought of as the reflection of solar light by a rough surface like the earth. Thus, in 1607, Galileo changed his opinion on reflection.

It is difficult to establish to what extent Leonardo's notes influenced Galileo's change of his conception of reflection from 1606 to 1607. There is no doubt that Galileo knew notes of Leonardo. In the 'Dialogo', he remarked that 'others enjoy all the precepts of da Vinci, and yet do not know how to paint a stool', no doubt, a reference to one of the compilations of notes of Leonardo.[68] However, they were different compilations of notes circulating, and it is not known to what kind of compilation that Galileo had access. The above quoted note of Leonardo on the light of the moon might not have been a part of the compilation of notes seen by Galileo. However, Galileo certainly had access to some of Leonardo's numerous notes on reflection, that were the important core of Leonardo's 'Trattato di Pittura' that was the most widely diffused part of Leonardo's notes. Galileo's change of opinion on reflection in 1607 was based on contemporaneous observations of mirror reflections that were particularly Leonardo-like.

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[66] MacCurdy, Edward, *The Notebooks of Leonardo Da Vinci*. Vol. 1. New York: Reynal & Hitchcock, 1938, p. 313.

[67] 'Quello che si vede manifestamente nella Luna è che le parti più oscure son tutte pianure, con pochi scogli e argini dentrovi, ma pur ve ne son alcuni: il restante più chiaro è tutto pieno di scogli, montagne, arginetti rotondi e di altre figure'. Galileo, Dialogo sopra i due massimi sistemi del mondo, in Galileo, *Opere*, Vol. 7, p. 125, translation in Galileo, *Dialogue Concerning the Two Chief World Systems - Ptolemaic & Copernican*, p. 99.

[68] 'altri posseggono tutti i precetti del Vinci, e non saprebber poi dipignere uno sgabello'. Galileo, Dialogo sopra i due massimi sistemi del mondo, in Galileo, *Opere*, Vol. 7, p. 60, translation in Ibid., p. 35.

In his 'Dialogo', Galileo made an observation of the reflection of light by a mirror that was particularly important to establish that smooth surfaces are not good reflectors. Therefore, this must have been a part of Galileo's and Sarpi's observations of 1607. Galileo noted that the image of the sun reflected in a mirror is very tiny. Thus, the moon cannot reflect light like a smooth mirror.[69] There appears to be nothing exceptional about this argument, because, as already discussed, that only a limited part of the reflection of light reaches the eye of the observer was a standard argument of the medieval commentators on Aristotle to establish that the moon is not a smooth mirror. However, Galileo's argument was much more sophisticated, because he took into account irradiation, considered to be an effect of the eye. Initially, a mirror, a smooth and flat surface, appears to be a good reflector, just as Galileo had argued in the 'Considerations of Alimberto Mauri'. However, Galileo argued that while a mirror appears to reflect much light, most of this light is not due to the primary reflection of the incident rays of the sun on the mirror, but to irradiation. Thus, if irradiation is taken into account, a mirror is not a good reflector.

> Let us take a very large gilded plate; it will show to a distant eye the image of the sun occupying only a part of the plate, that from which the reflection of the incident rays comes. It is true that on account of the vividness of the light such an image would appear crowned with many rays, and would therefore seem to occupy a much larger part of the plate than it really did. To verify this, one might note the exact place on the plate from which the reflection came, and likewise figuring how large the shining space appears, cover the major part of this space leaving only the middle revealed; the size of the apparent brilliance would not be a whit diminished, but it would be seen widely spread over the cloth or material used for the covering.[70]

Consequently, if the reflected light is stripped of its irradiation, the actual image of the sun in the mirror is small. Galileo argued that at a distance this small image is not seen. Thus, if the moon reflects light like a mirror, it would be invisible to us, and, consequently, the moon cannot reflect light like a mirror. Thus, he argued that 'if then the moon were smooth as a mirror, only a very small part would show itself to the eyes of a particular person as illuminated by the sun, although an entire hemisphere would be exposed to the sun's rays. The rest would remain, to this observer's eyes, unilluminated and therefore invisible. To conclude, the whole moon would be invisible, since that particle which gave the reflection would be lost by reason of its smallness and great distance'.[71] Galileo's procedure in the 'Dialogo' suggests that the irradiation is an effect of the eye, and not a direct reflection of light by the mirror. In his 'Trattato di Pittura', Leonardo

[69] Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, pp. 99-100, translation in Ibid., pp. 74-5.

[70] 'E per meglio dichiararmi, intendasi una piastra dorata piana e grandissima esposta al Sole:mostrerarsi a un occhio lontano l' immagine del Sole occupare una parte di tal piastra solamente, cioè quella donde viene la reflessione de i raggi solari incidenti; ma è vero che per la vivacità del lume tal immagine apparirà inghirlandata di molti raggi, e però sembrerà occupare maggior parte assai della piastra che veramente ella non occuperà. E che ciò sia vero, notato il luogo particolare della piastra donde viene la reflessione, e figurato parimente quanto grande mi si rappresenta lo spazio risplendente, cuoprasi di esso spazio la maggior parte, lasciando solamente scoperto intorno al mezo: non però si diminuirà punto la grandezza dell' apparente splendore a quello che di lontano lo rimira, anzi si vedrà egli largamente sparso sopra il panno o altro con che si ricoperse'. Galileo, Dialogo sopra i due massimi sistemi del mondo, in Galileo, *Opere*, Vol. 7, pp. 103-4, translation in Ibid., p. 79.

[71] 'Quando dunque la Luna fusse tersa come uno specchio, piccolissima parte si mostrerebbe a gli occhi di un particulare illustrata dal Sole, ancorchè tutto un emisferio fusse esposto a' raggi solari, ed il resto rimarrebbe all' occhio del riguardante come non illuminato e perciò invisibile, e finalmente invisibile ancora del tutto la Luna, avvenga che quella particella onde venisse la riflessione, per la sua piccolezza e gran lontananza si perderebbe'. Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, pp. 99-100, translation in Ibid., p. 75.

made similar observations. Leonardo made a difference between light, the direct reflection of light on a smooth surface, and luster, generated by the eye of the observer.

> The difference between luster and light is that luster is always more powerful than light, and light is of greater extent than luster. Luster moves with the eye, or with its cause, or with both, but light is fixed in a definite place, if the cause that creates it does not move.[72]

As Galileo noted, the image of the sun in a mirror appears larger than it is, because it is 'crowned with many rays'. In a convex mirror, which Galileo used as his primary instrument because it is comparable to the convex surface of the moon, the sun appears much like a star. What Galileo and Leonardo noted is that a bright object spreads its light over its surroundings, and, consequently, it will appear larger than it actually is. The effect is particularly noticeable with a bright object against a dark background, like the flame of a candle in the dark or a star. In the nineteenth century, Helmholtz attributed this irradiation to the inequalities in the parts of the human eye, like the crystalline lens, that diffuse the incident light in all directions.[73] As will become evident, Leonardo's and Galileo's explanation of irradiation is somewhat different, although they both considered it to be an effect of the eye. In his 'Dialogo', Galileo explicitly identified the sun's image crowned with rays in a mirror with the apparent rays around a star.

> First of all, that brilliance which you see so vividly on the mirror, and which seems to you to occupy such a large part of it, is not such a big piece. It is really very tiny, but its extreme brightness causes an adventitious irradiation of your eyes … It is like the little hat that seems to be seen around the flame of a candle at some distance; or you may want to compare it with the apparent rays around a star.[74]

In his 'Trattato di Pittura', Leonardo argued that luster around bright objects and the reflections of images in a mirror is only seen when the pupil of the eye is larger than the object. Therefore, if the object is larger or matches the size of the pupil, luster would not be noted.

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[72] 'La differenzia ch' è dal lustro al lume, è che sempre il lustro è più potente che 'l lume, et il lume è di maggiore quantità che 'l lustro; e 'l lustro si move insieme co' l' occhio o colla sua causa, o co' l' uno o co' l' altra; ma il lume è stabilito al loco terminato, non rimovendosi la cause che lo genera'. Leonardo da Vinci. *Libro di Pittura*. Carlo Predetti ed. Vol. 9, *Biblioteca della Scienza Italiana*. Firenze: Giunti, 1995, p. 446. Translation in Da Vinci, Leonardo. *Treatise on Painting [Codex Urbinas Latinus 1270]*. Translated by A. Philip McMahon and Ludwig H. Heydenreich. Princeton, New Jersey: Princeton University Press, 1956, p. 261.

[73] 'When even the healthiest human eye is examined by powerful light, the best being a pencil of sunlight concentrated on the side by a condensing lens, it is seen that the sclerotica and crystalline lens are not perfectly clear. If strongly illuminated, they both appear whitish and as if rendered turbid by a fine mist. Both are, in fact tissues of fibrous structure, and are not therefore so homogeneous as a pure liquid or a pure crystal. Every inequality, however small, in the structure of a transparent body can, however, reflect some of the incident light– that is, can diffuse it in all directions'. Warren, Richard M., and Roslyn P. Warren. *Helmholtz on Perception: Its Physiology and Development*. New York: John Wiley & Sons, Inc., 1968, p. 160, taken from Helmholtz, Popular Scientific Lectures, 1871-1873. I would like to thank Marc De Mey for pointing me to Helmholtz' discussion of irradiation.

[74] 'E prima, quello splendore così vivo che voi vedete sopra lo specchio, e chi vi par che ne occupi assai buona parte, non è cosi grande a gran pezzo, anzi è piccolo assai assai; ma lua sua vivezza cagiona nell' occhio vostro … una irradiazione avventizia, simile a quel capillizio che ci par di vedere intorno alla fiamella di una candela posta alquanto lontana, o vogliate assimigliarla allo splendore avventizio di una stella'. Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, p. 101, translation in Galileo, *Dialogue Concerning the Two Chief World Systems,* p. 76.

> Among the lusters created on spheres equally distant from the eye, that will have the smallest shape which is created on the sphere of least size. See how on little grains of quicksilver, which are almost imperceptible in quantity, the lusters are equal in size to the grains, and this occurs because the visual power of the pupil of the eye is larger than the little grain, and as has been stated surrounds it for that reason.[75]

Leonardo considered the pupil to be the cornea, the complete front surface of the eye, if not limited by narrowing the eyelids or by looking through a cardboard pinhole, that receives images in the same way a convex mirror receives images.[76] Leonardo is not an isolated case in considering the cornea receptive to images comparable to a convex mirror. In his commentary on Vignola's 'Le Due Regole della Prospettiva Practica' (1583), Danti considered the eye along the same lines, stating that the cornea, that functions like a mirror, of which the back is darkened by the uvea, first received the image.[77] Consequently, Leonardo's concept of the eye and vision was well known at the end of the sixteenth century, no doubt also to Galileo, even if it would not have been explicitly present in the compilation of Leonardo's notebooks to which Galileo had access. This concept of the eye led Leonardo to the idea that as the aperture of the eye contracts, by narrowing the eyelids or artificially by looking through a pinhole aperture, bright objects look smaller, and as it expands, they look larger.[78] A diagram, showing three stars a, b and c in the night sky, illustrates this point. (Figure 7.3)

[75] 'Delli lustri generati sopra li sperici equalmente distanti dall' occhio, quallo sarà di minore figura, che si genererà sopra sperico di minore grandezza. Vedasi ne' graniculi de l' argento vivo, li quali son quasi di quantità insensibili, li loro lustri essere equali alla grandezza d' essi grani; e questo nasce ché la virtù visiva della popilla è maggiore d' esso graniculo, e per questo lo circonda com' è detto'. Leonardo da Vinci, *Libro di Pittura*, p. 446, translation in Leonardo da Vinci, *Treatise on Painting [Codex Urbinas Latinus 1270]*, p. 261.

[76] For Leonardo's theory of the eye, in particular, the pupil, and vision, see Kemp, Martin. 'Leonardo and the Visual Pyramid.' *Journal of the Warburg and Courtauld Institutes* 40 (1977): 128-49; Ackerman, James S. 'Leonardo's Eye.' *Journal of the Warburg and Courtauld Institutes* 41 (1978): 108-46. See also Keele, K. D. 'Leonardo Da Vinci on Vision.' *Proceedings of the Royal Society of Medicine* 48 (1955): 384-90; Keele, Kenneth D. *Leonardo Da Vinci's Elements of the Science of Man*. New York: Academic Press, 1983, pp. 201-14.

[77] Danti, E. *Le Due Regole della Prospettiva Practica: A Reproduction of the Copy in the British Library*. Alburgh: Archival Facsimiles Limited, 1987, pp. 11-3. For discussion, see Frangenberg, Thomas. 'Egnatio Danti's Optics: Cinquecento Aristotelianism and the Medieval Tradition.' *Nuncius* 1 (1988): 3-38, pp. 12-20.

[78] Leonardo, MS F, f. 32v. For discussion, see Ackerman, 'Leonardo's Eye', pp. 133-4.

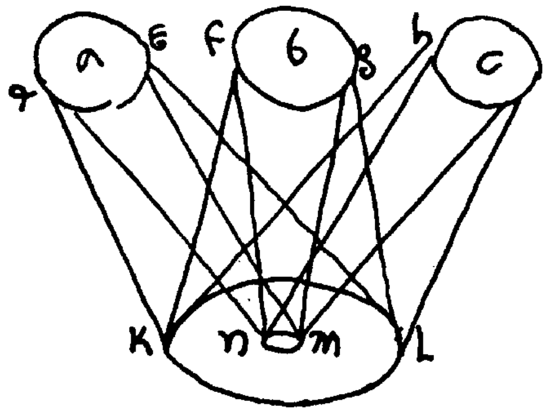

Figure 7.3

If they are observed with the whole eye kl they look proportionally bigger than if observed through the pinhole aperture nm in a piece of cardboard that allows you to see as much of the night sky, but over a more restricted area of the cornea.

In MS D, Leonardo attributed the cause of the irradiation shown by bright objects, apparently enlarging the size of a star, to reflections bouncing off the eyelids to the eye's pupil that creates a diffusive light similar to the diffusive light created by the reflections of the waves of the sea in different directions in Leonardo's discussion of the solar light reflected by the rough seas of the moon. Leonardo asked 'why luminous bodies reveal that their contours are full of straight rays'.

> Rays which reveal the limits of luminous bodies do not originate from these bodies but from their simulacra which imprint themselves upon the thickness of the lids of the eyes that look upon these bodies. This is proved first in an obvious way as the eye when wide open does not show us such rays around luminous bodies, and that if the simulacrum of a star or other light should pass to the eye through a very small hole made in a paper placed before the eye, such luminous bodies would always be without rays. But the true proof is shown by the ninth [proposition] of Perspective in which it is stated: 'the angle of incidence is always equal to the angle of reflection'. The rays, therefore, which seem as though they extend from the luminous body and make contact with the eye that sees them, originate with the eye, being almost closed, looks through the slit that intervenes between the eyelids at the luminous body of which the simulacrum is reflected in the thickness of the edges which terminate the eyelids. After making such an impression the image is then reflected to the pupil of the eye which thus receives three simulacra from the same luminous body, i. e., two in the thick parts of the lids of the coverings of the eye and one in the pupil, and because these three simulacra are so near to one another, they seem to the eye to be continuous and joined to the simulacrum of the pupil.[79]

Thus, Leonardo argued that by looking through a pinhole aperture, stars look smaller because the pupil of the eye is restricted and, consequently, the rays under non-pinhole conditions reflected

[79] Leonardo, MS D, f. 9v, in Strong, Donald Sanderson. *Leonardo on the Eye: An English Translation and Critical Commentary of Ms. D in the Bibliothèque Nationale, Paris, with Studies on Leonardo's Methodology and Theories on Optics.* New York London: Garland Publishing, Inc., 1979, pp. 84-5.

by the eyelids cannot cause the stars' surrounding irradiation. Leonardo's eyelid-theory is not common, but also not completely original. Roger Bacon had formulated a similar theory.

> When the eye views a candle and lowers the eyelashes, it sees the candle project rays in the form of a radiant pyramid whose vertex is in the candle, and the dispersion of the rays is quite sensible to the eye. The reason for this is found in the fact that the rays of the candle fall on the lids and the lashes are polished, possessing the property of a mirror, and a reflection for this reason takes place from them to the eye, when they are so inclined that the eye is able to receive the rays reflected at angles equal to those of incidence. Therefore this phenomenon does not occur in any position at all of the lashes, but only in one determined.[80]

However, beside that Bacon appears to consider the eyelashes primarily responsible for the reflection causing irradiation, while Leonardo refers to eyelids, elswhere in the MS D, Leonardo gave a much more specific explanation of irradiation that has no equivalent in Bacon.[81] Leonardo attributed the cause of irradiation to the rays reflected from the moisture at the pupil that builds up there by capillary action. He considered this moisture equivalent to concave mirrors.

> And in drawing the eyelids together it is necessary that the watery substance, which keeps the lids continually moistened as they rub upon the eye, fills up the angle produced by the contact of the lids with the pupil. And the surface of this watery substance is concave … Thus as such an angle is found to be created by the contact between the eyelid and the pupil, it will be filled with a concave watery surface. 'And since every concave mirror shows within the pyramidal concourses of its rays the image of its object upside down ['] it follows therefore that the eyelashes or lids of the eye mirrored within this concavity together with the simulacrum of the luminous body will show the upper and lower eyelids inverted; and this is why the pupil, being within the concourse of the pyramidal rays of a concave mirror, sees the the pyramids of rays in the spaces between the lids inverted. And this is the true cause of why the rays of luminous bodies seem to enlarge as they near the eye.[82]

Leonardo's adjacent diagram does not show the invertion of the image by the 'concave watery surfaces' of which the reflections cause irradiation. (Figure 7.4)

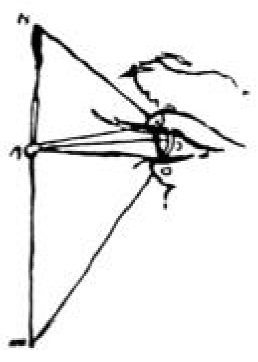

Figure 7.4

[80] Roger Bacon, Opus Majus, V. III. V, quoted from Ibid., p. 98.

[81] For Leonardo's discussion of the cause of irradiation, see Ibid., pp. 97-101, 128-9, 190-5.

[82] Leonardo, MS D, f. 1v, in Ibid., p. 43.

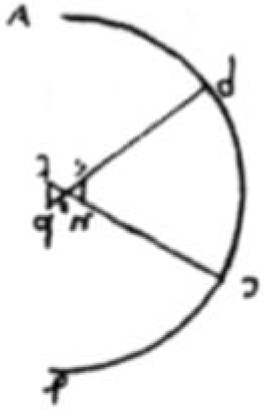

Figure 7.5

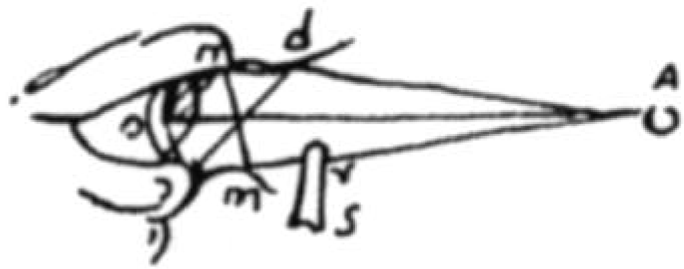

Figure 7.6

However, elsewhere in the MS D, Leonardo appears to have studied this point of inversion of a concave mirror, and another of his diagrams shows it applied to the reflections bouncing of the moisture at the eyelids.[83] (Figure 7.5 – Figure 7.6) Thus, Leonardo considered irradiation caused by the reflections bouncing off the moisture added up at the pupil by the eyelids, similar to concave mirrors. Since he argued, in agreement with sixteenth century optics, that the whole pupil was sensitive, acting like a convex mirror, the irradiation was not noted when either the direct image of an object matches the aperture of the eye or this aperture is artifically narrowed by looking through a pinhole.

There is evidence that such considerations were part of Galileo's observations of reflections from mirrors and the light of the moon in 1607. Two correspondents of Galileo who were around in 1607 noted the observations that the reflection of the sun in a convex mirror is actually very small. This corresponce is of a later date, but it should be noted that Galileo had not interferred by publishing the observation. It is not present in the 'Sidereus Nuncius'. First, in a letter to Leschassier of 27 April 1610, Sarpi wrote to Leschassier, after Galileo and Sarpi had lost contact after the publication of the 'Sidereus Nuncius' in March 1610, that 'when you place a round stone and a spherical mirror of the same size as the stone in front of the sun, far removed from you, you will see the hemisphere (of the stone) illuminated, while you see the whole of the mirror dark,

[83] Leonado, MS D, f. 3r; MS F, f. 30r. See Ibid., p. 99.

except for a very small part, where you 'll see a small sun'.[84] Second, in a letter of 2 September 1611, Daniello Antonini, a former student of Galileo around 1607 before he left for Flanders, and well acquainted with the Sarpi circle, mentioned to Galileo that he had 'never been convinced that the moon has a smooth surface, because we never would see its whole face illuminated, but only a little sun reflected in it, like is seen in convex mirrors'.[85] Consequently, the basis of Galileo's observations of the 'Dialogo' (1632) must be dated to around 1607.
This observation allowed to conclude that the moon only shines with reflected solar light, but not that planets shine with reflected light, while stars shine with their own light. The difference between planets and stars eluded Sarpi around 1578, in a passage in which he specifically compares the light of the stars with the light of the moon, based on his knowledge of mirrors.

> If the stars would receive their light from the Sun, being smooth and opaque, they will not only be very clearly seen by day, but the Sun will be seen in them. Neither can that they are not seen be attributed to their smallness, because every mirror of which the distance is proportional to its size is seen, and in it the Sun is seen; but the sizes of the stars are proportional to their distances, since they are seen by night. Neither can it be said that they are opaque but not smooth, because they will be seen like the Moon, notwithstanding their smallness, for the mentioned reason. Thus, they have their own light and therefore, because they are small, they can be occultated by the great light of the Sun, because both operating on the eye, the weak action of the smaller light is not sensible.[86]

Consequently, from his observations, equally based on the actual small size of the image of the sun in a mirror, Sarpi was able to conclude, otherwise than Galileo in 1606, that the stars shine with their own light, but he did not make a difference between stars and planets. However, from his study of mirrors and reflections, in the context of his investigations of the light of the moon, Galileo knew that looking through a pinhole might show stars without irradiation. Consequently, Galileo's study of the light of the moon in 1607 lead to the application of the diaphragm to his telescope, in order to further investigate the question of celestial light, with which, as has been shown, Galileo was already involved in 1604. From his acquaintance with the problem of celestial light, Galileo, or anyone else, could not predict that the application of a diaphragm to his telescope would allow seeing a difference between planets and stars, but he did know that a diaphragm might be able to strip irradiation of both. The diaphragm allowed seeing a difference between the stars and the planets, not noticeable with the naked eye or with a poor telescope

[84] 'si soli eminus a te obicias globum lapideum, ac globosum speculum magnitudinis eiusdem lapidis, hemispherium videbis illustre, totum speculum vero videbis obscurum, praeter eius particulam quandam minimam, in qua parvum quendam solem inspicies.' Sarpi to Leschassier, 27 April 1610. Sarpi, *Lettere ai Gallicani*, p.81.

[85] 'Insoma non mi son io mai persuaso che la luna sia di superficie liscia et pulita, perchè non potressimo mai vedere tutta la faccia di quella illuminata, ma vi vederessimo dentro un picciol sole reflesso, sicome ne' specchi convessi si suol vedere.' Antonini to Galileo, 2 September 1611. In Galileo, *Opere*, Vol. 11, p. 204. On Antonini's contacts with the circle of Sarpi, see the letter of Micanzio to Galileo, 26 February 1611, in Galileo, *Opere*, Vol. 11, p. 57.

[86] 'Se le stelle ricevessero lume dal Sole, sendo terse ed opache, non solo gagliardamente si vedrebbon ellono di giorno, ma il Sole in lor si vedrebbe. Né alla loro piccolezza si può ascrivere il non vederle: perché ogni specchio la cui distanza sia proporzionale alla grandezza vedesi, e in lui si vede il Sole; ma la grandezza delle stelle si è alla distanza lor proporzionale, poiché si veggono di notte; dunque. Né tampoco si può dire che sien opache, ma non terse, perché si vedrebbon come la Luna, non ostante la lor piccolezza, per la detta ragione. Son ellono dunque lumi propi e così possono, come piccoli, venire occultati dal gran lume del Sole, perché, perando amendue nell' occhio, non resta sensibile l' azion debole del minor lume'. Sarpi, *Pensieri Naturali, Metafisici e Matematici*, pp. 38-9.

without a diaphragm, making it possible to revise his earlier views on the light of the planets and the stars. In his 'Sidereus Nuncius' (1610), Galileo noted that his telescope resolved the planets, unlike the fixed stars, into disk-like little globes, similar to the appearance of the moon.

> The difference between the appearance of the planets and fixed stars also seems worthy of notice. For the planets present entirely smooth and exact circular globes that appear *as little moons*, entirely covered with light, while the fixed stars are not seen bounded by circular outlines but rather as pulsating all around with certain bright rays.[87]

As to the explanation of the working of his telescope, Galileo referred to the irradiation that made stars observed with the naked eye much larger, and, consequently, because his telescope allowed to see them without irradiation, they were not magnified to the same extent as the moon.

> And first, it is worthy of notice that when they are observed by means of the spyglass, stars, fixed as well as wandering, are seen not to be magnified in size in the same proportion in which other objects, and also the Moon herself are increased. In the stars, the increase appears much smaller so that you may believe that a glass capable of multiplying other objects, for example, by a ratio of 100 hardly multiplies stars by a ratio of 4 or 5. The reason for this is that when the stars are observed with the naked eye, they do not show themselves according to their simple and, so to speak, naked size, but rather surrounded by a certain brightness and crowned by twinkling rays, especially as the night advances. Because of this they appear much larger than if they were stripped of these extraneous rays, for the visual angle is determined not by the primary body of the star but by the widely surrounding brilliance. … The spyglass ... takes away the borrowed and accidental brightness from the stars and thereupon it enlarges their simple globes (if indeed their figures are globular), and therefore they appear increased by a much smaller ratio.[88]

Thus, Galileo's argument was that large objects, like the moon, unlike small bright objects like stars, does not show irradiation, because their images match the size of the aperture of the eye, and, consequently, as Leonardo, Galileo argued that irradiation is not seen, because of the primary reflection of the image of the object on the cornea of the eye.[89] However, as to the

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[87] 'Adnotatione quoque dignum videtur esse discrimen inter Planetarum atque fixarum Stellarum aspectus. Planetae enim globulos suos exacte rotundos ac circinatos obiiciunt, ac, veluti Lunulae quaedam undique lumine perfusae, orbiculares apparent: fixae vero Stellae peripheria circulari nequaquam terminatae conspiciuntur, sed veluti fulgores quidam radios circumcirca vibrantes atque admodum scintillantes'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 76, translation in Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 58.

[88] 'Ac primo illud animadversione dignum est, quod scilicet Stellae, tam fixae, quam errabundae, dum adhibito Perspicillo spectantur, nequaquam magnitudine augeri videntur iuxta proportionem eandem, secundum quam obiecta reliqua, et ipsamet quoque Luna, acquirunt incrementa: verum in Stellis talis auctio longe minor apparent; adeo ut Perspicillum, quod reliqua obiecta secundum centuplam, gratia exempli, rationem multiplicare potens erit, vix secundum quadruplam aut quintuplam Stellas multiplices reddere credeas. Ratio autem huius est, quod scilicet Astra, dum libera ac naturali oculorum acie spectantur, non secundum suam simplicem nudamque, ut ita dicam, magnitudinem sese nobis offerunt, sed fulgoribus quibusdam irradiata, micantibusque radiis crinita, idque potissimum cum iam increverit nox; ex quo longe maiores videntur, quam si ascititiis illis crinibus essent exuta: angulus enim visorius, non a primario Stellae corpuscolo, sed a late circumfuso splendore, terminatur. … Perspicillum … adscititios accidentalesque a Stellis fulgores adimit, illarum inde globulos simplices (si tamen figura fuerint globosa) auget; atque adeo secundum minorem multiplicatem adaucta videntur'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, pp. 75-6, translation in Ibid., pp. 57-8.

[89] See Brown, Harold I. 'Galileo on the Telescope and the Eye.' *Journal of the History of Ideas* 46 (1985): 487-501.

problem of celestial light, his telescopic observations of the 'Sidereus Nuncius' did not allow any definite conclusions. In his 'Dissertatio cum Nuncio Sidereo', Kepler commented that from the difference observed by Galileo between the planets and the stars nothing else could be concluded than that stars shine with their own light, while the planets only reflect light.[90] However, that Kepler later had to admit to Galileo, as already shown, that, notwithstanding his comment in the 'Dissertatio cum Nuncio Sidereo', he thought that at least Venus had some light of its own, shows that no definite conclusions could be reached in March 1610. The question of celestial light was no doubt on Galileo's mind when writing in the 'Sidereus Nuncius' that the planets appear 'as little moons'. Since he had just stressed that the moon only reflects light, it would have been very unwise to suggest that the planets and the moon are similar as concerns the problem of celestial light, without jeopardizing his own arguments about the moon, unless he was convinced that also the planets only reflect light. However, Galileo never explicitly said so. He was much less inclined than Kepler to jump forward to conclusions, because of two reasons. First, from his caveat on the fixed stars in the 'Sidereus Nuncius', 'if indeed their figures are globular', it is evident that he played with the thought that his telescope would eventually be able to resolve the shape of the fixed stars, and, consequently, that he still held to the idea of his 'Considerations of Alimberto Mauri' that the fixed stars also reflect light. Second, in March 1610, Galileo had not been able to observe the globular shapes of all the planets. From his acquaintance with the problem of celestial light, he would have considered the possibility of observing the phases of Venus the most important element of such an argumentation that would show beyond doubt that planets do not have light of their own. Thus, the phases of Venus were as important to solving the problem of celestial light as to the development of an argument in favor of Copernicanism.

In July 1610, Galileo was able to resolve the shape of Saturn. In a letter of 30 July to Belisario Vinta, the secretary of the Grand Duke of Tuscany, he announced that 'the star of Saturn is not a single star, but is a composite of three, which almost touch each other, never change or move relative to each other, and are arranged in a row along the zodiac, the middle one being three times larger than the two lateral ones, and they are situated in this form oOo'.[91] After the discovery of four satellites of Jupiter in January 1610, already published in the 'Sidereus Nuncius', Galileo thought that he had discovered two satellites of Saturn. However, different from the satellites of Jupiter, the companions of Saturn did not appear to move. However, that Galileo was able to resolve the shape of Saturn brought him again one step closer to the confirmation that the planets only reflect solar light. Resolving Saturn's shape was not an easy task. Contemporary reports show that telescopes of lesser quality than Galileo's show Saturn to have an oblong shape.[92] Moreover, a painting of Rubens of around 1636 shows that to his contempories it was not always easy to grasp Galileo's argument that resolving the globe-like

[90] Kepler, Dissertatio cum Nuncio Sidereo, in Galileo, *Opere*, Vol. 3.1, p. 118, translated in Rosen, Edward. *Kepler's Conversation with Galileo's Sidereal Messenger.* Translated by Edward Rosen. Vol. 5, *The Sources of Science*. New York London: Johnson Reprint Corporation, 1965, p. 34.

[91] 'la stella di Saturno non è una sola, ma un composto di 3, le quali quasi si toccano, nè mai tra di loro si muovono o mutano; et sono poste in fila secondo la lunghezza del zodiaco, essendo qualle di mezzo circa 3 volte maggiore delle altre 2 laterali: et stanno in questa forma oOo'. Galileo to Belisario Vinta, 30 July 1610, in Galileo, *Opere*, Vol. 10, p. 410. For translation of this passage and Galileo's discovery of the companions of Saturn, see Van Helden, Albert. 'Saturn and His Anses.' *Journal for the History of Astronomy* 5 (1974): 105-21, p. 105, in particular, pp. 105-11. For the discovery that the Galilean companions were the ring of Saturn, see Van Helden, Albert. ''Annulo Cingitur': The Solution of the Problem of Saturn.' *Journal for the History of Astronomy* 5 (1974): 155-74.

[92] See Galileo to Giuliano de' Medici, 13 November 1610, in Galileo, *Opere,* Vol. 10, p. 474.

appearance of a planet in a telescope was evidence that planets only reflect light.[93] The painting shows the god Saturn devouring his own child to safeguarde his dominion. (Figure 7.7)

Figure 7.7

[93] For discussion of this painting and the identification of the planet Saturn, see Baudouin, Frans. 'Saturnus.' In *De Vlaamse Schilderkunst in het Prado*, edited by A. Balis, M. Díaz Padrón, C. Van de Velde and H. Vlieghe. Antwerpen: Mercatorfonds, 1989, p. 178; Díaz Padrón, Matías. *El Siglo de Rubens en el Museo del Prado: Catalogo Razonado de Pintura Flamenca del Siglo XVII*. Madrid: Editorial Prensa Ibérica Museo del Prado, 1995, pp. 941-2.

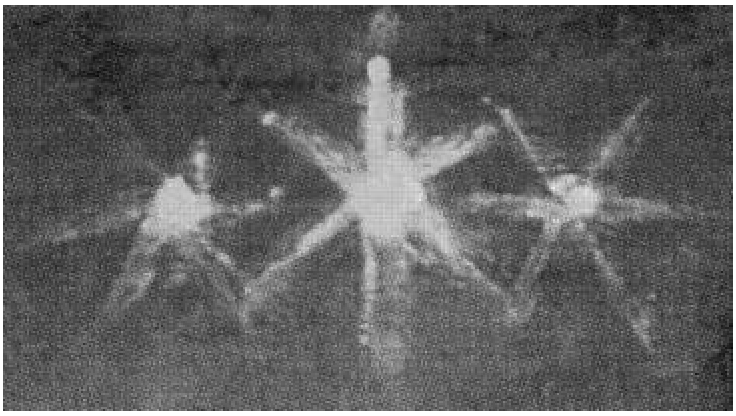


Figure 7.8

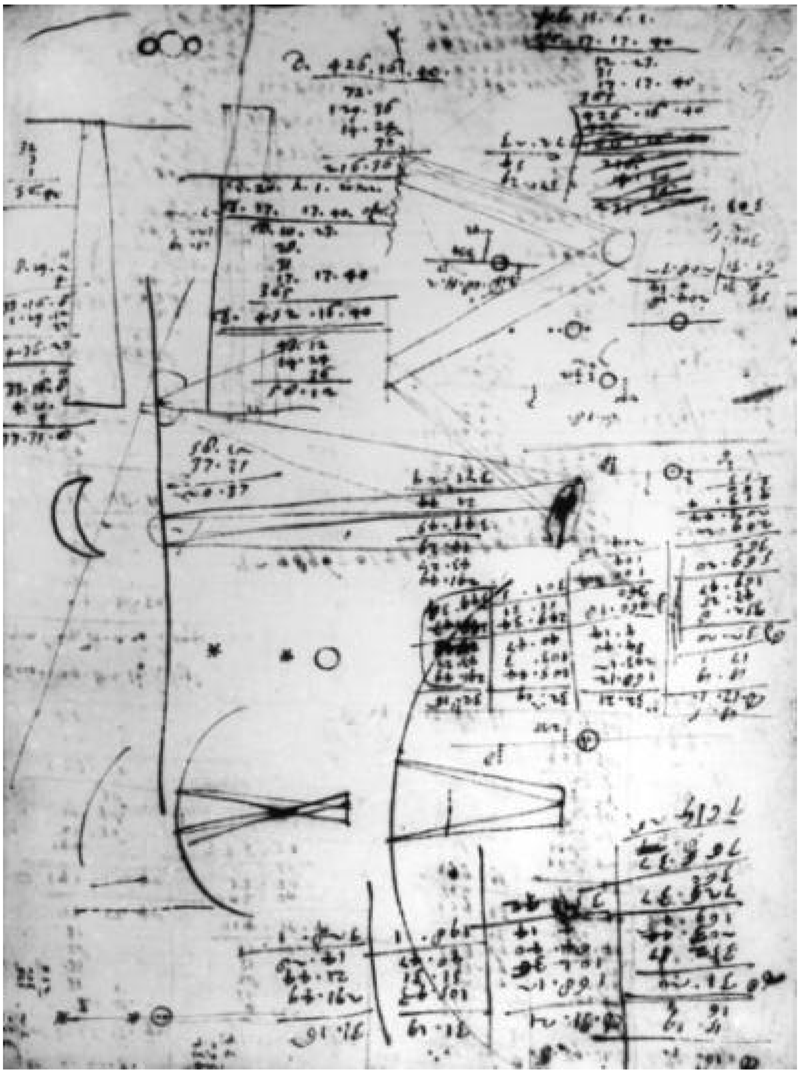


Figure 7.9

Above the god Saturn, Rubens, who might have met Galileo as early as 1605, shows the planet Saturn with its two Galilean companions.[94] (Figure 7.8) In an impossible combination with the resolution of the ‘satellites’ of Saturn, Rubens shows both Saturn and its two Galilean companions with the brilliant rays, that Galileo associated with the irradiation of planets and stars when observed without artificially narrowing the aperture by a pinhole or a telescopic diaphragm.
Saturn and its companions are shown on the top of a folio among Galileo’s notes on the calculations of periods of satellites of Jupiter. (Figure 7.9) Favaro has not published the folio, presumably because he considered the other drawings, above which Galileo has made calculations, of little importance.[95] However, these drawings show that around the same time in the summer of 1610 that he discovered Saturn’s companions, he returned to his study of the reflection of light, that he had undertaken in 1607. On the left, Galileo drew the crescent of the moon. To the right, there is a drawing that appears to be a study of angles of reflection, and above it, there is a drawing of the reflection of the sun on a smooth and a rough surface. Galileo’s drawings are reminiscent of Leonardo’s study of reflection from rough and smooth surfaces that Galileo had been involved with in 1607, and that he during the period following the publication of the ‘Sidereus Nuncius’ appears to have taken up again to take the format of the First Day of the ‘Dialogo’, where he had his spokesman Salviati asking the imaginary Aristotelian Simplicio ‘tell me, Simplicio; if you had to paint a picture of that wall with the mirror hanging on it, where would you use the darkest colors? In depicting the wall or the mirror?’, and his response.[96]

> As to vividness, you see that the reflection of that little flat mirror, where it is thrown there under the balcony, shines strongly; and the rest of the wall, which receives a reflection from the wall to which the mirror is attached, is not lighted up to any great extent (as is the small part struck by the reflection of the mirror). If you wish to understand the whole matter, consider how the surface of this rough wall is composed of countless very small surfaces placed in an innumerable diversity of slopes, among which of necessity many happen to be arranged so as to send the rays they reflect to one place, and many others to another. In short, there is no place whatever which does not receive a multitude of rays reflected from very many little surfaces dispersed over the whole surface of the rough body upon which the luminous rays fall. From all this it necessarily follows that reflected rays fall upon every part of any surface opposite that which receives the primary incident rays, and it is accordingly illuminated.[97]

[94] On the meeting of Rubens and Galileo around 1605, see Reeves, *Painting the Heavens: Art and Science in the Age of Galileo*, pp. 68-76. See also Huemer, Frances. ‘Rubens and Galileo 1604: Nature, Art and Poetry.’ In *Wallraf-Richartz-Jahrbuch Wesdeutsches Jahrbuch FürKunstgeschichte*, 175-96. Köln: Dumont Buchverlag, 1983; Gerstenberg, Kurt. ‘Rubens im Kreise seiner Römischen Gefährten.’ *Zeitschrift für Kunstgeschichte* 1 (1932): 99-109; Friedlaender, Walter. ‘Early to Full Baroque: Cigoli and Rubens.’ In *Studien zur Toskanischen Kunst: Festschrift für Ludwig Heinrich Heydenreich*, 65-82. Munich: Prestel-Verlag, 1964.

[95] Biblioteca Nazionale (Florence), Gal. 50, f. 64.

[96] ‘Ditemi ora qual vi si rappresenta più chiara: quella del muro e quella del specchio?’ Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, p. 96, translation in Galileo, *Dialogue Concerning the Two Chief World Systems*, p. 72.

[97] ‘quanto alla vivezza, voi vedete che la reflessione di quello specchietto piano, dove ella ferisce là sotto la loggia, illumina gagliardamente, ed il restante della parete, che riceve la reflession del muro, dove è attaccato lo specchio, non è gran segno illuminato coma la piccola parte dove arriva il reflesso dello specchio. E se voi desiderate intender l’ intero di questo negozio, considerate come l’ esser la superficie di quel muro aspra, è l’ istesso che l’ esser composta di innumerabili superficie piccolissime, disposte secondo innumerabili diversità di inclinazioni, tra le quali di necessità accade che ne sieno molte disposte a mandare i raggi, reflessi da loro, in un tal luogo, molte altre in altro; ed in somma non è luogo alcuno al quale non arrivino moltissimi raggi reflessi da moltissime supoerficiette sparse per tutta l’ intera superficie del corpo scabroso, sopra il quale cascano i raggi luminosi: dal che segue di necessità che

Consequently, after the publication of the 'Sidereus Nuncius', Galileo returned to his earlier study of reflection, for the obvious interest to his ongoing study of celestial light. He first referred to these studies in print in his third letter on the sunspots of 1 December 1612 to defend his view on the secondary light of the moon against Scheiner, when he writes that he is 'certain, contrary to common opinion, that when the Moon is polished and smooth like a mirror, it would not only not reflect the light of the Sun, as it does, but it would stay absolutely invisible, as if it did not exist, what I will show in due time with clear demonstrations'.[98] These demonstrations, no doubt, were his observations of 1607 that he had taken up again over the past months.

The lower part of the folio shows two drawings that are at first sight puzzling, if they were made in the context of Galileo's study of celestial light. The drawings represent the reflection by concave mirrors, showing how they the rays cross in the point of inversion. Did Galileo entertain an explanation of the cause of irradiation similar to Leonardo, who, as has been shown, considered it to be the diffusive light reflected from the moisture built up by the eyelids, comparable to reflections of concave mirrors? Galileo's explanation of the cause of irradiation was indeed remarkably similar to Leonardo's. He referred to it in three different publications. First, in his 'Discorso delle Comete' (1619), written for Mario Guiducci, he refuted an explanation of irradiation as an effect of the air between the eye and the object, by arguing that 'this irradiation is not around the luminous object anyway, but is so close to us that if indeed it is not actually within our eyes, it is at least upon their surfaces; perhaps it is caused by the principal light from the object being refracted in that moisture which is always maintained upon the pupil of the eye by the eyelid'.[99] Second, in his 'Il Saggiatore' (1623), Galileo argued less hesitantly that the irradiation 'is produced by reflection of the primary rays in the moisture at the edges of the eyelids, and it extends over the convexity of the pupil'.[100] Third, in his 'Dialogo' (1632), Galileo argued that this irradiation both around the image of the sun in a mirror, concerning the light of the moon, or around a star, was caused by reflection from the moisture at the edges of the eyelids.

> First of all, that brilliance which you see so vividly on the mirror, and which seems to you to occupy such a large part of it, is not such a big piece. It is really very tiny, but its extreme brightness causes an adventitious irradiation of your eyes through the reflection made in the moisture at the edges of your eyelids, which

sopra qualsivoglia parte di qualunque superficie opposta a quella che riceve i raggi primarii incidenti, pervengano raggi reflessi, ed in conseguenza l' illuminazione'. Galileo, Dialogo sopra i due massimi sistemi del mondo, in Galileo, *Opere*, Vol. 7, p. 102, translation in Ibid., p. 77.

98 'ed io son molto ben sicuro, contro alla comune opinione, che quando la Luna fosse polita e tersa come uno specchio, ella non solamente non ci refletterebbe, come fa, il lume del Sole, ma ci resterebbe assolutamente invisibile, come se la non fosse al mondo; il che a suo luogo con chiare dimostrazioni farò manifesto'. Galileo, Istoria e dimostrazioni intorno alle macchie solari, in Galileo, *Opere*, Vol. 5, p. 222.

99 'tale irragiamento non è altrimenti intorno all' oggetto luminoso, ma è cosi vicino a noi, che, se non è dentro all' occhio nosro stesso, almeno è nella sua superficie, forse cagionato dal lume principal dell' oggetto rifratto in quella umidità che continuamente è sopra la pupilla dell' occhio mantenuta dalle palpebre'. Galileo, Discorso delle Comete, in Galileo, *Opere*, Vol. 6, p. 84, translation in Galileo Galilei, Horatio Grassi, Mario Guiducci, and Johann Kepler. *The Controversy on the Comets of 1618*, p. 47.

100 'il qual si produce per reflessione de' raggi primarii fatta nell' umidità de gli orli ed estremità delle palpebre, la qual reflessione si distende sopra 'l convesso della pupilla'. Galileo, Il Saggiatore, in Galileo, *Opere*, Vol. 6, p. 357, translation in Ibid., p. 319.

> extends over the pupils. It is like the little hat that seems to be seen around the flame of a candle at some distance; or you may want to compare it with the apparent rays around a star.[101]

Thus, Galileo attributed the cause of irradiation, just as Leonardo, to the reflections in the moisture at the edges of the eyelids. That Galileo never referred in print to this moisture as similar to a concave mirror, is most likely due to his consideration, as in 'Il saggiatore', that the moisture is not only at the edges, but extends over the whole pupil. However, Galileo must have considered the analogy with concave mirrors, and the drawings on the folio refer to such a study of concave mirrors in the context of an exploration of the cause of irradiation. If Galileo did not have the appropriate passages of Leonardo's notebooks on the cause of irradiation before him, it cannot be denied that he followed a remarkably similar path of argument, most likely triggered by clues in Leonardo's notebook, as for example concerning luster, to which he did have access.[102]

Conclusive evidence that planets only shine with reflected solar light came during the last months of 1610, when Galileo observed Venus. Between the beginning of October 1610 and the end of December 1610, Galileo saw Venus taking different shapes. By the end of December, Galileo saw the transition of Venus from a gibbous to a crescent phase, which is only reconcilable with a system in which Venus revolves around the sun.[103] Galileo immediately announced that he not only had visual evidence in favor of Copernicanism, but also as concerns the question of celestial light, to two correspondents, who had shown interest in the question, Kepler following up his claims in the 'Dissertatio cum Nuncio Sidereo' and Sarpi with which Galileo had collaborated on the question in 1607. In a letter of 1 January 1611, writing to Kepler via Giuliano de' Medici, the ambassador of the Tuscan court in Prague, Galileo claimed that he had 'a sensible and certain demonstration of two great questions, about which there haven been doubts among the greatest minds of the world'. These questions were, first, that 'all planets by their nature are opaque (to Mercury the same happens as to Venus), and, second, that 'Venus by ecissity revolves around the Sun, just as Mercury and all the other planets'.[104] Galileo had actually not been able to resolve the phases of Mercury, but he assumed that Mercury shows phases just as Venus. This is evident from his letter to Sarpi on 12 February 1611, in which he wrote that he was certain that 'we will

[101] 'E prima, quello splendore così vivo che voi vedete sopra lo specchio, e chi vi par che ne occupi assai buona parte, non è cosi grande a gran pezzo, anzi è piccolo assai assai; ma lua sua vivezza cagiona nell' occhio vostro, mediante la reflessione fatta nell' umido de gli orli delle palpebre, la quale si distende sopra la pupilla, una irradiazione avventizia, simile a quel capillizio che ci par di vedere intorno alla fiamella di una candela posta alquanto lontana, o vogliate assimigliarla allo splendore avventizio di una stella'. Galileo, Dialogo, in Galileo, *Opere*, Vol. 7, p. 101, translation in Galileo, *Dialogue Concerning the Two Chief World Systems,* p. 76.

[102] Leonardo's study of the cause of irradiation is not a part of the 'Trattato di Pittura' as it is known today. However, in the sixteenth century, a treatise of Leonardo, now apparently lost, was circulating of which Leonardo's study of the eye and irradiation might have been a part. See Strong, *Leonardo on the Eye*, pp. 233-5.

[103] Palmieri, Paolo. 'Galileo and the Discovery of the Phases of Venus', pp. 109-117; Gingerich, Owen. 'Galileo and the Phases of Venus.' *Journal for the History of Astronomy* 15 (1984): 98-104, correcting Westfall's dating, see Westfall, Richard S. 'Science and Patronage: Galileo and the Telescope.' *Isis* 76 (1985): 11-30.

[104] 'haviamo sensate e certa dimostrazione di due gran questioni, state sin qui dubbie tra' maggiori ingegni del mondo. L' una è, che i pianeti tutti sono di loro natura tenebrosi (accadendo anco a Mercurio l' istesso che a Venere): l' altra, che Venere necessariisimamente si volge intorno al sole, come Mercurio et tutti li altri pianeti'. Galileo to Giuliano de' Medici, 1 January 1611, in Galileo, *Opere*, Vol. 11, p. 12.

see Mercury make the same changes'.[105] Moreover, Galileo went beyond his letter to Kepler in claiming that the stars, unlike the planets, have their own light.

> Moreover, by these same appearances of Venus, we have been convinced that all the planets receive the light of the sun, being by their nature opaque. But above that I am, by necessary demonstration most certain that the fixed stars are by theirselves most lucid, nor need the irradiation of the sun.[106]

What was this necessary demonstration? As will become evident, Galileo's observations of Saturn allowing him to resolve the shape of this planet were most important to his argument. In his next letter to Giuliano de' Medici and Kepler, he returned to the question of celestial light.

> The principal basis of my argument is that from most cleary observing with my telescope that those planets that are nearer to us or to the Su n, receive a much greater splendor and more outspoken reflect this light towards us. Thus Mars, at its perigee, when it is nearest to us, is seen much brighter than Jupiter, though its size is less than Jupiter and one can hardly deprive it of the irradiation [irradiazione] that prevents us from observing its disk bounded and round, what not happens in Jupiter, which is seen perfectly circular. Next, due to its great distance, Saturn is seen precisily bounded, the large star in the middle as well as the two very small one on its side; and its light appears languid and blinding, without any irradiation that impedes to distinguish its three small bounded globes. Now, since we clearly see that the Sun greatly illuminates Mars when it is near, while the light from Jupiter is much more languid (notwithstanding that without the instrument it appears rather bright, what happens because of the size and the radiant whiteness of the star), and that of Saturn more languid and darkened, since the latter is much further, how should the fixed stars, incredibly further than Saturn, appear, if they received light from the Sun? Certainly, very weak, dark, and pallid. Yet the opposite is true.[107]

Consequently, Galileo's argument was that irradiation is dependent upon brightness. Any source of light causes irradiation, but the amount of irradiation is depended on the amount of light, affected by the distance between light source and illuminated body, in the case of reflection, and the distance between the observer and the illuminated body. Consequently the farther a planet is from the sun, the less bright it appears and the less irradiation occurs. Thus, the resolving of the shape of Saturn was most important to Galileo's argument, because it was the planet most distant

[105] 'E l'istesse mutazioni son sicuro che vedremo fare a Mercurio'. Galileo to Paolo Sarpi, 12 February 1611, in Galileo, *Opere*, Vol. 11, p. 48.

[106] 'Sia mo in oltre da queste medesime apparizioni di Venere fatti certi come i pianeti tutti ricevono il lume dal sole, essendo per lor natura tenebrosi. Ma io di più sono, per dimostrazione necessaria, sicurissimo che le stelle fisse sono per sè medesime lucidis sime, nè hanno bisogno dell' irradiazione del sole'. Ibid., p. 49.

[107] 'Il principale fondamento del mio discorso è nell' osservare io molto evidentemente con l' occhiali, che quelli pianeti, di mano in mano che si trovano più vicini a noi a al sole, ricevono maggiore splendore, et più illustramente ce lo riverberano: et perciò Marte perigeo, et a noi vicinissimo, si vede assai inferiore; et difficilmente se gli può con l' occhiale levare quella irradiazione che impedisce il vedere il suo disco terminato et rotondo, il che in Giove non accade, vedendosi esquisitamente circolato: Saturno poi, per la sua gran lontananza, si vede essattamente terminato, si la stella maggiore di mezo come le due laterali piccolissime; et appare il suo lume languido et abacinato, senza niuna irradiazione che impedisca il distinguere i suoi 3 piccoli globi terminatissimi. Hora, poichè apertissimamente veggiamo che il sole molto splendidamente illustra Marte vicino, et che molto più languido è il lume di Giove (se bene senza lo strumento appare assai chiaro, il che accade per la grandezza et candore della stella), languidissimo et fosco quello di Saturno, come molto più lontano, quali doveriano apparirci le stelle fisse, lontano indicibilmente più di Saturno, quando il lume derivasse dal sole? Certamente debolissime, torbide e smorte. Ma tutto l' opposito si vede'. Galileo to Giuliano de' Medici, February 1611, in Galileo, *Opere*, Vol. 11, pp. 61-2.

of the sun. Since the stars are even further from the sun, his telescope would have been perfectly capable of resolving their globular shape, if they only reflected solar light and thus be little enlarged by irradiation because of the weakness of their light. Since his telescope did not allow resolving the shape of the stars, they must shine with their own light. Therefore, Galileo needed to reconsider the cause of the twinkling of the stars, not to be confused with irradiation, which he in his 'Considerations of Alimberto Mauri' had attributed to reflection of solar light by the stars.

> Thus I believe that we should philosophize correctly and assign the cause of the twinkling [scintillazione] of the fixed stars to the vibration of the splendour native to their intimate substance, whereas on the surface of the planets the light coming from the Sun terminates and is reflected.[108]

In 1611, irradiation became such a central concept for Galileo that he used it to reinterpret his earlier observations as concerns the light of the moon. In a letter to Grienberger, one of the astronomers of the Collegio Romano, he described the little bright spots in the darker part of the moon, which he had taken to be the illuminated summits of lunar mountains in his 'Sidereus Nuncius', as little stars. He argued that 'in the non-illuminated part of the moon, somewhat distant from the boundary of the light, some bright little parts appear in the form of stars, that grow piece by piece to join the boundary of the light which likewise goes toward them, when the moon is waxing; and as concerns the opposite, when the moon is waning, similar little stars are separated more and more, and finally they extinguish and are lost'.[109] With the description of the little bright spots as stars, Galileo intended to make clear that these little bright spots had the same properties of irradiation as stars. After reviewing almost verbatim his discussion of the light of the planets and the stars in his letter to Kepler, he again considered irradiation to be dependent upon brightness, but, more than in his letter to Kepler, he stressed the effect of contrast. He argued that 'also the surroundings highly alter these same effects: these same lucid bodies, surrounded by a dark field, are crowned by many and long rays; but situated in clear spaces, they are seen encircled by few and very little rays'.[110] Moreover, as concerns the little bright spots on the moon, he argued that 'many illuminated summits, that are very near to the boundary of the light, appear joined with it, although some are actually separated from it by some small piece of darkness interposed'.[111] Thus, he claimed that some bright spots in the dark part appear to be joined with the illuminated part, if not too far removed of this brighter part, due to irradiation.

However, with his reinterpretation of his lunar observations in function of irradiation, Galileo primarily aimed at the elaboration of an argument that the circumference of the moon is seen

[108] Et per tanto i stimo che bene filosoferemo referendo la causa della scintallazione delle stelle fisse al vibrare che elle fanno dello splendore proprio et nativo dall' intima loro sustanza, dove che nella superficie de i pianeti termina più presto et si finisce la illuminazione che dal sole deriva et si parte'. Ibid., p. 62.

[109] 'Et finalmente, dentro a la parte non illuminata di essa Luna, alquanto lontano dal termine della luce, appariscono in guisa di stelle alcune particelle illustrate, le quali crescendo appoco appoco si vanno a congiugnere col termine della luce, che parimente camina verso di quelle, quando però la Luna è crescente; et per l' opposito, nella decrescente simili stellete si separano più e più, et finalmente si estinguono e si perdono'. Galileo to Grienberger, 1 September 1611, in Galileo, *Opere*, Vol. 11, pp. 183-4.

[110] 'L' ambiente ancora altera grandissimamente questi medesimi effetti: imperò essi medesimi corpi lucidi, circondati da un campo tenebroso, di molti et lunghi raggi si incoronano; ma situati in spatii chiari, da pochi e piccolissimi raggi si veggono inghirlandati'. Ibid., p. 195.

[111] 'molte cuspidi illuminate, et vicinissime al termine della luce, apparischino ad esso congiunte, ben che per avventura siano veramente talvolta da quello separate per qualche angusta interpostione di tenebre'. Ibid., p. 198.

without irregularities, although the lunar mountains extend all the way to the visible circumference of the moon. As discussed in chapter 5, Galileo had previously given an explanation in the 'Sidereus Nuncius', based on a lunar atmosphere, and, in this same letter to Grienberger, another explanation, based on perspective and anamorphosis. He now turned to a third explanation on the basis of irradiation that he considered to be entirely convincing. Galileo argued that the moon also was seen with irradiation, although less than Venus, because the moon was more distant from the sun. However, surrounding the lunar circumference, it made any small irregularities that might be noted there as a consequence of illuminated summmits of mountains invisible. To prove his point, he proposed an experiment.[112] He set up a target, consisting of two slits in an iron plate, one with even boundaries, the other with uneven boundaries, in the circular shape of the lunar circumference. (Figure 7.10)

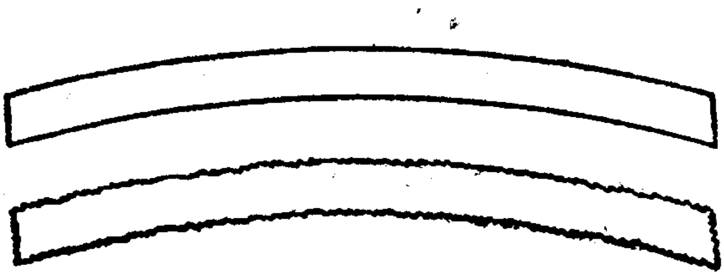


Figure 7.10

A light source is placed behind the slits in the plate. From nearby and with the naked eye, the difference between the two slits is seen. Further away, however, the uneven boundaries are no longer visible and both slits appear with even outlines, because of irradiation. However, with the telescope the difference is again visible. Finally, he argued for a critical distance, even further away, 1000 or 1500 braccia, from which even with the telescope the unevenness of the boundary of one of the slits is no longer visible. Galileo claimed the latter case pertinent to telescopic observation of the visible circumference of the moon.

In September 1611 Galileo's exploration of the question of celestial light came to an end. He had arrived at a theory of celestial light that was different from his initial theory in 1604. While in 1604, he considered the planets and the stars to reflect light, after his telescopic observations, he concluded that while planets reflect light, stars shine with their own light. However, Galileo's discovery pattern of a solution to the question of celestial light was much more complex than what the macro-level of revision of an initial theory by observation suggests. Galileo's exploration of light is not only of interest, because it shows that the development of the telescope itself, in particular, the application of an aperture stop to its objective lens, followed from Galileo's earlier investigations with mirrors in the context of his study of the light of the moon. Galileo's exploration of the question of celestial light is also of interest, because it suggests a much more complex trajectory of investigation than the theory- or observation/experiment-driven models of research that, as discussed in chapter 1, Galileo has been assumed by Koyré and Drake to use in the context of his research on mechanics. Galileo's exploration of the question of light, as it has been developed in this chapter, is summarized in the table below. Instead of the two

[112] Ibid., pp. 198-99.

categories of theory and observation, I believe that a distinction in five categories should be made. One category represents Galileo's theory of celestial light, in which a distinction is made between whether a theory was presented as conclusive or only as a probable clue for further research. A second category represents observation, but it needs to be stressed that these are observations with an instrument, and thus cannot be considerent as self-evident. Consequently, a third category represents the instrument itself. A fourth category concerns the appraisal or interpretations of observations. Finally, a fifth category presents Galileo's considerations on irradiation. As concerns chronology, the table is to be read from left to right and from the top to the bottom.

Galileo's Exploration of the Question of Celestial Light

Date	Theory of Celestial Light	Appraisal of Observations	Instrument	Observation with Instrument	Considerations on Irradiation
1604-1606	Moon, Planets, and Stars Reflect Light	Smooth Surface is Better Reflector than Rough Surface			
1607			Mirror	Reflection of Sunlight Spreads over the Mirror	Leonardo: Luster and Irradiation are an Effect of the Eye – Theory of the Eye shared by Galileo – Pinhole Shows Stars without Irradiation
				Solar Image in a Mirror is Small	
		Rough Surface is Better Reflector than Smooth Surface – Secondary Light of the Moon			
Summer 1609			Development of a Telescope with Higher Magnifications		
November 1609		Rough Surface of the Moon Reflects Light	Telescope		

January 1610			Telescope with Diaphragm	Planets show Globular Shape - Jupiter	Pinhole Shows Stars without Irradiation – No Irradiation when Magnified Image of the Telescope Matches Pupil
March 1610	*Planets and Stars Reflect Light, therefore the Shape of all Planets will be Globular and Venus will show Phases*				
July 1610				Saturn and its Companions	
		Reconsideration of the Light of the Moon	Mirror		Cause of Irradiation in Reflection from Moisture at the Edges of the Eyelids – Leonardo?
December 1610			Telescope with Diaphragm	Phases of Venus	
January 1611	Planets Reflect Light				
February 1611	Stars Shine with Their Own Light				Irradiation dependent on Brightness
September 1611		Reconsideration of the Light of the Moon			Irradiation dependent on Contrast

I would like to stress three conclusions that can be drawn from Galileo's exploration of the question of celestial light. First, Galileo's trajectory is non-linear. On two occasions, he returned to his investigations of 1607 on the light of the moon. In the summer of 1610, he took up again his investigations of the light of the moon, because of their renewed importance in the context of his ongoing research on the light of the planets and the stars. He again returned to the question of the light of the moon in September 1611, re-interpreting his earlier observations in function of irradiation, stressing that irradiation depends on contrast. Second, although the context of Galileo's research was the question of celestial light, sometimes the instrument and his considerations on irradiation assumed their own development, without having any direct bearing on the theoretical issue of celestial light. There are three examples of such an independent development. First, research with mirrors in 1607 was related with investigations of irradiation and these investigations allowed the application of the diaphragm to the telescope. Second, in the summer of 1610, Galileo investigated the cause of irradiation. Third, his reconsideration of the light of the moon in 1611 was meant to elaborate his thoughts on irradiation. A third conclusion is that the instrument caused its own dynamic. The telescope caused Galileo to take up again the question of the light of the moon, about which he had already reached definitive conclusions in 1607. More interestingly, while in 1606 Galileo argued that the stars and the planets both reflect light, he still held to that theory in March 1610. However, the availability of the instrument had caused him to be less certain about this theory, notwithstanding that his first telescopic observations confirmed his initial theory. In March 1610, Galileo thought more telescopic observations necessary to come to definitive conclusions on the light of the planets and the stars.

To conclude, as has been shown, the telescope was integrated in a broader network of research. On the one hand, the application of the diaphragm to the telescope was dependent on Galileo's earlier research on the question of celestial light. On the other hand, the telescope caused a new dynamic to Galileo's research on this question. In this context, Galileo developed an understanding of the optics of his telescope. As shown in the previous chapter, Galileo had found out why a telescope magnified and he had used it to improve the magnification of the telescope. As shown in this chapter, Galileo understood the second important element of his telescope, the diaphragm, in terms of irradiation. Consequently, his telescopic observations are as much a development of his understanding of irradiation as that they established visual evidence in favor of Copernicanism. However, he did not deliberately set up experiments in order to establish a new science of optics, as his understanding of the telescope only developed in the course of his astronomical observations. As discussed in chapter 3, mathematical practitioners involved in the design of optical instruments, like Ettore Ausonio, did not deliberately set up experiments, but they only worked towards the understanding of an optical instrument. The telescope of Galileo, mathematical practitioner, is no exception to this practice of mathematical practitioners.

VIII. Postscript: Big History, Small History

What I have suggested is that the problem of Galileo's optics at the micro-level and on the short term reflects a problem of the history of optics at the macro-level and on the long term. The failure to explain Galileo's development of the telescope mirrors the failure to assign the telescope its appropriate place in the history of optics. At the macro-level of the history of optics it has been argued that seventeenth century optics is the continuation of medieval optics, and that Kepler's 'Paralipomena', and by extension his 'Dioptrice', Kepler's opus magnum on the telescope, was written using the mathematical perspectivistic means of medieval optics but applied to the telescope. Therefore, there is no apparently good reason why anyone with mathematical skills similar to Kepler's, did not invent a telescope once the appropriate optical components became available. Consequently, the telescope, the real thing, has no place in the long-term history of optics. At the micro-level of Galileo, it has been argued that Galileo was not acquainted with optics. His development of the telescope, improvement of magnification and application of the aperture stop, has appeared unjustified unproblematical. Galileo's ignorance of optics however makes his development of the telescope unexplainable. Moreover, his attitude toward and style of research in optical matters then appears quite different from what we know of it as concerns his work on mechanics. I have argued that the problems of the big history of optics and the small history of Galileo's optics are resolved, if the optics of sixteenth century mathematical practitioners is analyzed on its own terms. Sixteenth century optics must be considered the representation of skill appropriated within a context provided by medieval optics.
As concerns the big history of optics, it has been shown that there was appropriation instead of continuity in the sixteenth century. Our example par excellence has been Ettore Ausonio. On the one hand, a mathematical practitioner and instrument designer like Ausonio was fully acquainted with medieval optics. On the other hand, his involvement with instrument design functioned as the looking glasses through which he saw medieval optics. As has been shown, he considered optics an instrumental, though not necessarily an experimental science. The practice that he represented in the notion of point of inversion in his work on concave spherical mirrors was not an experimental practice, but a representation of skill. It has been argued that this point of inversion was not present in medieval optics. Nevertheless, Ausonio did not see a contradiction between the point of inversion and image formation according to the cathetus rule taken from medieval optics. That no such contradiction was perceived explains why in his 'De Telescopio' Della Porta, as shown, well aware of the point of inversion, unsuccessfully tried to give an account of image formation in lenses and a Galilean telescope on the basis of the cathetus rule.
By recognizing the relevance of the point of inversion in sixteenth century optics for the invention of the telescope, the telescope is assigned a place in the history of optics. Notwithstanding the age-old idea of telescopic magnification and the availability of the appropriate optical components to make a telescope, no telescopes appear to have been made before the 1570s and 1580s. It has been argued that this delay of the invention of the telescope was due to the absence of the notion of point of inversion, which made for the first time a connection between the focal point, coming out of the rising sixteenth century tradition of burning mirrors, and vision or image formation in a concave mirror, and slightly later, in a convex lens. Bourne's discussion of Digges' reflecting telescope clearly showed the dependence of the telescope on the point of inversion. However, Digges' reflecting telescopes did not cause the take-off of the telescope, most likely because he was not able to control the magnification of

his instrument. As shown, the point of inversion suggested that magnification was dependent on the diameter of the convex lens, and not on the focal length of the objective lens.

While our primary focus has been on sixteenth century optics versus medieval optics, there might have been opened up some research questions regarding the instrumental approach of optics in the sixteenth century, the telescope and its relation to seventeenth century optics, in particular, as concerns Kepler. First, that as has been suggested Kepler adopted in his 'Dioptrice' the Galilean principle that the magnification of the telescope primarily depends upon the focal length of the objective lens and that seventeenth century practice of telescope-making concentrated on making objective lenses of progressively longer focal lengths shows that the common assumption about the interaction between instrument and optical theory, with optical theory always following instrumental practice in the seventeenth century, might need some refinement. Second, why didn't Kepler invent a telescope in his 'Paralipomena'? As shown, Kepler was well aware of the point of inversion, making references to the work of Della Porta. The tentative answer that follows from our considerations on the point of inversion is that where for example Digges and Bourne saw a practical opportunity in the point of inversion, Kepler saw a conceptual difficulty. His conceptual distinction between an image and a picture, depending on his concept of focus, in the 'Paralipomena' destroyed the point of inversion as it was known and used in the sixteenth century by Digges and Bourne to make a reflecting telescope.

As concerns Galileo's telescope, we have raised three questions in our introduction, (1) how did Galileo understand the telescope, (2) how did he improve its magnification, and (3) why did he apply a diaphragm to his objective lens? There might be more truth in Galileo's words in the 'Sidereus Nuncius' that he arrived at a duplication of the Dutch telescope 'on the basis of the science of refraction' than is usually assumed.[1] What Galileo had in mind was the science of refraction as found in Della Porta's 'De Refractione', which gave an analysis of the point of inversion of a convex lens and the insight that concave lenses make faraway things appear sharper. With the help of the rumors on the design of the Dutch telescope, he would have been able to duplicate the Dutch instrument and his acquaintance with sixteenth century optics would have made him understand the optical working of the telescope along the lines of the study of refraction in lenses in Della Porta' 'De Refractione'. It has been shown that Galileo was familiar with contemporaneous optics as appropriated by sixteenth century mathematical practitioners. His copy of Ausonio's 'Theorica', showing the point of inversion of a concave spherical mirror, is in this respect important evidence that he was familiar with the point of inversion.

As concerns our second question, it has been shown that Galileo did not improve the telescope by finding out that the magnification of his instrument was dependent upon the proportion of the focal lengths of the convex and the concave lenses. His familiarity with the point of inversion of a convex lens, and the absence of such a point for concave lenses in sixteenth century optics, made him consider magnification to be primarily dependent upon the convex lens. However, finding out which parameter of the convex lens determined magnification was not a self-evident task. As has been shown, the point of inversion in sixteenth century rather suggested that magnification was determined by the diameter of the convex lens, not by its focal length. However, Galileo found out the relationship between focal length and magnification, because it was embodied in a contemporaneous workshop procedure to test the curvature of lenses, which appears as a procedure to test the magnification of a telescope in Galileo's own 'Sidereus

[1] 'doctrinae de refractionibus innixus'. Galileo, Sidereus Nuncius, in Galileo, *Opere*, Vol. 3.1, p. 60. Translation in Galileo, *Sidereus Nuncius or the Sidereal Messenger*, p. 37.

Nuncius'. Moreover, the principle of angular magnification was completely within the boundaries of the Euclidean framework established by sixteenth century mathematical practitioners.

As to our third question why Galileo introduced the diaphragm, it has been shown that it was dependent upon Galileo's introduction of the telescope in an already established research context on the question of celestial light. That Galileo considered the optics of the telescope to be important to this question fits the pattern established by sixteenth century mathematical practitioners of considering optics capable of solving natural philosophical questions. It has been shown that Galileo did not consider the diaphragm to be the means to limit the optical aberrations of his lenses, but that he instead connected it with irradiation. Moreover, his first investigations of irradiation occurred in a context of research on celestial light with mirrors prior to his involvement with the telescope, but subsequently transferred to the telescope. His knowledge of the diaphragm and irradiation appears to have been to some extent borrowed from Leonardo's notebooks. Although no doubt Galileo's pathway of investigation on celestial light could be more refined, if we would have the appropriate manuscript sources, from what we can reconstruct, it is clearly shown that he followed an 'experimental trajectory' which appears to be very similar to the research patterns he is known to have used in other contexts of investigation. However, an important difference with his mechanics and dynamics, is that he never used the instrument to establish an optical theory. His 'experimental' work stayed strictly within the boundaries of the development of an instrument, and as such Galileo's optics never transcended the optics of a mathematical practitioner, who designs an instrument without deliberately experimenting. The sixteenth century mathematical practitioners considered Archimedes an ingenious instrument designer who had invented the burning mirror. Galileo became the Archimedes of the telescope.

Appendix I:
Ausonio, Fragments

1. Ausonio di una nuova invenzione d' uno specchio, B. A. M., A 71 Inf., ff. 20r-21v

/20r/ Questo specchio rappresenta prima l' imagine delle cose in cinque luoghi succesivamente distinti: cioe di dietro il specchio, nella superficie del specchio, tra lo specchio e la vera forma di cui e l' imagine, nel proprio luogo della vera forma, e quello che è sopramodo maraviglioso rappresenta l' imagine dietro la vera forma: talmente che se io mi affacierò allo specchio in una certa misura l' imagine mia mi anderà dopo le spalle, et io non mi vederò nello specchio: ma quelli che saranno dietro à me vederanno la mia faccia, che li andera à ritrovare, e s' io parlero piano nello specchio: quella mia imagine apparerà che parli; si che quelli che saranno da dietro lontani un buon passo mi udiranno à parlare: e quelli che non vederanno la mia imagine se ben fussero piu dappresso dietro à me non m' intenderanno. Queste cinque imagini se vedeno bene con una candela la quale quando e buon sole si puo accendere per virtù dello specchio, e con essa poi si fanno molte rappresentationi, cioe di queste cinque imagini, e di far sentire che la imagine del lume esce piu di dui passa lontana e scalda si che ognuno la puo sentire con il lume acceso ancora si puo far vedere la notte piu di sei passa lontano ponendo la candela appresso lo specchio in un certo ponto, e la imagine del lume va lontano circa sette passa, e nella oscurità della notte fa si gran luce che si puo discernere benissimo. Questa estate le persone dormivano con le fenestre aperte senza lume per non essere vedute, et io pigliava, il mio specchio, e con una candela mandava il lume nella camera delli vicini, e vedeva tutta la camera, echi gli era dentro come se fusse stato di giorno perche dove è pui oscuro fa miglior vista. Ancora mi son ritrovato io stare nell' oscuro la notte et essendo nel vicinato un lume acceso, che io poteva vedere posi d' incontro quel lume il specchio, e dove fa un splendore posi la carta scritta e lessi la lettera benissimo a parte à parte. Se con un pugnale si da negli occhi delle imagini: la imagine del pugnale esce, et va à ritrovar il vero occhio, eli fa qualche risentimento, si che bisogna, tirarsi a dietro per paura. Acconcio una carta scritta in un certo modo /20v/ latino, e volgare, e la mostro al specchio in una distanza, e mi fa vedere una di quelle cose scritte: poniamo che sia la latina; muto la distanza, e dispaiono le lettere latine, e si accadono le volgari italiane.

Ancora scrivo in una carta, e pongo la parte scritta verso me, e l' altra verso il specchio; nondimeno le lettere comparevano benissimo nello specchio.

Al sole questo specchio fa un splendore lontano , che faria discernere ogni minutezza e posta al specchio una carta con lettere tagliate potemo scrivere sopra un muro qualche cosa.

Questo specchio accende il fuoco in qualsivoglia cosa: si che fonda il piombo et abbrugia le pietre, e fa varie cose sopra questa virtù di riscaldare con gran forza et abbrugiare. Si puo con un poco di tempo mostrare con il specchio la causa del flusso, e reflusso del mare: ponendo in uno canale lungo del' acqua, et accommodando lo specchio al sole in modo che il punto, che abbrugia dia nell''acqua et in quel ponto ponendo delle pagliuzze, e lasciando cosi per un pezzo l' acqua in quel ponto incominciare di estuare, edi gonfiarsi e manderà le pagliuzze lontane dal punto riscaldato, e da poi movendosi il sole alla contraria parte la sera, poneremo lo specchio dell' altra parte, e riscaldaremo il luogo dove saranno ite le pagliuzze et ivi l' acqua ritornerà ad estuare e gonfiarsi e le pagliuzze ritorneranno dove erano la matina, e da questa apparenze si puo intendere qualmente li lumi del cielo con il sito delli fondi del mare, e delli liti, montagne, e simil cose atte à far riflesso, et aggregationi de raggi possono in diversi luoghi del mare fare questi ponti, che per il caldo si gonfiano, e fanno il flusso, et il reflusso delli mari con tanta diversità quanta si vede. Il specchio dunque puo servire, e per delettatione nel mostrare varie mostruosità e per modello di fare cose grandissime nella guerra, et in diverse ordino, et anco per potere dalla isperienza filosofare e ritrovare la vera causa di molte cose, che per ignoranza della scienza di perspettiva li studiosi delli nostri tempi sapore /21r/ non possano. Queste sono parte delle virtù di questo specchio; ma non tacerò queste due altre che sono grandi, e belle. Se l' huomo si serrerà in una camera all' oscuro, e farà che da un solo bucolino entri il lume del sole e sopra di questo bucolino vi accomodi una carta bianca, accommodando lo specchio con certa misura dravanzi lo bucolino egli rappresentarà nella carta una vaga, e polita pittura minutissima che ponerà in modo di perspettiva tutto quello, che di fuori si potria vedere e se vi saranno huomini, overo animali, che si movino nella pittura le imagini si moveranno con grandissima nostra delettatione. La grandezza dello specchio essendo accommodata ad altro uso, fa che questa vaga apparenza non puo essere ben compresa, se non da un solo: ma perche la causa e l' aggregatione delli raggi, con uno retto si puo fare questa operatione piu godevole, et anco con uno di questi specchi concavi piu piccioli.

Cosa maravigliosa è per questa via che nella carta compare il sole, et il cielo con le nuvoli, e quando il sole s' eclissa nel cielo anco nella carta si puo vedere le eclissi senza impedimento della vista. Ma bisogna notare, che se nella carta le eclisse appare da una parte nel cielo egli si fa nella contraria, perche il specchio mostra allo roverscio la cosa in uno certo sito, e questo è il secondo miracolo, che per questo specchio si deve annotare.

Se in una sala vi saranno vinti persone lontane dallo specchio dui o tre passa nello specchio si vederanno tutti alla roverscia, ma se porremo poi l' occhio nostro in uno punto non potremo vedere se non un occhio solamente, e se passaremo quel punto verso il specchio non si vedrà altro che una faccia grande alla dritta, come di gigante, e le donne che si pelano le ciglia e la barba sogliono adoperarlo perche dimostra li peli separati come spini.

Ancora con questo specchio si puo vedere chiaramente la causa dell' estate e dello verno e delle stagioni di mezo accommodandolo in diverso modo verso il sole secondo le altezze meridiane del sole, nel verno nella primavera, e nella estate perche si sente la varietà delli calori.

/21v/ Ma accioche sappiate come questo specchio operi tenerete il specchio al sole è nella camera farete fare un poco di profumo, e per questo fumo apparirà chiaramente come li raggi del sole percotendo nel specchio fanno una piramide nel sommo della quale si vede una linea luminosa, che puo dar segno della apparenza della via lattea, causata per simile aggregatione, et in quel punto abbrugia, e fa le cose, e havremo detto, fuori di quel punto le imagini appiano riverscie, dentro di quel punto si vedono diritte, secondo che Euclide nella sua optica, e catoptrica con vive ragioni ha dimostrato, e come per l' esperienza confirmare si puo.

Di tutte queste cose havemo le sue ragioni demostrative che rendono le cause di tutte predette apparenze ma perche saria cosa di maggior speculatione basterà saperne gli effetti per adesso come vi ho detto.

2. Per vedere in una camera tutto quello che si fà da lontano in una piazza o altro luogo per forza di specchio et prospettiva, B. A. M., G 120 Inf., ff. 74r-75v

In prima bisogna ellegere una torre o campanile alto che sia tanto appresso la piazza che si hà da vedere che stando uno alla sommità di quella possi vedere una torre lontana da quella e che sia al verso della camera dove alla fine si vole vedere. Anchora l'istesso possi vedere tutta la piazza dove si ha da fare detta fusta o bagordo. In quel luogo si deve mettere uno speglio piano et grande molto bene visibile. Il sito del quale deve essere acconcio come si dirà di sotto.

Modo di acconciar il primo specchio

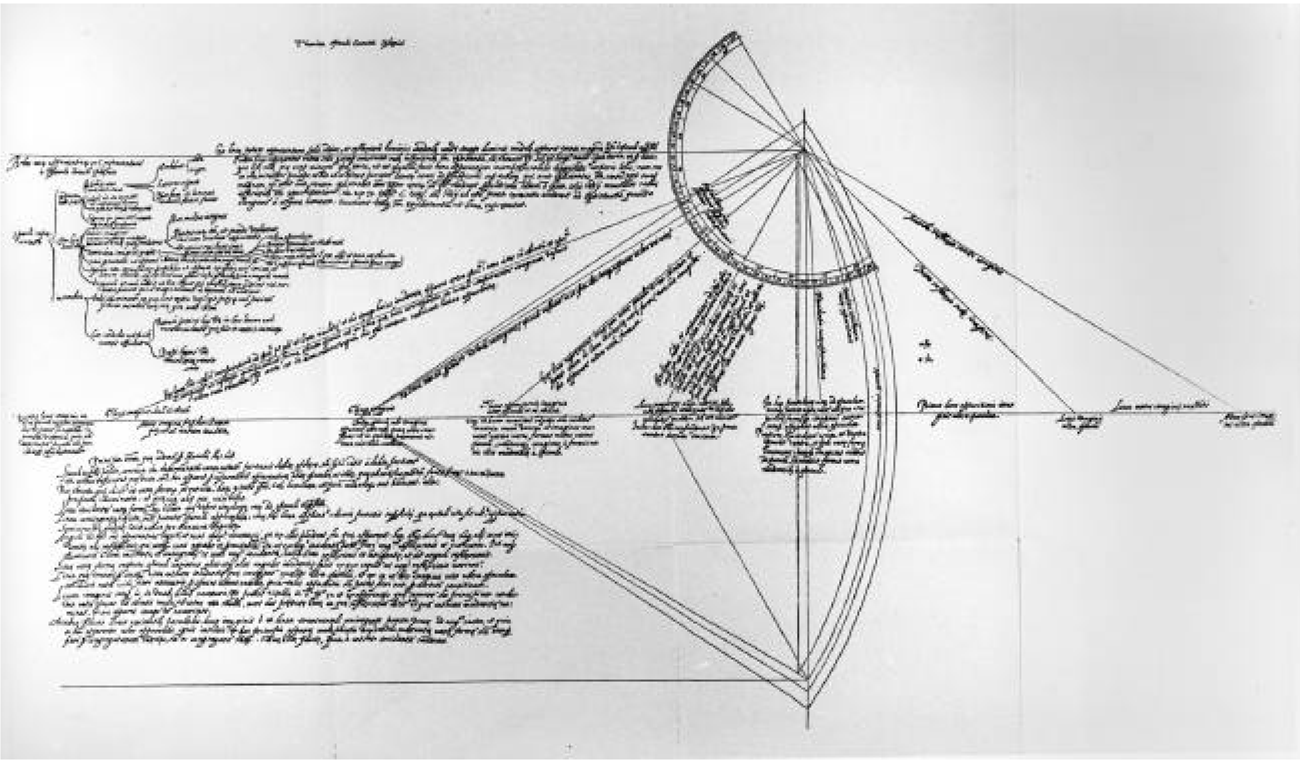
Habbi uno Astrolabio con il quale misurrai prima l'angolo che per il profondo può venire in quella piaccia, et però prima si pigli il minor sensibile da vedere le cose che si faranno a piede la torre. Da poi retta il maggior sensibile et questo quanto alla profondità come nel essempio si vede. AB sarà il minore, AC il maggior. Similmente con la chorda et squadra angolare pigliorai li angoli laterali maggiori, cioè Ad et Ae et li uguali angoli li cognoscerai sopra l'Astrolabio. Questi angoli a croce retta cognosciuti noterai a parte.

Et all'hora torrai la misura del'angolo che fa la predetta croce con l'altra torre che si può vedere dalla prima perche overo ella sarà più alta o più bassa o uguale: quando ancho la distantia della prima torre alla camera fusse picciola siche si potesse discorrere il speglio et le imagini non accadri usar mezzano torre, ma si vorria solamente gli angoli alla camera.

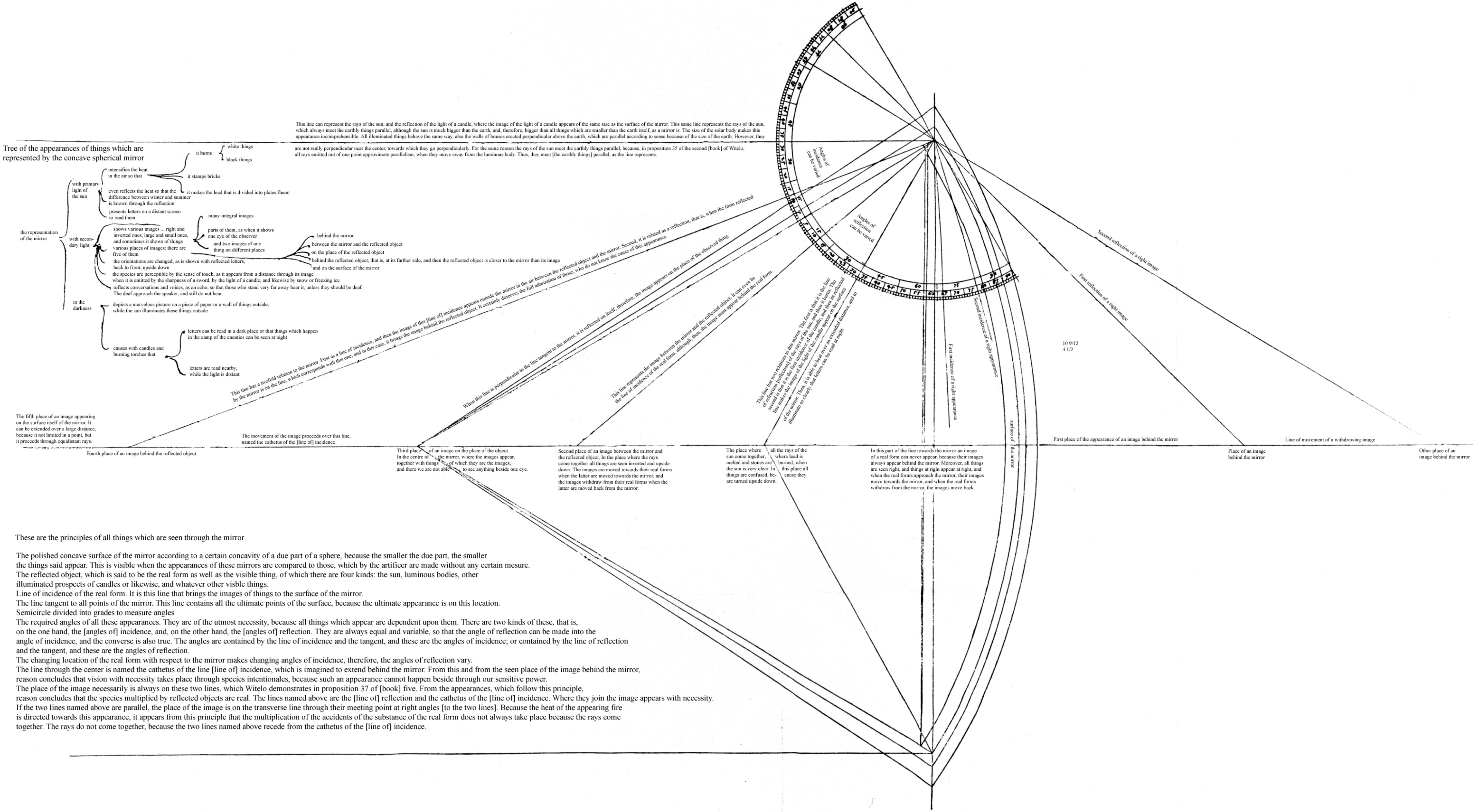
Il modo di pigliar gli angoli da torre a torre, o da la torre alla camera: é questo sopra la prima torre metterai duo chorde a croce, l'una ad archipendolo, l'altra ad angoli retti per il traverso di modo che possi far l'angolo con tutta la piazza per il largo et possi medesimamente far angolo con l'altra torre; et quando la torre seconda elletta non [ricovesse] tal sito ellegene una che facci angolo con quella chorda: Misura dunque dal ponto della incrociaciono da tanta chorda quanto può essere il speglio largo et alto, li angoli che vengono da torre a torre et fanne comparatione agli angoli predetti della profondità tolti da principio et fa la torre 2.a havesse luogo alto come dalla prima quanto è l'angolo della profondità della piazza. Le sue chorde a croce [stiano] bene quanto all'altezza et fa li angoli [aresti] della larghezza della piazza corrisponderanno alli angoli della larghezza della seconda torre starà similmente bene. Ma nota che quando la piazza sia /75v/ maggiore di speglio si havera bisogno di più specchi, et più fatica, per che il speglio piglia tanto quanto può far l'istesso angolo et non più essendo piano ma fagli è convesso. Ricoverrai il tutto.

Appendix II:
Transcription and Translation
Ausonio's 'Theorica Speculi Concavi Sphaerici'

Biblioteca Nazionale (Florence), Gal. 83, f. 4r



Theory of the concave spherical mirror



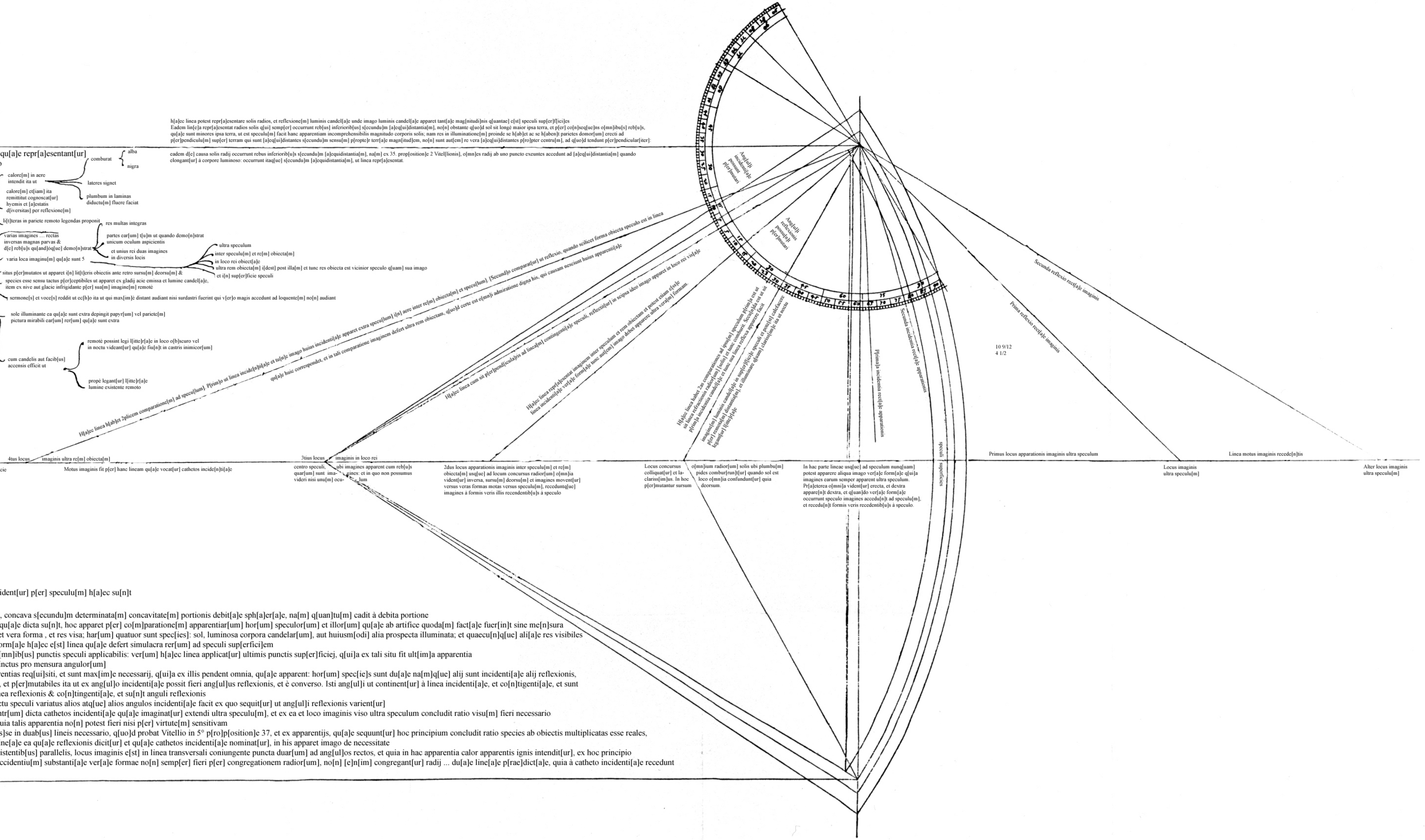
These are the principles of all things which are seen through the mirror

The polished concave surface of the mirror according to a certain concavity of a due part of a sphere, because the smaller the due part, the smaller the things said appear. This is visible when the appearances of these mirrors are compared to those, which by the artificer are made without any certain mesure.
The reflected object, which is said to be the real form as well as the visible thing, of which there are four kinds: the sun, luminous bodies, other illuminated prospects of candles or likewise, and whatever other visible things.
Line of incidence of the real form. It is this line that brings the images of things to the surface of the mirror.
The line tangent to all points of the mirror. This line contains all the ultimate points of the surface, because the ultimate appearance is on this location.
Semicircle divided into grades to measure angles
The required angles of all these appearances. They are of the utmost necessity, because all things which appear are dependent upon them. There are two kinds of these, that is, on the one hand, the [angles of] incidence, and, on the other hand, the [angles of] reflection. They are always equal and variable, so that the angle of reflection can be made into the angle of incidence, and the converse is also true. The angles are contained by the line of incidence and the tangent, and these are the angles of incidence; or contained by the line of reflection and the tangent, and these are the angles of reflection.
The changing location of the real form with respect to the mirror makes changing angles of incidence, therefore, the angles of reflection vary.
The line through the center is named the cathetus of the line [line of] incidence, which is imagined to extend behind the mirror. From this and from the seen place of the image behind the mirror, reason concludes that vision with necessity takes place through species intentionales, because such an appearance cannot happen beside through our sensitive power.
The place of the image necessarily is always on these two lines, which Witelo demonstrates in proposition 37 of [book] five. From the appearances, which follow this principle, reason concludes that the species multiplied by reflected objects are real. The lines named above are the [line of] reflection and the cathetus of the [line of] incidence. Where they join the image appears with necessity.
If the two lines named above are parallel, the place of the image is on the transverse line through their meeting point at right angles [to the two lines]. Because the heat of the appearing fire is directed towards this appearance, it appears from this principle that the multiplication of the accidents of the substance of the real form does not always take place because the rays come together. The rays do not come together, because the two lines named above recede from the cathetus of the [line of] incidence.

Theorica speculi concavi sph[a]erici

Arbor ear[um]apparentiar[um] qu[a]e repr[a]esentant[ur] a speculo co[n]cavo sph[a]erico

speculi repr[a]esentatio
- cu[m] prima luce solis
  - calore[m] in aere intendit ita ut
    - comburat
      - alba
      - nigra
    - lateres signet
    - plumbum in laminas diductu[m] fluere faciat
  - calore[m] et[iam] ita remittitut cognoscat[ur] hyemis et [a]estatis d[iversitas] per reflexione[m]
  - li[t]teras in pariete remoto legendas proponit
- cum luce secu[n]da
  - varias imagines .... rectas inversas magnas parvas & d[e] reb[u]s qu[and]oq[ue] demo[n]strat
    - res multas integras
    - partes ear[um] t[u]m ut quando demo[n]strat unicum oculum aspicientis
    - et unius rei duas imagines in diversis locis
  - varia loca imaginu[m] qu[a]e sunt 5
    - ultra speculum
    - inter speculu[m] et re[m] obiecta[m]
    - in loco rei obiect[a]e
    - ultra rem obiecta[m] i[dest] post illa[m] et tunc res obiecta est vicinior speculo q[uam] sua imago
    - et i[n] sup[er]ficie speculi
  - situs p[er]mutatos ut apparet i[n] lit[t]eris obiectis ante retro sursu[m] deorsu[m] &
  - species esse sensu tactus p[er]ceptibiles ut apparet ex gladij acie emissa et lumine candel[a]e, item ex nive aut glacie infrigidante p[er] sua[m] imagine[m] remoté
  - sermone[s] et voce[s] reddit ut ec[h]o ita ut qui max[im]e distant audiant nisi surdastri fuerint qui v[er]o magis accedunt ad loquente[m] no[n] audiant
- in tenebris
  - sole illuminante ea qu[a]e sunt extra depingit papyr[um] vel pariete[m] pictura mirabili ear[um] rer[um] qu[a]e sunt extra
  - cum candelis aut facib[us] accensis efficit ut
    - remoté possint legi l[itte]r[a]e in loco o[b]scuro vel in noctu videant[ur] qu[a]e fiu[n]t in castris inimicor[um]
    - propè legant[ur] l[itte]r[a]e lumine existente remoto



Principia o[mn]ium qu[a]e vident[ur] p[er] speculu[m] h[a]ec su[n]t

Speculi sup[er]f[ici]es polita, concava s[ecundu]m determinata[m] concavitate[m] portionis debit[a]e sph[a]er[a]e, na[m] q[uan]tu[m] cadit à debita portione t[anto] minus rep[ra]esentat qu[a]e dicta su[n]t, hoc apparet p[er] co[m]paratione[m] apparentiar[um] hor[um] speculor[um] et illor[um] qu[a]e ab artifice quoda[m] fact[a]e fuer[in]t sine me[n]sura

Res obiecta qu[a]e dicit[ur] et vera forma , et res visa; har[um] quatuor sunt spec[ies]: sol, luminosa corpora candelar[um], aut huiusm[odi] alia prospecta illuminata; et quaecu[n]q[ue] ali[a]e res visibiles

Linea incidenti[a]e ver[a]e form[a]e h[a]ec e[st] linea qu[a]e defert simulacra rer[um] ad speculi sup[erfici]em

Linea contigenti[a]e dicta, o[mn]ib[us] punctis speculi applicabilis: ver[um] h[a]ec linea applicat[ur] ultimis punctis sup[er]ficiej, q[ui]a ex tali situ fit ult[im]a apparentia

Semicirculus gradib[u]s distinctus pro mensura angulor[um]

Anguli ad has o[mn]es apparentias req[ui]siti, et sunt max[im]e necessarij, q[ui]a ex illis pendent omnia, qu[a]e apparent: hor[um] spec[ie]s sunt du[a]e na[m]q[ue] alij sunt incidenti[a]e alij reflexionis, qui semp[er] sunt [a]equales, et p[er]mutabiles ita ut ex ang[ul]o incidenti[a]e possit fieri ang[ul]us reflexionis, et è converso. Isti ang[ul]i ut continent[ur] à linea incidenti[a]e, et co[n]tigenti[a]e, et sunt anguli incidenti[a]e, vellà linea reflexionis & co[n]tingenti[a]e, et su[n]t anguli reflexionis

Situs ver[a]e form[a]e respectu speculi variatus alios atq[ue] alios angulos incidenti[a]e facit ex quo sequit[ur] ut ang[ul]i reflexionis varient[ur]

Linea qu[a]e transit p[er] centr[um] dicta cathetos incidenti[a]e qu[a]e imaginat[ur] extendi ultra speculu[m], et ex ea et loco imaginis viso ultra speculum concludit ratio visu[m] fieri necessario p[er] species intentionales, quia talis apparentia no[n] potest fieri nisi p[er] virtute[m] sensitivam

Locum imaginis semp[er] [es]se in duab[us] lineis necessario, q[uo]d probat Vitellio in 5° p[ro]p[osition]e 37, et ex apparentijs, qu[a]e sequunt[ur] hoc principium concludit ratio species ab obiectis multiplicatas esse reales, sunt aut[em] p[ra]edict[a]e line[a]e ea qu[a]e reflexionis dicit[ur] et qu[a]e cathetos incidenti[a]e nominat[ur], in his apparet imago de necessitate

Duabus p[rae]dictis lineis existentib[us] parallelis, locus imaginis e[st] in linea transversali coniungente puncta duar[um] ad ang[ul]os rectos, et quia in hac apparentia calor apparentis ignis intendit[ur], ex hoc principio apparet multiplicatio[nem] accidentiu[m] substanti[a]e ver[a]e formae no[n] semp[er] fieri p[er] congregationem radior[um], no[n] [e]n[im] congregant[ur] radij ... du[a]e line[a]e p[rae]dict[a]e, quia à catheto incidenti[a]e recedunt

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